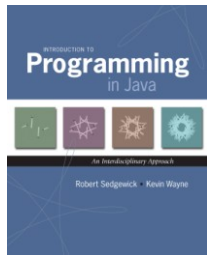


# Encryption



Introduction to Programming in Java: An Iterative Approach · Robert Sedgewick · Kevin Wayne · Copyright ©2002-2010 · 19 Feb 2012 19:24:23

## Hidden flaw jeopardizes millions of online transactions

Mathematicians discover weakness in commonly used encryption schemes

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By Paul Wagenseil



updated 2/15/2012 6:01:37 PM ET

Mathematicians based in Switzerland and the United States have discovered a small but the most commonly used digital encryption schemes, a flaw that could undermine the secure communication.

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### IT Blogwatch

A Daily Digest of IT Blogs from Richi Jennings  
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February 16, 2012 - 6:00 A.M.

## RSA crypto: 'flawed', 'risky', 'quagmire of vulnerabilities'

2 Comments

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TAGS: certificate, crypto, cryptography, Diffie-Hellman, encryption, key management, RSA, SSL  
 IT TOPICS: Applications, Cybercrime & Hacking, Financial IT, Government & Regulation, Internet, Privacy, Security, Security Hardware & Software

A group of six academic researchers have concluded that real-world RSA encryption keys are riskier than Diffie-Hellman-based ones. It seems that some of the random numbers used to generate them weren't arm, random. In [IT Blogwatch](#), bloggers wonder if the sky is falling.

Hardware Software Music & Media Networks Security Cloud Public Sector Business Science  
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## 'Predictably random' public keys can be cracked - crypto boffins Battling researchers argue over whether you should panic

By John Leyden · [Get more from this author](#)

Posted in Enterprise Security, 16th February 2012 13:38 GMT

**Analysis** Cryptography researchers have discovered flaws in the key generation that underpins the security of important cryptography protocols, including SSL.

## Secure Chat

Alice wants to send a secret message to Bob?

- Sometime in the past, they exchange a **one-time pad**.
- Alice uses the pad to encrypt the message.
- Bob uses the same pad to decrypt the message.



**Key point.** Without the pad, Eve cannot understand the message.



## Encryption Machine

**Goal.** Design a machine to encrypt and decrypt data.

S E N D M O N E Y

g x 7 6 w 3 v 7 k

S E N D M O N E Y

↓ encrypt

↓ decrypt

## Encryption Machine

Goal. Design a machine to encrypt and decrypt data.

S E N D M O N E Y

encrypt

g x 7 6 w 3 v 7 k

decrypt

S E N D M O N E Y

Enigma encryption machine.

- "Unbreakable" German code during WWII.
- Broken by Turing bombe.
- One of first uses of computers.
- Helped win Battle of Atlantic by locating U-boats.



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## A Digital World

Data is a sequence of bits. [bit = 0 or 1]

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

File formats. txt, pdf, java, exe, docx, pptx, jpeg, mp3, divx, ...



computer with a lens



computer with earbuds



computer with a radio

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## A Digital World

Data is a sequence of bits. [bit = 0 or 1]

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

File formats. txt, pdf, java, exe, docx, pptx, jpeg, mp3, divx, ...



computer with a cash dispenser

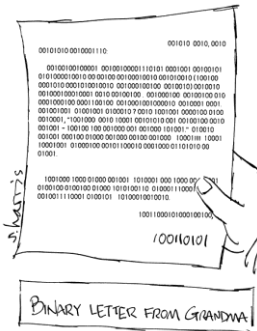


computer with a ballot box



computer with a heating element

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Copyright 2004, Sidney Harris, <http://www.sciencecartoonplus.com>

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## A Digital World

Data is a sequence of bits. [bit = 0 or 1]

- Text.
- Programs, executables.
- Documents, pictures, sounds, movies, ...

Base64 encoding. Use 6 bits to represent each alphanumeric symbol.

very weak type of encryption

Binary Char	Binary Char	Binary Char	Binary Char	Binary Char	Binary Char
000000	A	001011	L	010110	W
000001	B	001100	M	010111	X
000010	C	001101	N	011000	Y
000011	D	001110	O	011001	Z
000100	E	001111	P	011010	a
000101	F	010000	Q	011011	b
000110	G	010001	R	011010	c
000111	H	010010	S	011101	d
001000	I	010011	T	011110	e
001001	J	010100	U	011111	f
001010	K	010101	V	100000	g
				101011	h
				101100	i
				101101	j
				101110	k
				101111	l
				110000	m
				110001	n
				110010	o
				110011	p
				110100	q
				110101	r
				110110	s
				110111	t
				111000	u
				111001	v
				111010	w
				111011	x
				111100	y
				111101	z
				111110	+
				111111	/

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## One-Time Pad Encryption

Encryption.

- Convert text message to N bits.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
W	52	001110
...	...	...

S	E	N	D	M	O	N	E	Y
010010	000100	001101	000011	001100	001110	001101	000100	011000

message

base64

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### One-Time Pad Encryption

#### Encryption.

- Convert text message to N bits.
- Generate N random bits (one-time pad).

