Lecture 6

CIS 341: COMPILERS

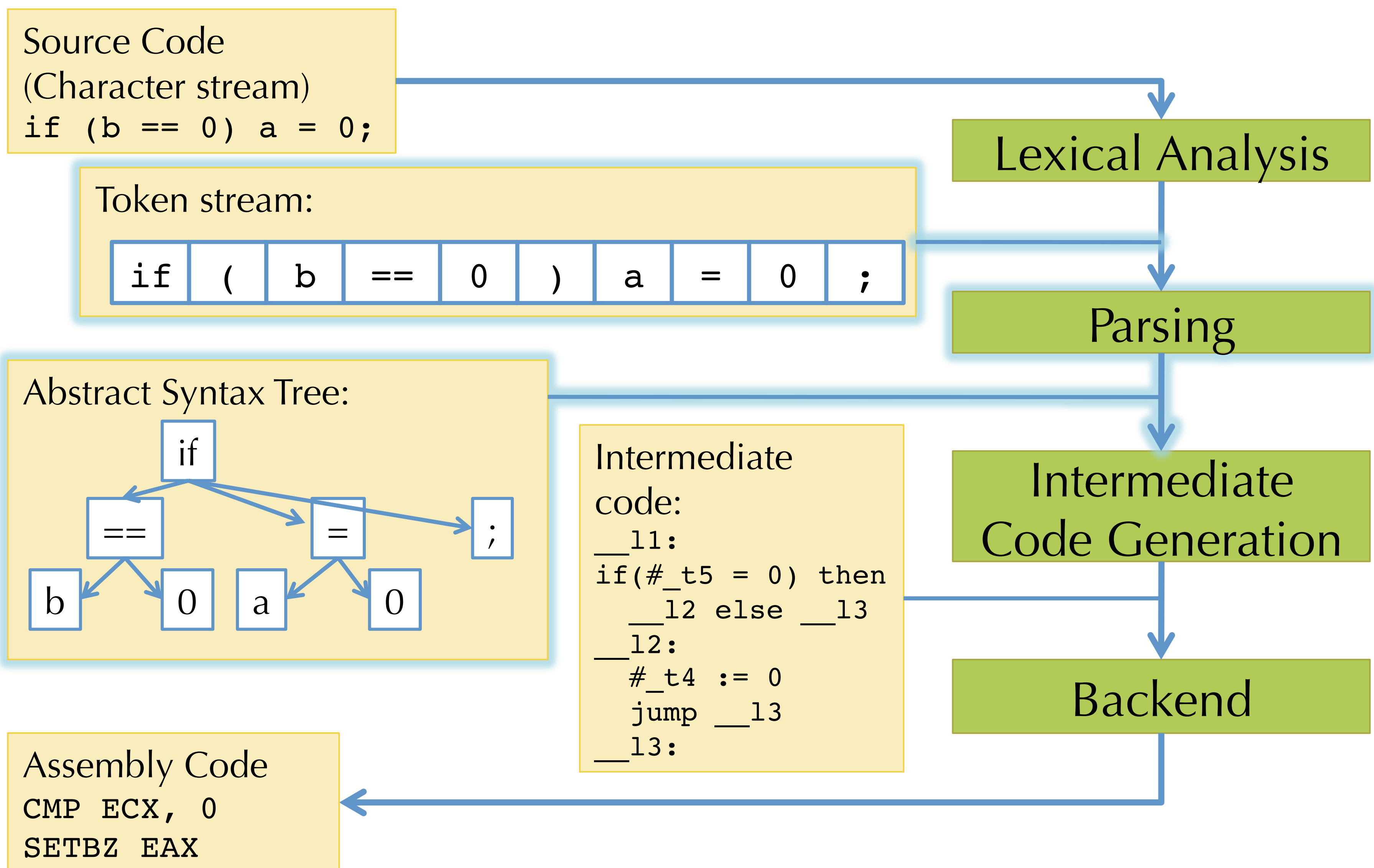
Announcements

- Project 1: X86lite
  - Due: Tonight at 11:59:59 pm
- Project 2: Parsing and Compiling Expressions
  - Available soon!
- Slightly abbreviated office hours for Dr. Zdancewic today
  - 4:00-4:45

Creating an abstract representation of program syntax.

