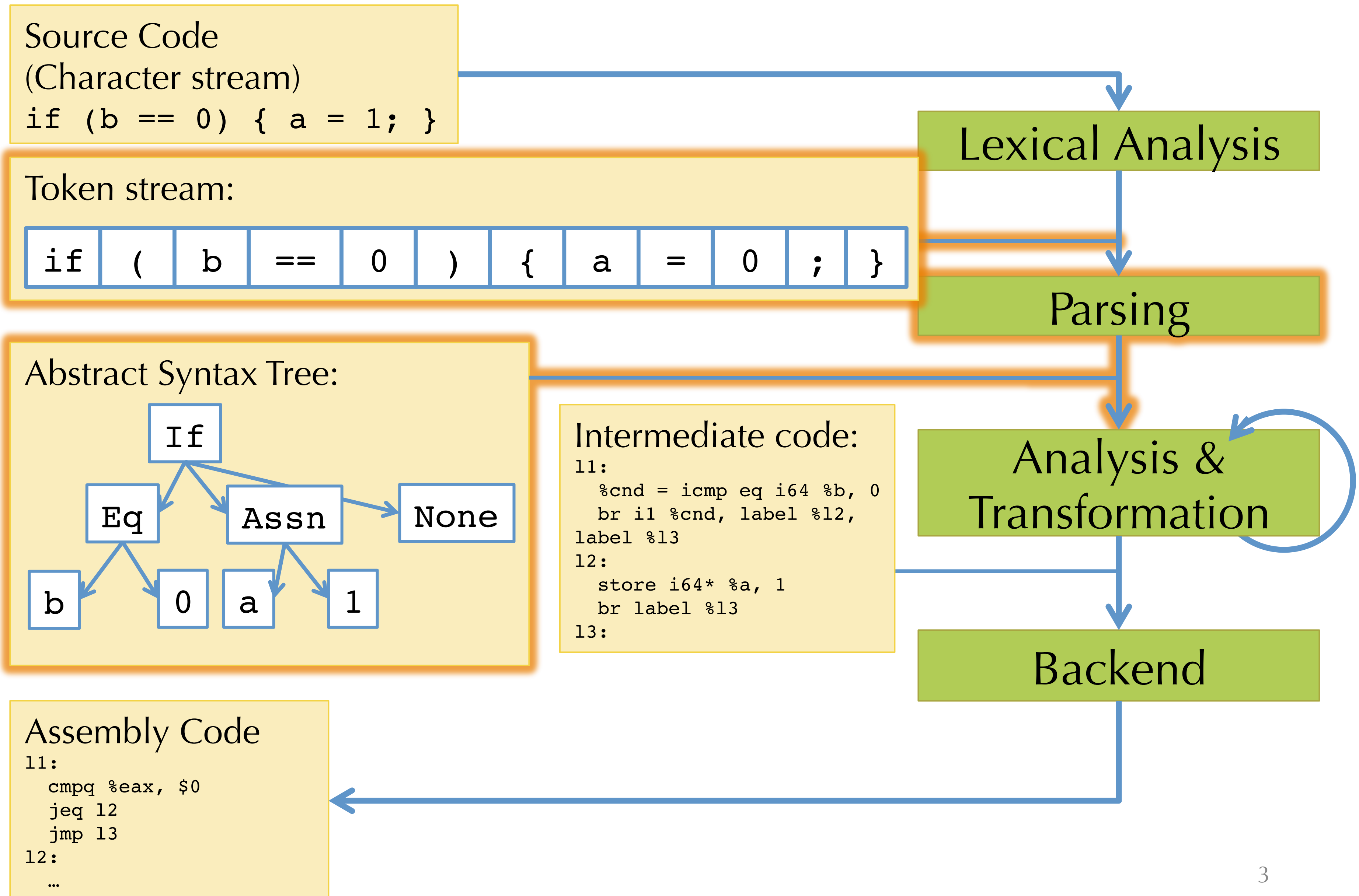
Lecture 10

CIS 341: COMPILERS

Creating an abstract representation of program syntax.

