Programming Languages and Techniques (CIS120)

Lecture 9

Lists and Higher-order functions

Lecture notes: Chapter 9

Announcements

- Homework 3 available
  - Due next Tuesday at 11:59 pm
  - Practice with BSTs, generic functions, first-class functions and abstract types
  - *Start early!*

- Reading: Chapters 8, 9, and 10 of the lecture notes

- Midterm 1: Friday, September 27th
  - Coverage: up to Monday, Sept. 23 (Chs. 1-10)
  - During lecture (001 @ 11am, 002 @ noon)
    Last names: A – L Leidy Labs 10
    Last names: M – Z Stitler (STIT) B6

Anonymous, First-class Functions

`fun (x : T_in) -> e`

Named function values

A standard function definition…

```
let add_one (x:int) : int = x+1
```

really has two parts:

```
let add_one : int->int = fun (x:int) -> x+1
```

define a name for the value

create a function value

The two definitions have the same type and behave exactly the same.
(The first is actually just an abbreviation for the second.)

Function Types

- Functions have types that look like this:

$$t_{in} \;\; \texttt{->} \;\; t_{out}$$

- Examples:

```
int -> int

int -> bool * int

int -> int -> int              int input

(int -> int) -> int            function input
```

Function Types

- Functions have types that look like this:

$$t_{in} \;\; \texttt{->} \;\; t_{out}$$

- Examples:

```
int -> int

int -> (bool * int)

int -> (int -> int)

(int -> int) -> int
```

int input

function input

Parentheses matter!

int -> int -> int   is equivalent to int -> (int -> int) but not to (int -> int) -> int

Function Types

Hang on... did we just say that

```
int -> int -> int
```

and

```
int -> (int -> int)
```

mean the same thing??

Yes!

Multiple Arguments

We can decompose a standard function definition

```
let sum (x : int) (y:int) : int = x + y
```

into parts

```
let sum = fun (x:int) -> fun (y:int) -> x + y
```

define a variable with that value

create a function value

that returns a function value

The two definitions have the same type and behave exactly the same

```
let sum : int -> int -> int
```

Partial Application

```
let sum (x : int) (y:int) : int = x + y
```

```
sum 3
⟼ (fun (x:int) -> fun (y:int) -> x + y) 3        definition
⟼  fun (y:int) -> 3 + y                   substitute 3 for x
```

What is the value of this expression?

```
let f (x:bool) (y:int) : int =
   if x then 1 else y in

f true
```

A. 1

B. True

C. fun (y:int) -> if true then 1 else y

D. fun (x:bool) -> if x then 1 else y

What is the value of this expresssion?

```
let f (x:bool) (y:int) : int =
    if x then 1 else y in

f true
```

1. `1`
2. `true`
3. `fun (y:int) -> if true then 1 else y`
4. `fun (x:bool) -> if x then 1 else y`

Answer: 3

What is the value of the this expression?

```
let f (g : int->int) (y: int) : int =
   g 1 + y in

f (fun (x:int) -> x + 1) 3
```

1

2

3

4

5

6

What is the value of this expression?

```
let f (g : int->int) (y: int) : int =
   g 1 + y in

f (fun (x:int) -> x + 1) 3
```

1. 1
2. 2
3. 3
4. 4
5. 5

Answer: 5

What is the type of this expression?

```
let f (g : int->int) (y: int) : int =
   g 1 + y in

f (fun (x:int) -> x + 1)
```

1. int
2. int -> int
3. int -> int -> int
4. (int -> int) -> int -> int
5. Ill-typed

What is the type of this expression?

```
let f (g : int->int) (y: int) : int =
   g 1 + y in

f (fun (x:int) -> x + 1)
```

1. `int`
2. `int -> int`
3. `int -> int -> int`
4. `(int -> int) -> int -> int`
5. `ill-typed`

Answer: 2

List transformations

A fundamental design pattern
using first-class functions

Phone book example

```
type entry = string * int
let phone_book = [ ("Pat", 2155559092); … ]

let rec get_names (p : entry list) : string list =
  begin match p with
  | ((name, num)::rest) -> name :: get_names rest
  | [] -> []
  end

let rec get_numbers (p : entry list) : int list =
  begin match p with
  | ((name, num)::rest) -> num :: get_numbers rest
  | [] -> []
  end
```

Can we use first-class functions to refactor code to share common structure?