PARSING

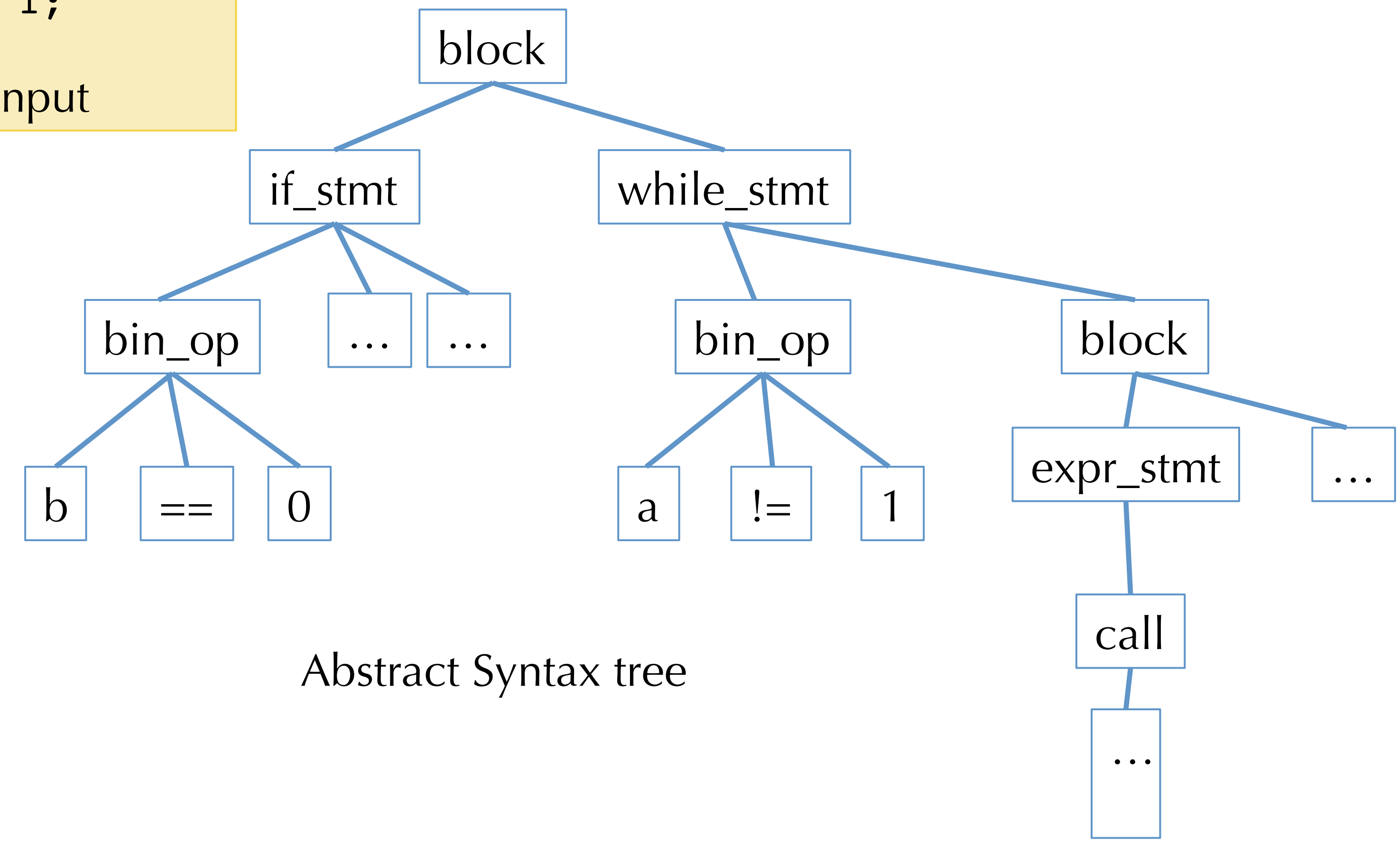
Today: Parsing I



Parsing: Finding Syntactic Structure

```
{
  if (b == 0) a = b;
  while (a != 1) {
    print_int(a);
    a = a – 1;
  }
}
```

