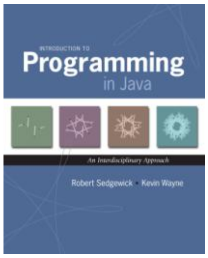
3.2 Creating Data Types



Data Types

Data type. Set of values and operations on those values.

Basic types.

Data Type	Set of Values	Some Operations
`boolean`	`true, false`	not, and, or, xor
`int`	-2^{31} to $2^{31} - 1$	add, subtract, multiply
`String`	sequence of Unicode characters	concatenate, compare

Last time. Write programs that use data types.
Today. Write programs to create our own data types.

Defining Data Types in Java

To define a data type, specify:
- Set of values.
- Operations defined on those values.

Java class. Defines a data type by specifying:
- Instance variables. (set of values)
- Methods. (operations defined on those values)
- Constructors. (create and initialize new objects)

Point Charge Data Type

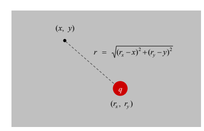
Goal. Create a data type to manipulate point charges.

Set of values. Three real numbers. [position and electrical charge]

Operations.
- Create a new point charge at (r_x, r_y) with electric charge q.
- Determine electric potential V at (x, y) due to point charge.
- Convert to string.

$$V = k \frac{q}{r}$$

r = distance between (x, y) and (r_x, r_y)
k = electrostatic constant = $8.99 \times 10^{9}\ \mathrm{N \cdot m^2 / C^2}$



Point Charge Data Type

Goal. Create a data type to manipulate point charges.

Set of values. Three real numbers. [position and electrical charge]

API.

public class Charge	
`Charge(double x0, double y0, double q0)`	
`double potentialAt(double x, double y)`	*electric potential at (x, y) due to charge*
`String toString()`	*string representation*

Charge Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args) {
   double x = Double.parseDouble(args[0]);
   double y = Double.parseDouble(args[1]);
   Charge c1 = new Charge(.51, .63, 21.3);
   Charge c2 = new Charge(.13, .94, 81.9);
   double v1 = c1.potentialAt(x, y);
   double v2 = c2.potentialAt(x, y);
   StdOut.println(c1);
   StdOut.println(c2);
   StdOut.println(v1 + v2);
}
```

automagically invokes the `toString()` method

```
% java Charge .50 .50
21.3 at (0.51, 0.63)
81.9 at (0.13, 0.94)
2.74936907085912e12
```

Anatomy of Instance Variables

Instance variables. Specifies the set of values.
- Declare outside any method.
- Always use access modifier **private**.
- Use modifier **final** with instance variables that never change.

stay tuned

```
public class Charge
{
   private final double rx, ry;
   private final double q;
   .
   .
   .
}
```

instance variable declarations

modifiers

Anatomy of a Constructor

Constructor. Specifies what happens when you create a new object.

```
public Charge ( double x0 , double y0 , double q0 )
{
   rx = x0;
   ry = y0;
   q = q0;
}
```

access modifier; no return type; constructor name (same as class name); argument variables; signature; instance variable names; body

Calling a constructor. Use **new** operator to create a new object.

```
Charge c1 = new Charge(.51, .63, 21.3);
Charge c2 = new Charge(.13, .94, 81.9);
```

create and initialize object; invoke constructor

Anatomy of an Instance Method

Instance method. Define operations on instance variables.

```
public double potentialAt(double x, double y)
{
   double k = 8.99e09;
   double dx = x - rx;
   double dy = y - ry;
   return k * q / Math.sqrt(dx*dx + dy*dy);
}
```

access modifier; return type; method name; argument variables; signature; local variables; argument variable name; instance variable name; call on a static method; local variable name

Invoking an instance method. Use dot operator to invoke a method.

```
double v1 = c1.potentialAt(x, y);
double v2 = c2.potentialAt(x, y);
```

object name; invoke method

Anatomy of a Class

```
public class Charge
{
   private final double rx, ry;
   private final double q;

   public Charge(double x0, double y0, double q0)
   {  rx = x0; ry = y0; q = q0;  }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }

   public String toString()
   {  return q +" at " + "("+ rx + ", " + ry +")";  }

   public static void main(String[] args)
   {
      double x = Double.parseDouble(args[0]);
      double y = Double.parseDouble(args[1]);
      Charge c1 = new Charge(.51, .63, 21.3);
      Charge c2 = new Charge(.13, .94, 81.9);
      double v1 = c1.potentialAt(x, y);
      double v2 = c2.potentialAt(x, y);
      StdOut.printf("%.1e\n", (v1 + v2));
   }
}
```

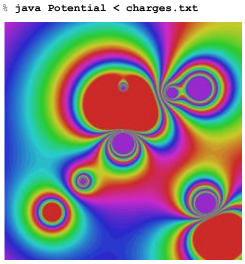
class name; instance variables; constructor; instance methods; instance variable names; test client; create and initialize object; invoke constructor; object name; invoke method

Potential Visualization

Potential visualization. Read in N point charges from standard input; compute total potential at each point in unit square.