Refactoring

```
let rec helper (f: entry->'b) (p: entry list) : 'b list =
  begin match p with
  | (e::rest) -> f e :: helper f rest
  | [] -> []
  end

let get_names (p: entry list) : string list =
  helper fst p
let get_numbers (p: entry list) : int list =
  helper snd p
```

fst and snd are functions that access the parts of a tuple:

```
let fst (x,y) = x
let snd (x,y) = y
```

The argument `f` determines what happens with the entry at the head of the list

Going even more generic

```
let rec helper (f: entry->'b) (p: entry list) : 'b list =
  begin match p with
  | (e::rest) -> f e :: helper f rest
  | [] -> []
  end

let get_names (p: entry list) : string list =
  helper fst p
let get_numbers (p: entry list) : int list =
  helper snd p
```

Now let's make it work for *all* lists, not just lists of entries…

Going even more generic

```
let rec helper (f: 'a->'b) (p: 'a list) : 'b list =
  begin match p with
  | (e::rest) -> f e :: helper f rest
  | [] -> []
  end

let get_names (p: entry list) : string list =
  helper fst p
let get_numbers (p: entry list) : int list =
  helper snd p
```

‘a stands for (string*int)
‘b stands for int

snd : (string*int) -> int

Transforming Lists

```
let rec transform (f: 'a->'b) (l:'a list) : 'b list =
  begin match l with
  | []    -> []
  | h::t -> (f h)::(transform f t)
  end
```

List transformation

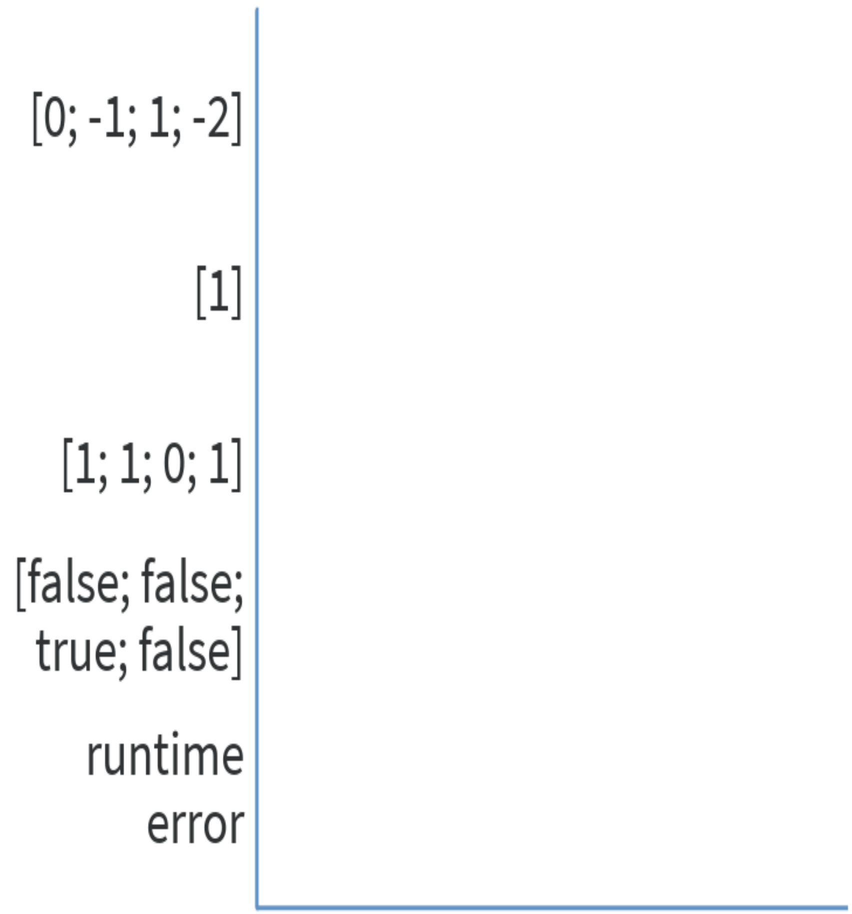
(a.k.a. "*mapping* a function across a list")

- foundational function for programming with lists
- used over and over again
- part of OCaml standard library (called List.map)

*many languages (including OCaml) use the terminology "map" for the function that transforms a list by applying a function to each element. Don't confuse List.map with "finite map".