Source input



Abstract Syntax tree

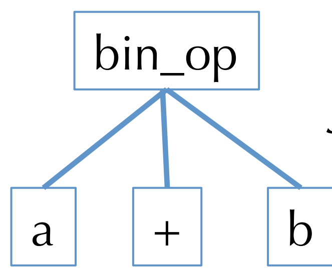
Syntactic Analysis (Parsing): Overview

- Input: stream of tokens (generated by lexer)
- Output: abstract syntax tree

- Strategy:
  – Parse the token stream to traverse the "concrete" syntax
  – During traversal, build a tree representing the "abstract" syntax

- Why abstract? Consider these three *different* concrete inputs:

```
a + b
(a + ((b)))
((a) + (b))
```



Same abstract syntax tree

- Note: parsing doesn't check many things:
  – Variable scoping, type agreement, initialization, …

Specifying Language Syntax

- First question: how to describe language syntax precisely and conveniently?
- Last time: we described tokens using regular expressions
  - Easy to implement, efficient DFA representation
  - Why not use regular expressions on tokens to specify programming language syntax?

- Limits of regular expressions:
  - DFA's have only finite # of states
  - So… DFA's can't "count"
  - For example, consider the language of all strings that contain balanced parentheses – easier than most programming languages, but not regular.

- So: we need more expressive power than DFA's

CONTEXT FREE GRAMMARS

Context-free Grammars

- Here is a specification of the language of balanced parens:

$$S \longmapsto (S)S$$
$$S \longmapsto \varepsilon$$

- The definition is *recursive* – S mentions itself.

- Idea: "derive" a string in the language by starting with S and rewriting according to the rules:
  - Example: $S \longmapsto (S)S \longmapsto ((S)S)S \longmapsto ((\varepsilon)S)S \longmapsto ((\varepsilon)S)\varepsilon \longmapsto ((\varepsilon)\varepsilon)\varepsilon = (())$

- You can replace the "nonterminal" S by its definition anywhere
- A context-free grammar accepts a string iff there is a derivation from the start symbol

CFGs Mathematically

- A Context-free Grammar (CFG) consists of
  - A set of *terminals* (e.g., a token or ε)
  - A set of *nonterminals* (e.g., S and other syntactic variables)
  - A designated nonterminal called the *start symbol*
  - A set of productions: LHS $\longmapsto$ RHS
    - LHS is a nonterminal
    - RHS is a *string* of terminals and nonterminals

- Example: The balanced parentheses language:

$$S \longmapsto (S)S$$

$$S \longmapsto \varepsilon$$

- How many terminals? How many nonterminals? Productions?

Another Example: Sum Grammar

- A grammar that accepts parenthesized sums of numbers:

$$
\begin{array}{lcll}
S & \longmapsto & E + S & \mid\ E \\
E & \longmapsto & \text{number} & \mid\ (\ S\ )
\end{array}
$$

e.g.: (1 + 2 + (3 + 4)) + 5

- Note the vertical bar '|' is shorthand for multiple productions:

$S \longmapsto E + S$
$S \longmapsto E$
$E \longmapsto \text{number}$
$E \longmapsto (S)$

4 productions
2 nonterminals: S, E
4 terminals: (,), +, number
Start symbol: S

Derivations in CFGs

- Example: derive (1 + 2 + (3 + 4)) + 5
- <u>**S**</u> ⟼ <u>**E**</u> + S
  ⟼ (<u>**S**</u>) + S
  ⟼ (<u>**E**</u> + S) + S
  ⟼ (1 + <u>**S**</u>) + S
  ⟼ (1 + <u>**E**</u> + S) + S
  ⟼ (1 + 2 + <u>**S**</u>) + S
  ⟼ (1 + 2 + <u>**E**</u>) + S
  ⟼ (1 + 2 + (<u>**S**</u>)) + S
  ⟼ (1 + 2 + (<u>**E**</u> + S)) + S
  ⟼ (1 + 2 + (3 + <u>**S**</u>)) + S
  ⟼ (1 + 2 + (3 + <u>**E**</u>)) + S
  ⟼ (1 + 2 + (3 + 4)) + <u>**S**</u>
  ⟼ (1 + 2 + (3 + 4)) + <u>**E**</u>
  ⟼ (1 + 2 + (3 + 4)) + 5