```
% more charges.txt
9
.51 .63 -100
.50 .50   40
.50 .72   10
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50
```

```
% java Potential < charges.txt
```



Potential Visualization

Arrays of objects. Allocate memory for the array with **new**; then allocate memory for each individual object with **new**.

```
% more charges.txt
9
.51 .63 -100
.50 .50   40
.50 .72   10
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50
```

```
// read in the data
int N = StdIn.readInt();
Charge[] a = new Charge[N];
for (int i = 0; i < N; i++) {
   double x0 = StdIn.readDouble();
   double y0 = StdIn.readDouble();
   double q0 = StdIn.readDouble();
   a[i] = new Charge(x0, y0, q0);
}
```

Potential Visualization

```
// plot the data
int SIZE = 512;
Picture pic = new Picture(SIZE, SIZE);
for (int i = 0; i < SIZE; i++) {
   for (int j = 0; j < SIZE; j++) {
      double V = 0.0;
      for (int k = 0; k < N; k++) {
         double x = 1.0 * i / SIZE;
         double y = 1.0 * j / SIZE;
         V += a[k].potentialAt(x, y);
      }
      Color color = getColor(V);
      pic.set(i, SIZE-1-j, color);
   }
}
pic.show();
```

$V = \sum_k (k\, q_k / r_k)$

compute color as a function of potential V

(0, 0) is upper left

Turtle Graphics

Turtle Graphics

Goal. Create a data type to manipulate a turtle moving in the plane.
Set of values. Location and orientation of turtle.

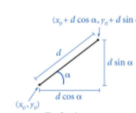
API. public class Turtle

	Turtle(double x0, double y0, double a0)	*create a new turtle at (x_0, y_0) facing a_0 degrees counterclockwise from the x-axis*
void	turnLeft(double delta)	*rotate* delta *degrees counterclockwise*
void	goForward(double step)	*move distance* step*, drawing a line*

```
// draw a square
Turtle turtle = new Turtle(0.0, 0.0, 0.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
```


Turtle Graphics

```
public class Turtle {
   private double x, y;   // turtle is at (x, y)
   private double angle;  // facing this direction

   public Turtle(double x0, double y0, double a0) {
      x = x0;
      y = y0;
      angle = a0;
   }

   public void turnLeft(double delta) {
      angle += delta;
   }

   public void goForward(double d) {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }
}
```



Turtle trigonometry

N-gon

```
public class Ngon {
   public static void main(String[] args) {
      int N        = Integer.parseInt(args[0]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < N; i++) {
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

Spira Mirabilis

```
public class Spiral {
   public static void main(String[] args) {
      int N        = Integer.parseInt(args[0]);
      double decay = Double.parseDouble(args[1]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < 10 * N; i++) {
         step /= decay;
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

Spira Mirabilis in Nature

Complex Numbers

Complex Number Data Type

Goal. Create a data type to manipulate complex numbers.
Set of values. Two real numbers: real and imaginary parts.

API.

`public class Complex`

	`Complex(double real, double imag)`	
`Complex`	`plus(Complex b)`	*sum of this number and* `b`
`Complex`	`times(Complex b)`	*product of this number and* `b`
`double`	`abs()`	*magnitude*
`String`	`toString()`	*string representation*

$a = 3 + 4i,\ b = -2 + 3i$
$a + b = 1 + 7i$
$a \times b = -18 + i$
$|\,a\,| = 5$

Applications of Complex Numbers

Relevance. A quintessential mathematical abstraction.

Applications.
- Fractals.
- Impedance in RLC circuits.
- Signal processing and Fourier analysis.
- Control theory and Laplace transforms.
- Quantum mechanics and Hilbert spaces.
- …

Complex Number Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args) {
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   Complex c = a.times(b);
   StdOut.println("a = " + a);
   StdOut.println("b = " + b);
   StdOut.println("c = " + c);
}
```

result of `c.toString()`

```
% java TestClient
a = 3.0 + 4.0i
b = -2.0 + 3.0i
c = -18.0 + 1.0i
```

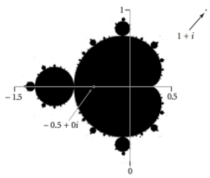
Remark. Can't write `c = a * b` since no operator overloading in Java.