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits

### One-Time Pad Encryption

#### Encryption.

- Convert text message to N bits.
- Generate N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.

XOR Truth Table

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

sum corresponding pair of bits: 1 if sum is odd, 0 if even

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR

$0 \oplus 1 = 1$

### One-Time Pad Encryption

#### Encryption.

- Convert text message to N bits.
- Generate N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	x	7	6	w	3	v	7	k	encrypted

### Secure Chat (review)

Alice wants to send a secret message to Bob?

- Some time in the past, they exchange a one-time pad.
- Alice uses the pad to encrypt the message.
- Bob uses the same pad to decrypt the message.



Key point. Without the pad, Eve cannot understand the message.



### One-Time Pad Decryption

#### Decryption.

- Convert encrypted message to binary.

g	x	7	6	w	3	v	7	k	encrypted
---	---	---	---	---	---	---	---	---	-----------

### One-Time Pad Decryption

#### Decryption.

- Convert encrypted message to binary.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

g	x	7	6	w	3	v	7	k	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64

### One-Time Pad Decryption

#### Decryption.

- Convert encrypted message to binary.
- Use **same** N random bits (one-time pad).

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits

### One-Time Pad Decryption

#### Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
010010	000100	001101	000011	001101	001110	001101	000100	011000	XOR

1 ^ 1 = 0

### One-Time Pad Decryption

#### Decryption.

- Convert encrypted message to binary.
- Use same N random bits (one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
M	12	001100
...	...	...

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
010010	000100	001101	000011	001101	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

### Why Does It Work?

Crucial property. Decrypted message = original message.

Notation	Meaning
a	original message bit
b	one-time pad bit
^	XOR operator
a ^ b	encrypted message bit
(a ^ b) ^ b	decrypted message bit

#### Why is crucial property true?

- Use properties of XOR.
- $(a \wedge b) \wedge b = a \wedge (b \wedge b) = a \wedge 0 = a$

associativity of ^      always 0      identity

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

### One-Time Pad Decryption (with the wrong pad)

#### Decryption.

- Convert encrypted message to binary.

g	x	7	6	w	3	v	7	K	encrypted
---	---	---	---	---	---	---	---	---	-----------

### One-Time Pad Decryption (with the wrong pad)

#### Decryption.

- Convert encrypted message to binary.

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64

### One-Time Pad Decryption (with the wrong pad)

#### Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits

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### One-Time Pad Decryption (with the wrong pad)

#### Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR

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### One-Time Pad Decryption (with the wrong pad)

#### Decryption.

- Convert encrypted message to binary.
- Use **wrong** N bits (bogus one-time pad).
- Take bitwise XOR of two bitstrings.
- Convert back into text: **Oops.**

g	x	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR
I	L	O	V	E	O	K	R	A	wrong message

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### Goods and Bads of One-Time Pads

#### Good.

- Easily computed by hand.
- Very simple encryption/decryption processes.
- Provably unbreakable if bits are truly random. [Shannon, 1940s]

← eavesdropper Eve sees only random bits

#### Bad.

- Easily breakable if pad is re-used.
- Pad must be as long as the message.
- Truly random bits are very hard to come by.
- Pad must be distributed **securely**.

← impractical for Web commerce



a Russian one-time pad

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### Pseudo-Random Bit Generator

#### Practical middle-ground.

- Let's make a "random"-bit generator gadget.
- Alice and Bob each get identical small gadgets.

← instead of identical large one-time pads

#### How to make small gadget that produces "random" bits.

- Enigma machine.
- Linear feedback shift register.
- Linear congruential generator.
- Blum-Blum-Shub generator.
- ...

"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin."

— Jon von Neumann (left)  
— ENIAC (right)

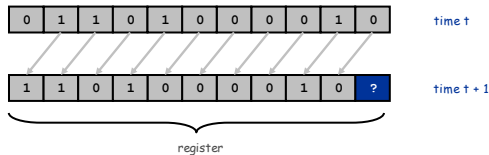


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### Shift Register

#### Shift register terminology.

- Bit: 0 or 1.
- Cell: storage element that holds one bit.
- Register: sequence of cells.
- Seed: initial sequence of bits.
- Shift register: when clock ticks, bits propagate one position to left.



time t

time t + 1

register

33

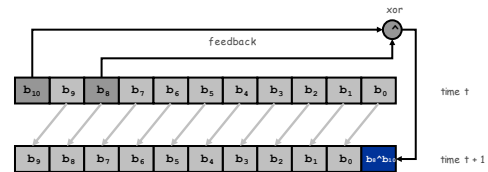
### Linear Feedback Shift Register (LFSR)

#### {8, 10} linear feedback shift register.

- Shift register with 11 cells.
- Bit  $b_0$  is XOR of previous bits  $b_8$  and  $b_{10}$ .
- Pseudo-random bit =  $b_0$ .