S ⟼ E + S | E
E ⟼ number | (S)

For arbitrary strings α, β, γ and production rule A ⟼ β
a single step of the derivation is:

$$\alpha A \gamma \longmapsto \alpha \beta \gamma$$

(*substitute* β for an occurrence of A)

In general, there are many possible derivations for a given string

Note: Underline indicates symbol being expanded.

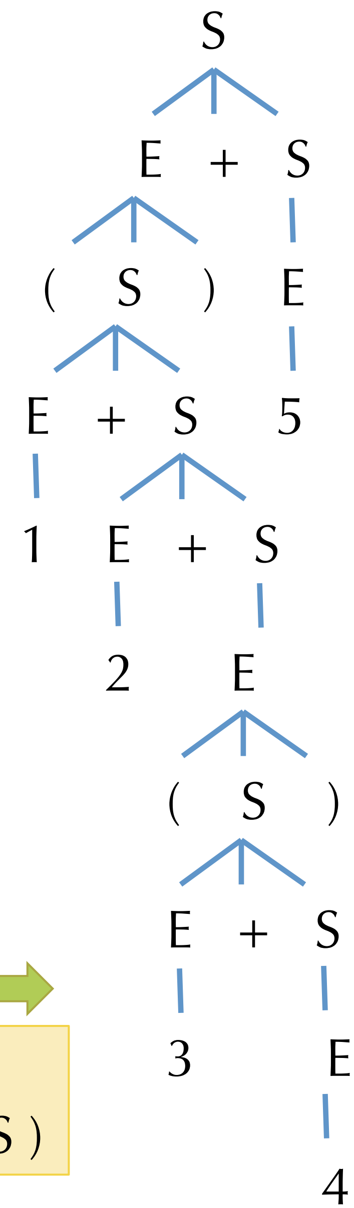
From Derivations to Parse Trees

- Tree representation of the derivation
- Leaves of the tree are terminals
  - In-order traversal yields the input sequence of tokens
- Internal nodes: nonterminals
- No information about the *order* of the derivation steps