What is the value of this expresssion?

```
transform (fun (x:int) -> x > 0)
    [0 ; -1; 1; -2]
```



What is the value of this expresssion?

```
transform (fun (x:int) -> x > 0)
    [0 ; -1; 1; -2]
```

1. [0; -1; 1; -2]
2. [1]
3. [1; 1; 0; 1]
4. [false; false; true; false]
5. runtime error

ANSWER: 4

The ‘fold’ design pattern

Refactoring code, again

- Is there a pattern in the definition of these two functions?

```
let rec exists (l : bool list) : bool =
   begin match l with
   | [] -> false
   | h :: t -> h || exists t
   end
```

```
let rec acid_length (l : acid list) : int =
   begin match l with
   | [] -> 0
   | h :: t -> 1 + acid_length t
   end
```

base case: Simple answer when the list is empty

combine step: Do something with the head of the list and the result of the recursive call

- Can we factor out this pattern using first-class functions?

Preparation

```
let rec exists (l : bool list) : bool =
   begin match l with
   | [] -> false
   | h :: t -> h || exists t
   end
```

```
let rec acid_length (l : acid list) : int =
   begin match l with
   | [] -> 0
   | h :: t -> 1 + acid_length t
   end
```

Preparation

```
let rec helper (l : bool list) : bool =
   begin match l with
   | [] -> false
   | h :: t -> h || helper t
   end

let exists (l : bool list) = helper l
```

```
let rec helper (l : acid list) : int =
   begin match l with
   | [] -> 0
   | h :: t -> 1 + helper t
   end

 let acid_length (l : acid list) = helper l
```

Abstracting with respect to Base

```
let rec helper (l : bool list) : bool =
   begin match l with
   | [] -> false
   | h :: t -> h || helper t
   end

let exists (l : bool list) = helper l
```

```
let rec helper (l : acid list) : int =
   begin match l with
   | [] -> 0
   | h :: t -> 1 + helper t
   end

 let acid_length (l : acid list) = helper l
```

Abstracting with respect to Base

```
let rec helper (base : bool) (l : bool list) : bool =
   begin match l with
   | [] -> base
   | h :: t -> h || helper base t
   end

let exists (l : bool list) = helper false l
```

```
let rec helper (base : int) (l : acid list) : int =
   begin match l with
   | [] -> base
   | h :: t -> 1 + helper base t
   end

 let acid_length (l : acid list) = helper 0 l
```

Abstracting with respect to Combine

```
let rec helper (base : bool) (l : bool list) : bool =
   begin match l with
   | [] -> base
   | h :: t -> h || helper base t
   end

let exists (l : bool list) = helper false l
```

```
let rec helper (base : int) (l : acid list) : int =
   begin match l with
   | [] -> base
   | h :: t -> 1 + helper base t
   end

 let acid_length (l : acid list) = helper 0 l
```

Abstracting with respect to Combine

```
let rec helper (base : bool) (l : bool list) : bool =
   begin match l with
   | [] -> base
   | h :: t -> h || helper base t
   end

let exists (l : bool list) = helper false l
```

```
let rec helper (base : int) (l : acid list) : int =
   begin match l with
   | [] -> base
   | h :: t -> 1 + helper base t
   end

 let acid_length (l : acid list) = helper 0 l
```

Abstracting with respect to Combine

```
let rec helper (combine : bool -> bool -> bool)
               (base : bool) (l : bool list) : bool =
   begin match l with
   | [] -> base
   | h :: t -> combine h (helper combine base t)
   end

let exists (l : bool list) =
  helper (fun (h:bool) (acc:bool) -> h || acc) false l
```

```
let rec helper (combine : acid -> int -> int)
               (base : int) (l : acid list) : int =
   begin match l with
   | [] -> base
   | h :: t -> combine h (helper combine base t)
   end

 let acid_length (l : acid list) =
    helper (fun (h:acid) (acc:int) -> 1 + acc) 0 l
```

Making the Helper Generic

```
let rec helper (combine : 'a -> 'b -> 'b)
               (base : 'b) (l : 'a list) : 'b =
   begin match l with
   | [] -> base
   | h :: t -> combine h (helper combine base t)
   end

let exists (l : bool list) =
  helper (fun (h:bool) (acc:bool) -> h || acc) false l
```

```
let rec helper (combine : 'a -> 'b -> 'b)
               (base : 'b) (l : 'a list) : 'b =
   begin match l with
   | [] -> base
   | h :: t -> combine h (helper combine base t)
   end

 let acid_length (l : acid list) =
    helper (fun (h:acid) (acc:int) -> 1 + acc) 0 l
```