Complex Number Data Type: Implementation

```
public class Complex {

   private final double re;
   private final double im;                      // instance variables

   public Complex(double real, double imag) {
      re = real;
      im = imag;
   }                                             // constructor

   public String toString() { return re + " + " + im + "i"; }

   public double abs() { return Math.sqrt(re*re + im*im); }

   public Complex plus(Complex b) {
      double real = re + b.re;
      double imag = im + b.im;                   // creates a Complex object, and returns a reference to it
      return new Complex(real, imag);
   }

   public Complex times(Complex b) {             // refers to b's instance variable
      double real = re * b.re - im * b.im;
      double imag = re * b.im + im * b.re;
      return new Complex(real, imag);
   }                                             // methods
}
```

Mandelbrot Set

Mandelbrot set. A set of complex numbers.
Plot. Plot (x, y) black if $z = x + y\,i$ is in the set, and white otherwise.

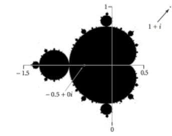


- No simple formula describes which complex numbers are in set.
- Instead, describe using an algorithm.

Mandelbrot Set

Mandelbrot set. Is complex number z_0 in the set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, then z_0 is not in set; otherwise z_0 is in set.



t	z_t
0	-1/2 + 0i
1	-1/4 + 0i
2	-7/16 + 0i
3	-79/256 + 0i
4	-26527/65536 + 0i
5	-1443801919/4294967296 + 0i

z = -1/2 is in Mandelbrot set

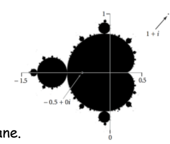
t	z_t
0	1 + i
1	1 + 3i
2	-7 + 7i
3	1 - 97i
4	-9407 – 193i
5	88454401 + 3631103i

z = 1 + i not in Mandelbrot set

Plotting the Mandelbrot Set

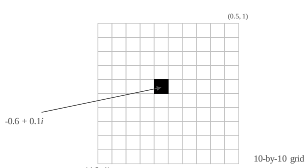
Practical issues.

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.



Approximate solution.

- Sample from an N-by-N grid of points in the plane.
- Fact: if $|z_t| > 2$ for any t, then z not in Mandelbrot set.
- Pseudo-fact: if $|z_{255}| \leq 2$ then z "likely" in Mandelbrot set.



Complex Number Data Type: Another Client

Mandelbrot function with complex numbers.

- Is z_0 in the Mandelbrot set?
- Returns white (definitely no) or black (probably yes).

```
public static Color mand(Complex z0) {
   Complex z = z0;
   for (int t = 0; t < 255; t++) {
      if (z.abs() > 2.0) return StdDraw.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return StdDraw.BLACK;
}
```

$z = z^2 + z_0$

More dramatic picture: replace `StdDraw.WHITE` with grayscale or color.

`new Color(255-t, 255-t, 255-t)`

Complex Number Data Type: Another Client

Plot the Mandelbrot set in gray scale.

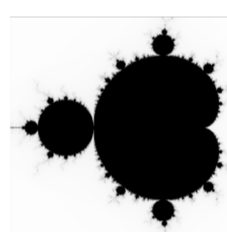
```
public static void main(String[] args) {
   double xc   = Double.parseDouble(args[0]);
   double yc   = Double.parseDouble(args[1]);
   double size = Double.parseDouble(args[2]);
   int N = 512;
   Picture pic = new Picture(N, N);

   for (int i = 0; i < N; i++) {
      for (int j = 0; j < N; j++) {
         double x0 = xc - size/2 + size*i/N;
         double y0 = yc - size/2 + size*j/N;
         Complex z0 = new Complex(x0, y0);
         Color color = mand(z0);
         pic.set(i, N-1-j, color);
      }
   }
   pic.show();
}
```