- (1 + 2 + (3 + 4)) + 5

$S \longmapsto E + S \mid E$

$E \longmapsto \text{number} \mid (S)$

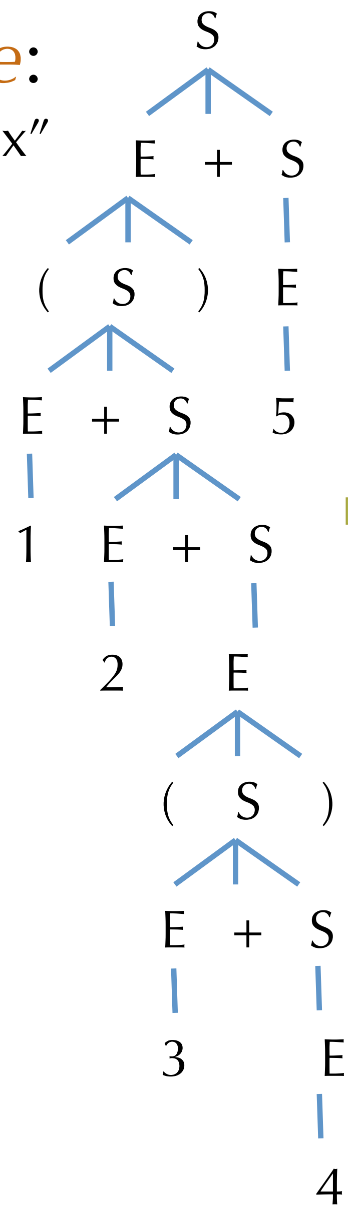


Parse Tree

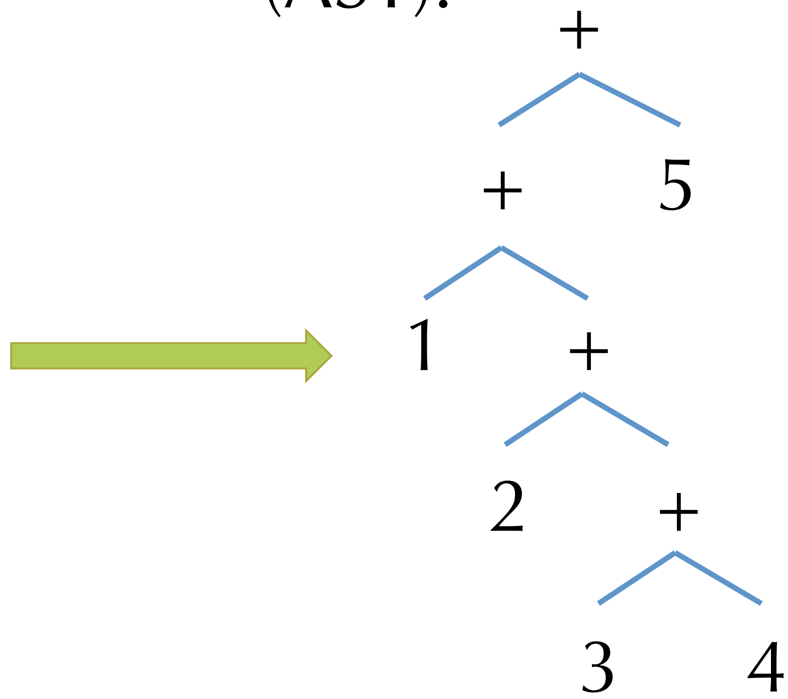
From Parse Trees to Abstract Syntax

- *Parse tree*:

"concrete syntax"



- *Abstract syntax tree* (AST):



- Hides, or *abstracts*, unneeded information.

Derivation Orders

- Productions of the grammar can be applied in any order.
- There are two standard orders:
  - *Leftmost derivation*: Find the left-most nonterminal and apply a production to it.
  - *Rightmost derivation*: Find the right-most nonterminal and apply a production there.

- Note that both strategies (and any other) yield the same parse tree!
  - Parse tree doesn't contain the information about what order the productions were applied.

Example: Left- and rightmost derivations

- Leftmost derivation:
- **S** ⟼ **E** + S
  - ⟼ (**S**) + S
  - ⟼ (**E** + S) + S
  - ⟼ (1 + **S**) + S
  - ⟼ (1 + **E** + S) + S
  - ⟼ (1 + 2 + **S**) + S
  - ⟼ (1 + 2 + **E**) + S
  - ⟼ (1 + 2 + (**S**)) + S
  - ⟼ (1 + 2 + (**E** + S)) + S
  - ⟼ (1 + 2 + (3 + **S**)) + S
  - ⟼ (1 + 2 + (3 + **E**)) + S
  - ⟼ (1 + 2 + (3 + 4)) + **S**
  - ⟼ (1 + 2 + (3 + 4)) + **E**
  - ⟼ (1 + 2 + (3 + 4)) + 5

Rightmost derivation:

S ⟼ E + **S**
- ⟼ E + **E**
- ⟼ **E** + 5
- ⟼ (**S**) + 5
- ⟼ (E + **S**) + 5
- ⟼ (E + E + **S**) + 5
- ⟼ (E + E + **E**) + 5
- ⟼ (E + E + (**S**)) + 5
- ⟼ (E + E + (E + **S**)) + 5
- ⟼ (E + E + (E + **E**)) + 5
- ⟼ (E + E + (**E** + 4)) + 5
- ⟼ (E + **E** + (3 + 4)) + 5
- ⟼ (**E** + 2 + (3 + 4)) + 5
- ⟼ (1 + 2 + (3 + 4)) + 5

S ⟼ E + S | E
E ⟼ number | (S)

Loops and Termination

- Some care is needed when defining CFGs
- Consider:

  $$S \longmapsto E$$
  $$E \longmapsto S$$

  - This grammar has nonterminal definitions that are "nonproductive". (i.e. they don't mention any terminal symbols)
  - There is no finite derivation starting from S, so the language is empty.