LFSR demo



time t

time t + 1

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### Random Numbers

Q. Are these 2000 numbers random? If not, what is the pattern?

```
11001000011101011000010101100110000011111010000001011001011100100100111111
1011000010010000001101000110000111000010011111101000000000000100000000000000
01010001000000001110001001010101011000100001011000101011000000001100000000000000
01101000000100000001000000000100000001000000000000000000000000000000000000000000000
11111110000011100010000101100011010011000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000000
01000000010001000101000000000000000000000000000000000000000000000000000000000000000
00001011000000111000000000000000000000000000000000000000000000000000000000000000000
00001011000000000000000000000000000000000000000000000000000000000000000000000000000
10010001101100101011000110000110011101100011000110000000000000000000000000000000000
01100100011101011000000000000000000000000000000000000000000000000000000000000000000
01100100111000000000000000000000000000000000000000000000000000000000000000000000000
01101011000000000000000000000000000000000000000000000000000000000000000000000000000
00001000000000000000000000000000000000000000000000000000000000000000000000000000000
01100110111000000000000000000000000000000000000000000000000000000000000000000000000
01100110111000000000000000000000000000000000000000000000000000000000000000000000000
10011000000000000000000000000000000000000000000000000000000000000000000000000000000
11000010110101101000100000000000000000000000000000000000000000000000000000000000000
11001110101000000000000000000000000000000000000000000000000000000000000000000000000
11010000000000000000000000000000000000000000000000000000000000000000000000000000000
01010100111000000001100000011010000011011001100111000000000000000000000000000000000
```

A. No. This is output of {8, 10} LFSR with seed 01101000010!

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### LFSR Encryption

#### LFSR encryption.

- Convert text message to N bits.
- Initialize LFSR with small seed.
- Generate N bits with LFSR.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	X	7	6	w	3	v	7	K	encrypted

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### LFSR Decryption

#### LFSR Decryption.

- Convert encrypted message to N bits.
- Initialize identical LFSR with same seed.
- Generate N bits with LFSR.
- Take bitwise XOR of two bitstrings.
- Convert binary back into text.

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
...	...	...
w	22	010110
...	...	...

g	X	7	6	w	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	E	N	D	M	O	N	E	Y	message

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### Goods and Bads of LFSR Encryption

#### Goods.

- Easily computed with simple machine.
- Very simple encryption/decryption process.
- Scalable: 20 cells for 1 million bits; 30 cells for 1 billion bits. [but need theory of finite groups to know where to put taps]



a commercially available LFSR

#### Bads.

- Still need secure, independent way to distribute LFSR seed.
- The bits are not truly random. [bits in our 11-bit LFSR cycle after  $2^{11} - 1 = 2047$  steps]
- Experts have cracked LFSR. [more complicated machines needed]

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### Other LFSR Applications

**What else can we do with a LFSR?**

- DVD encryption with CSS.
- DVD decryption with DeCSS!
- Subroutine in military cryptosystems.



DVD Jon  
(Norwegian hacker)

```

/* edit.c Author: Charles M. Harris <cmh@cs.cmu.edu> */
/* Usage: ls: cat tit1a-key.so rshell.d.vob | wdt t > clear.vob */

#define LFSR_LEN (16) /* 16 bits */

void signed_char * x = [5]; /* x[12-15] = main (
a) { for ( r=0; r<16; r++) { r=x[0]; r=x[20-48
}; while(1) ; /* } }
{ y=x [13]+8+20; /1614 == 1; [ int
1-16; 1117 *256 + m[0]; B, x == (2)
0; /* m[0]; 17* m[0]; /* x = 2+3-8
* 8, s =0, c =-2; for ( a[y]; --s;
--c); +c); s = a[25]; 1; s = 4 / 25; a[
<<24; for ( j= 1; j; ++j; c = m;
}
}
+ s = 1; /8 - 1; 201.2.
1; > 30; < 17; a = a > 14; y = a > 8; a < 0; a = a
>> 8; c < 0; s = 1; a = -79; c = 1; 218 * 18
47; 142 * c; c = a > 4; k = a / a; * 18; >> 12; * a * 207 /
8; a [ 14; * 0; 64 * 243 4; 6 * c = y
}
}

```

<http://www.cs.cmu.edu/~dst/DeCSS/Gallery>

### A Closing Profound Question

Q. What is a random number?

LFSR does not produce random numbers.

- It is a very simple deterministic machine.
- But not obvious how to distinguish the bits it produces from random.

Q. Are truly random processes found in nature?

- Motion of cosmic rays or subatomic particles?
- Mutations in DNA?

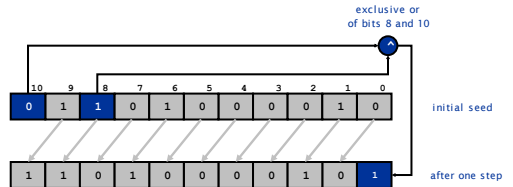
Q. Or, is the natural world a (not-so-simple) deterministic machine?

"God does not play dice."  
- Albert Einstein



### Extra Slides

### Linear Feedback Shift Register



One step of an 11-bit LFSR with initial seed 01101000010 and tap at position 8