scale to screen coordinates

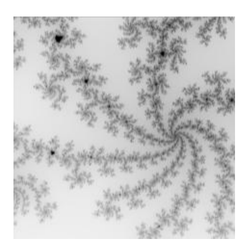
(0, 0) is upper left

Mandelbrot Set

`% java Mandelbrot –.5 0 2`

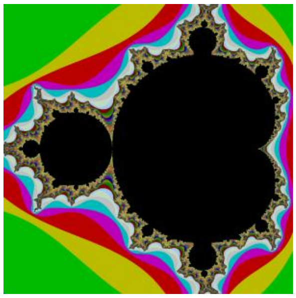


`% java Mandelbrot .1045 -.637 .01`



Mandelbrot Set

```
% java ColorMandelbrot -.5 0 2 < mandel.txt
```



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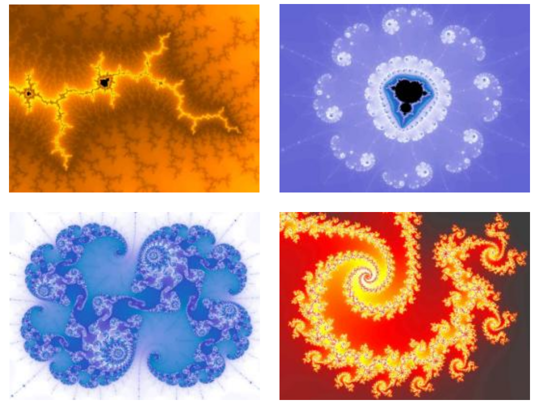

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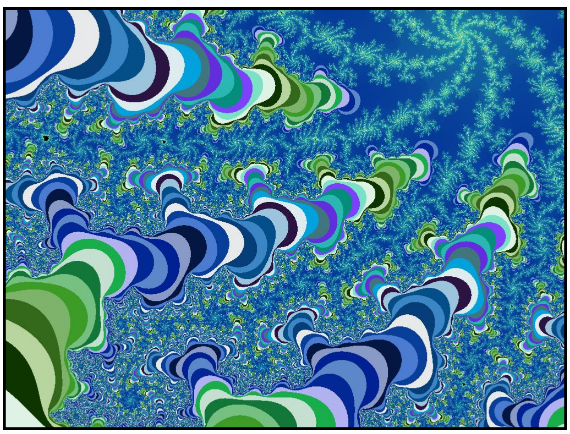

Mandelbrot Set Music Video

http://www.jonathancoulton.com/songdetails/Mandelbrot Set

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Applications of Data Types

Data type. Set of values and collection of operations on those values.

Simulating the physical world.
- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Ex: charged particle, molecule, COS 126 student, ...

Extending the Java language.
- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Ex: complex, vector, polynomial, matrix, ...

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3.2 Extra Slides

Example: Bouncing Ball in Unit Square

Bouncing ball. Model a bouncing ball moving in the unit square with constant velocity.

Example: Bouncing Ball in Unit Square

Ball.java

```
public class Ball {
   private double rx, ry;        ← instance variables
   private double vx, vy;
   private double radius;

   public Ball() {                                   constructor
      rx = ry = 0.5;
      vx     = 0.015 - Math.random() * 0.03;
      vy     = 0.015 - Math.random() * 0.03;
      radius = 0.01  + Math.random() * 0.01;
   }

   public void move() {
      if ((rx + vx > 1.0) || (rx + vx < 0.0)) vx = -vx;
      if ((ry + vy > 1.0) || (ry + vy < 0.0)) vy = -vy;
      rx = rx + vx;                              ↑ bounce
      ry = ry + vy;
   }

   public void draw() {
      StdDraw.filledCircle(rx, ry, radius);
   }                                                 methods
}
```


Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

addr	value
C0	0
C1	0
C2	0
C3	0
C4	0
C5	0
C6	0
C7	0
C8	0
C9	0
CA	0
CB	0
CC	0

main memory (64-bit machine)

Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

b1
C0

registers

addr	value
C0	0.50
C1	0.50
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0
C8	0
C9	0
CA	0
CB	0
CC	0

main memory (64-bit machine)

Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

b1
C0

registers

addr	value
C0	0.55
C1	0.51
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0
C8	0
C9	0
CA	0
CB	0
CC	0

main memory (64-bit machine)

Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

b1
C0

registers

addr	value
C0	0.60
C1	0.52
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0
C8	0
C9	0
CA	0
CB	0
CC	0

main memory (64-bit machine)

Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

registers	
b1	C0
b2	C7

addr	value
C0	0.60
C1	0.52
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0.50
C8	0.50
C9	0.07
CA	0.04
CB	0.04
CC	0

registers main memory (64-bit machine)

Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

registers	
b1	C0
b2	C7

addr	value
C0	0.60
C1	0.52
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0.57
C8	0.54
C9	0.07
CA	0.04
CB	0.04
CC	0

registers main memory (64-bit machine)

Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

registers	
b1	C0
b2	C0

addr	value
C0	0.60
C1	0.52
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0.57
C8	0.54
C9	0.07
CA	0.04
CB	0.04
CC	0

Data stored in C7 – CB for abstract bit recycler.

registers main memory (64-bit machine)

Object References

Object reference.
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

registers	
b1	C0
b2	C0

addr	value
C0	0.65
C1	0.53
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0.57
C8	0.54
C9	0.07
CA	0.04
CB	0.04
CC	0

Moving b2 also moves b1 since they are aliases that reference the same object.

registers main memory (64-bit machine)

Creating Many Objects

Each object is a data type value.
- Use new to invoke constructor and create each one.
- Ex: create N bouncing balls and animate them.

```
public class BouncingBalls {
   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      Ball balls[] = new Ball[N];
      for (int i = 0; i < N; i++)
         balls[i] = new Ball();

      while(true) {
         StdDraw.clear();
         for (int i = 0; i < N; i++) {
            balls[i].move();
            balls[i].draw();
         }
         StdDraw.show(20);
      }
   }
}
```

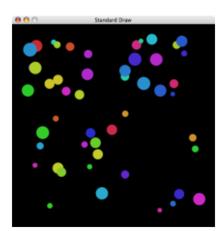
create and initialize N objects

animation loop

50 Bouncing Balls

Color. Associate a color with each ball; paint background black.

```
% java BouncingBalls 50
```



Scientific variations. Account for gravity, spin, collisions, drag, …

OOP Context

Reference. Variable that stores the name of a thing.

Thing	Name
Web page	www.princeton.edu
Bank account	45-234-23310076
Word of TOY memory	1C
Byte of computer memory	00FACADE
Home	35 Olden Street

Some consequences.
- Assignment statements copy references (not objects).
- The == operator tests if two references refer to same object.
- Pass copies of references (not objects) to functions.
  - efficient since no copying of data
  - function can change the object

Using a Data Type in Java

Client. A sample client program that uses the `Point` data type.

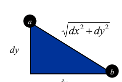
```
public class PointTest {
   public static void main(String[] args) {
      Point a = new Point();
      Point b = new Point();
      double distance = a.distanceTo(b);
      StdOut.println("a =  " + a);
      StdOut.println("b =  " + b);
      StdOut.println("distance =  " + distance);
   }
}
```

```
% java PointTest
a = (0.716810971264761, 0.0753539063358446)
b = (0.4052136795358151, 0.033848435224524076)
distance = 0.31434944941098036
```


Points in the Plane

Data type. Points in the plane.

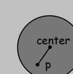
```
public class Point {
   private double x;
   private double y;

   public Point() {
      x = Math.random();
      y = Math.random();
   }

   public String toString() {
      return "(" + x + ", " + y + ")";
   }

   public double distanceTo(Point p) {
      double dx = x - p.x;
      double dy = y - p.y;
      return Math.sqrt(dx*dx + dy*dy);
   }
}
```

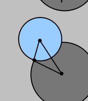


A Compound Data Type: Circles

Goal. Data type for circles in the plane.

```
public class Circle {
   private Point center;
   private double radius;

   public Circle(Point center, double radius) {
      this.center = center;
      this.radius = radius;
   }

   public boolean contains(Point p) {
      return p.dist(center) <= radius;
   }

   public double area() {
      return Math.PI * radius * radius;
   }

   public boolean intersects(Circle c) {
      return center.dist(c.center) <= radius + c.radius;
   }
}
```





Pass-By-Value

Arguments to methods are always passed by value.
- Primitive types: passes copy of value of actual parameter.
- Objects: passes copy of reference to actual parameter.

```
public class PassByValue {
   static void update(int a, int[] b, String c) {
      a    = 7;
      b[3] = 7;
      c    = "seven";
      StdO.println(a + " " + b[3] + " " + c);
   }
   public static void main(String[] args) {
      int a = 3;
      int[] b = { 0, 1, 2, 3, 4, 5 };
      String c = "three";
      StdOut.println(a + " " + b[3] + " " + c);
      update(a, b, c);
      StdOut.println(a + " " + b[3] + " " + c);
   }
}
```

```
% java PassByValue
3 3 three
7 7 seven
3 7 three
```