- Consider: $S \longmapsto (\ S\ )$
  - This grammar is productive, but again there is no finite derivation starting from S, so the language is empty

- Easily generalize these examples to a "chain" of many nonterminals, which can be harder to find in a large grammar

- Upshot: be aware of "vacuously empty" CFG grammars.
  - Every nonterminal should eventually rewrite to an alternative that contains only terminal symbols.

Associativity, ambiguity, and precedence.

GRAMMARS FOR PROGRAMMING LANGUAGES

Associativity

$S \longmapsto E + S \mid E$
$E \longmapsto \text{number} \mid (S)$

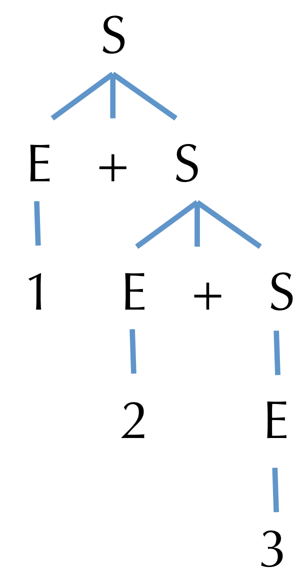
Consider the input: 1 + 2 + 3

Leftmost derivation:

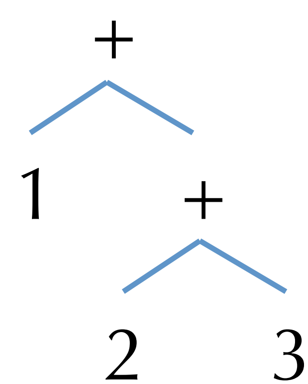
$$\begin{aligned}
\underline{\mathbf{S}} &\longmapsto \underline{\mathbf{E}} + S \\
&\longmapsto 1 + \underline{\mathbf{S}} \\
&\longmapsto 1 + \underline{\mathbf{E}} + S \\
&\longmapsto 1 + 2 + \underline{\mathbf{S}} \\
&\longmapsto 1 + 2 + \underline{\mathbf{E}} \\
&\longmapsto 1 + 2 + 3
\end{aligned}$$

Rightmost derivation:

$$\begin{aligned}
\underline{\mathbf{S}} &\longmapsto E + \underline{\mathbf{S}} \\
&\longmapsto E + E + \underline{\mathbf{S}} \\
&\longmapsto E + E + \underline{\mathbf{E}} \\
&\longmapsto E + \underline{\mathbf{E}} + 3 \\
&\longmapsto \underline{\mathbf{E}} + 2 + 3 \\
&\longmapsto 1 + 2 + 3
\end{aligned}$$



Parse Tree



AST

Associativity

- This grammar makes '+' *right associative*…
- The abstract syntax tree is the same for both 1 + 2 + 3 and 1 + (2 + 3)
- Note that the grammar is *right recursive*…

$$S \longmapsto E + S \mid E$$
$$E \longmapsto \text{number} \mid (S)$$

- How would you make '+' left associative?
- What are the trees for "1 + 2 + 3"?

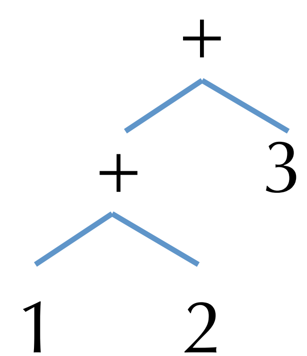
Ambiguity

- Consider this grammar:

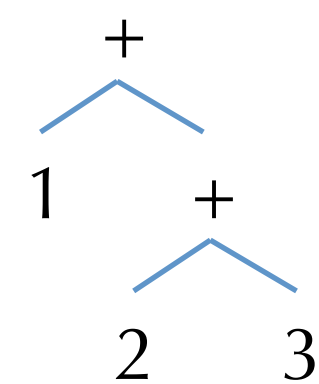
$$S \longmapsto S + S \;\mid\; (\,S\,) \;\mid\; \text{number}$$