List Fold

```
let rec fold (combine: 'a -> 'b -> 'b)
             (base:'b) (l : 'a list) : 'b =
   begin match l with
   | [] -> base
   | x :: t -> combine x (fold combine base t)
   end

let exists (l : bool list) : bool =
    fold (fun (h:bool) (acc:bool) -> h || acc) false l

let acid_length (l : acid list) : int =
    fold (fun (h:acid) (acc:int) -> 1 + acc) 0 l
```

- `fold` (a.k.a. "reduce")
  - Like transform, foundational function for programming with lists
  - Captures the pattern of recursion over lists
  - Also part of OCaml standard library (`List.fold_right`)
  - Similar operations for other recursive datatypes (`fold_tree`)

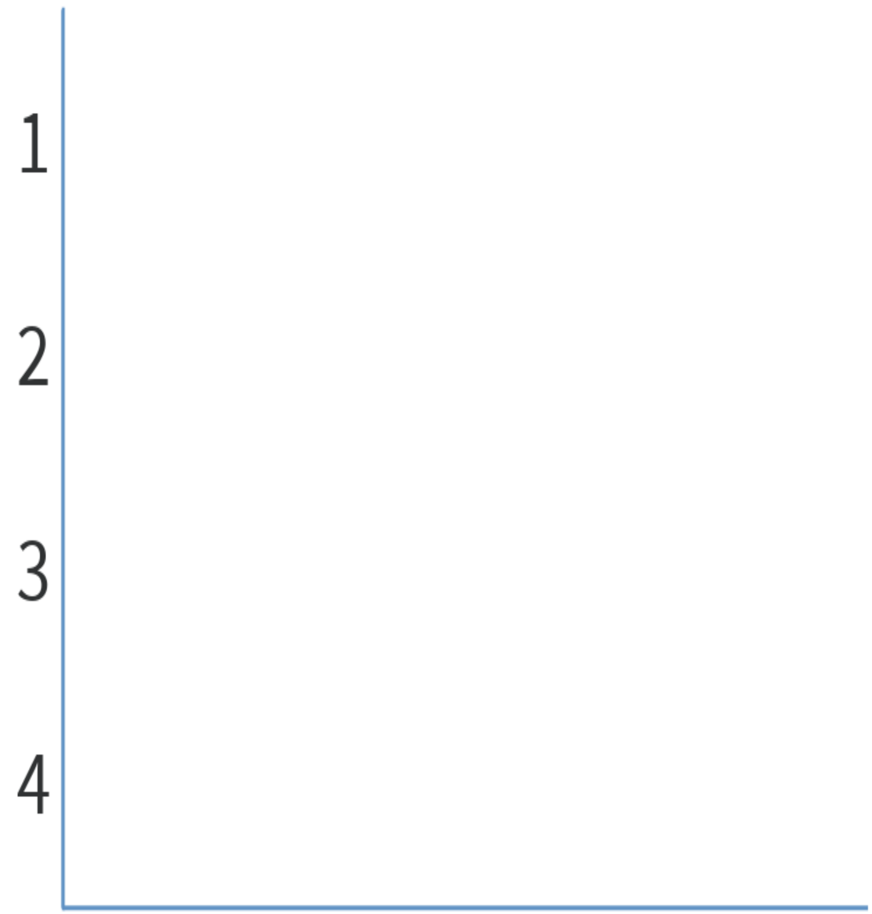
Rewrite using fold

How would you rewrite this function

```
let rec sum (l : int list) : int =
  begin match l with
  | [] -> 0
  | h :: t -> h + sum t
  end
```

using fold? What should be the arguments for base and combine?

1. combine is: `(fun (h:int) (acc:int) -> acc + 1)`
   base is: `0`
2. combine is: `(fun (h:int) (acc:int) -> h + acc)`
   base is: `0`
3. combine is: `(fun (h:int) (acc:int) -> h + acc)`
   base is: `1`
4. sum can't be written with `fold`.