- Claim: it accepts the *same* set of strings as the previous one.
- What's the difference?
- Consider these *two* leftmost derivations:
  - $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{S}} + S \longmapsto 1 + \underline{\mathbf{S}} \longmapsto 1 + \underline{\mathbf{S}} + S \longmapsto 1 + 2 + \underline{\mathbf{S}} \longmapsto 1 + 2 + 3$
  - $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{S}} + S \longmapsto \underline{\mathbf{S}} + S + S \longmapsto 1 + \underline{\mathbf{S}} + S \longmapsto 1 + 2 + \underline{\mathbf{S}} \longmapsto 1 + 2 + 3$

- One derivation gives left associativity, the other gives right associativity to '+'
  - Which is which?



AST 1



AST 2

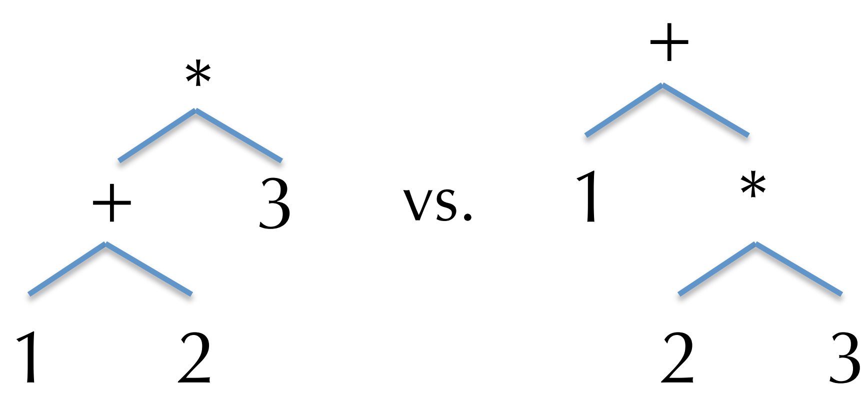
Why do we care about ambiguity?

- The '+' operation is associative, so it doesn't matter which tree we pick. Mathematically, $x + (y + z) = (x + y) + z$
  - But, some operations aren't associative. Examples?
  - Some operations are only left (or right) associative. Examples?

- Moreover, if there are multiple operations, ambiguity in the grammar leads to ambiguity in their *precedence*
- Consider:

$$S \longmapsto S + S \mid S * S \mid (S) \mid \text{number}$$

- Input: 1 + 2 * 3
  - One parse = (1 + 2) * 3 = 9
  - The other = 1 + (2 * 3) = 7



Eliminating Ambiguity

- We can often eliminate ambiguity by adding nonterminals and allowing recursion only on the left (or right) .
- Higher-precedence operators go *farther* from the start symbol.
- Example:

$$S \longmapsto S + S \;|\; S * S \;|\; (S) \;|\; \text{number}$$

- To disambiguate:
  - Decide (following math) to make '*' higher precedence than '+'
  - Make '+' left associative
  - Make '*' right associative
- Note:
  - S_2 corresponds to 'atomic' expressions

$$
\begin{aligned}
S_0 &\longmapsto S_0 + S_1 \;|\; S_1 \\
S_1 &\longmapsto S_2 * S_1 \;|\; S_2 \\
S_2 &\longmapsto \text{number} \;|\; (S_0)
\end{aligned}
$$

CFGs Summary

- Context-free grammars allow concise specifications of programming languages.
  - An unambiguous CFG specifies how to parse: convert a token stream to a (parse tree)
  - Ambiguity can (often) be removed by encoding precedence and associativity in the grammar.
- Even with an unambiguous CFG, there may be more than one derivation
  - Though all derivations correspond to the same abstract syntax tree.
- Still to come: finding a derivation
  - But first: yacc

parser.mly, lexer.mll, range.ml, ast.ml, main.ml

DEMO: BOOLEAN LOGIC