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   end
```

using fold? What should be the arguments for base and combine?

1. combine is: `(fun (h:int) (acc:int) -> acc + 1)`
   base is: `0`

2. combine is: `(fun (h:int) (acc:int) -> h + acc)`
   base is: `0`

3. combine is: `(fun (h:int) (acc:int) -> h + acc)`
   base is: `1`

4. sum can't be written with `fold`.

Answer: 2

Rewrite using fold

How would you rewrite this function

```
let rec reverse (l : int list) : int list =
  begin match l with
  | [] -> []
  | h :: t -> reverse t @ [h]
  end
```

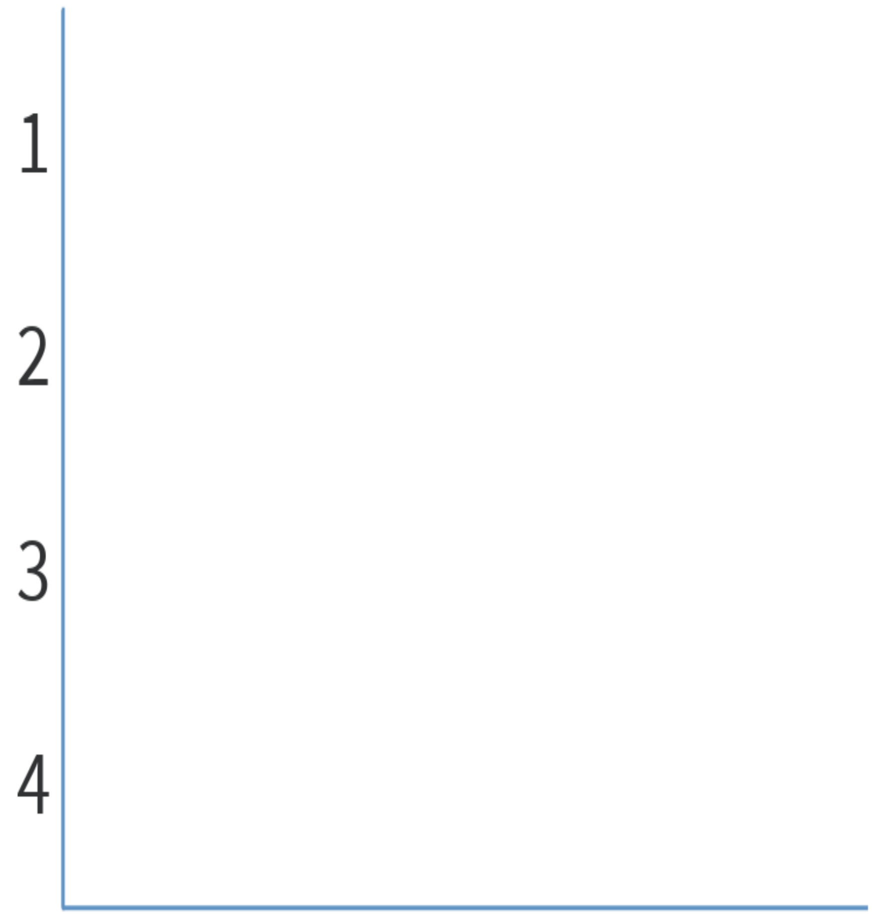
using fold? What should be the arguments for base and combine?

1. combine is: `(fun (h:int) (acc:int list) -> h :: acc)`
   base is: `0`

2. combine is: `(fun (h:int) (acc:int list) -> acc @ [h])`
   base is: `0`

3. combine is: `(fun (h:int) (acc:int list) -> acc @ [h])`
   base is: `[]`

4. reverse can't be written by with `fold`.



How would you rewrite this function

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   base is: `0`

2. combine is: `(fun (h:int) (acc:int list) -> acc @ [h])`
   base is: `0`

3. combine is: `(fun (h:int) (acc:int list) -> acc @ [h])`
   base is: `[]`

4. reverse can't be written by with `fold`.

Answer: 3

Functions as Data

- We've seen a number of ways in which functions can be treated as data in OCaml
- Everyday programming practice offers many more examples
  - objects bundle "functions" (a.k.a. methods) with data
  - iterators ("cursors" for walking over data structures)
  - event listeners (in GUIs)
  - etc.
- Also heavily used at "large scale": Google's MapReduce
  - Framework for transforming (mapping) sets of key-value pairs
  - Then "reducing" the results per key of the map
  - Easily distributed to 10,000 machines to execute in parallel!