PARSING

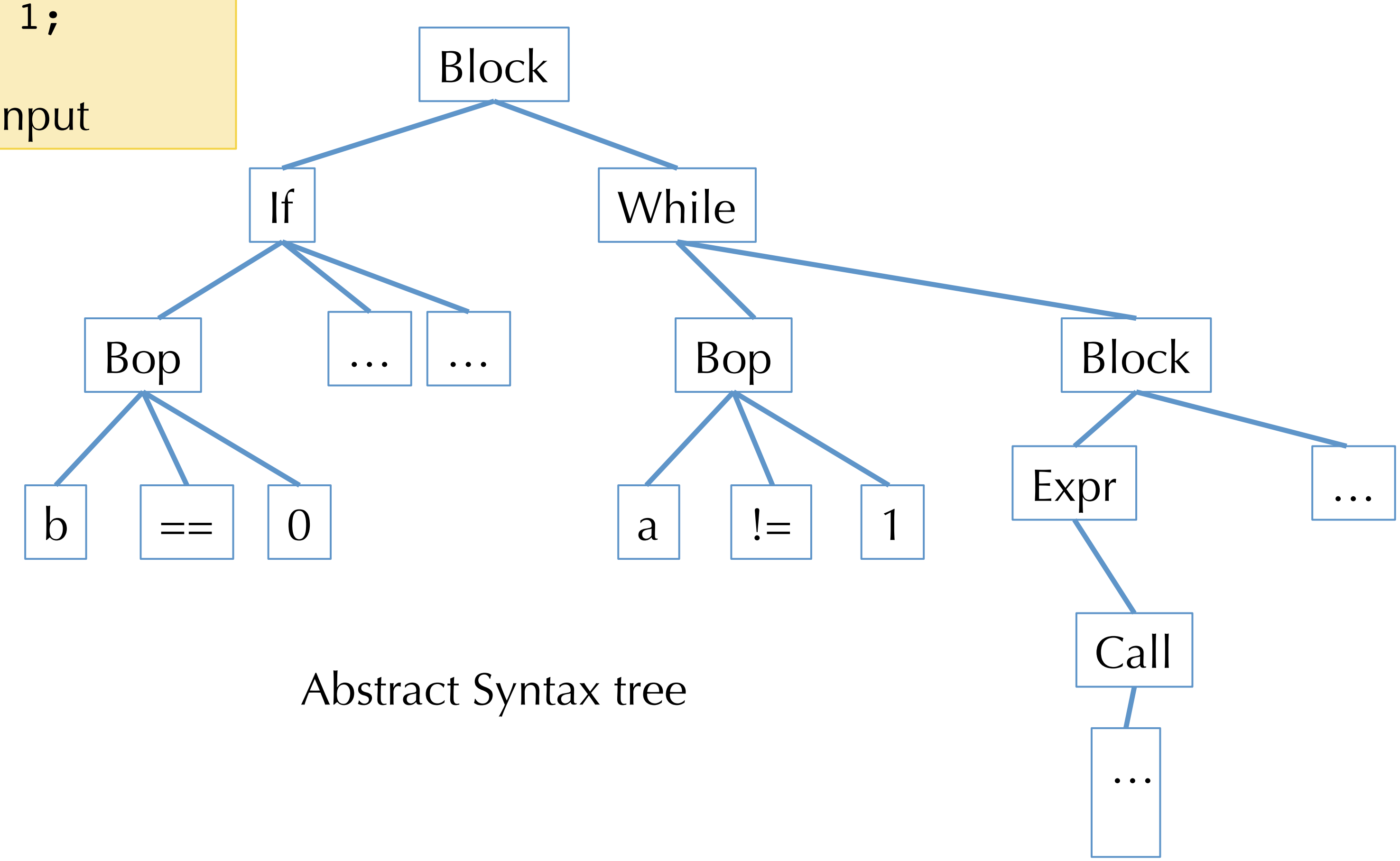
Today: Parsing



Parsing: Finding Syntactic Structure

```
{
  if (b == 0) a = b;
  while (a != 1) {
    print_int(a);
    a = a – 1;
  }
}
```

Source input



Abstract Syntax tree

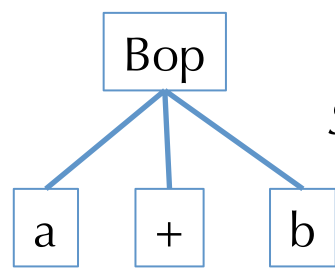
Syntactic Analysis (Parsing): Overview

- Input: stream of tokens (generated by lexer)
- Output: abstract syntax tree

- Strategy:
  - Parse the token stream to traverse the "concrete" syntax
  - During traversal, build a tree representing the "abstract" syntax

- Why abstract? Consider these three *different* concrete inputs:

```
a + b
(a + ((b)))
((a) + (b))
```



Same abstract syntax tree

- Note: parsing doesn't check many things:
  - Variable scoping, type agreement, initialization, …

Specifying Language Syntax

- First question: how to describe language syntax precisely and conveniently?
- Last time: we described tokens using regular expressions
  - Easy to implement, efficient DFA representation
  - Why not use regular expressions on tokens to specify programming language syntax?

- Limits of regular expressions:
  - DFA's have only finite # of states
  - So… DFA's can't "count"
  - For example, consider the language of all strings that contain balanced parentheses – easier than most programming languages, but not regular.

- So: we need more expressive power than DFA's

CONTEXT FREE GRAMMARS

Context-free Grammars

- Here is a specification of the language of balanced parens:

$$S \longmapsto (S)S$$
$$S \longmapsto \varepsilon$$

Note: Once again we have to take care to distinguish meta-language elements (e.g. "S" and "⟼") from object-language elements (e.g. "(").*

- The definition is *recursive* – S mentions itself.

- Idea: "derive" a string in the language by starting with S and rewriting according to the rules:
  - Example: $S \longmapsto (S)S \longmapsto ((S)S)S \longmapsto ((\varepsilon)S)S \longmapsto ((\varepsilon)S)\varepsilon \longmapsto ((\varepsilon)\varepsilon)\varepsilon = (())$

- You can replace the "nonterminal" S by its definition anywhere
- A context-free grammar accepts a string iff there is a derivation from the start symbol

* And, since we're writing this description in English, we are careful distinguish the meta-meta-language (e.g. words) from the meta-language and object-language (e.g. symbols) by using quotes.

CFGs Mathematically

- A Context-free Grammar (CFG) consists of
  - A set of *terminals* (e.g., a token or ε)
  - A set of *nonterminals* (e.g., S and other syntactic variables)
  - A designated nonterminal called the *start symbol*
  - A set of productions: LHS $\longmapsto$ RHS
    - LHS is a nonterminal
    - RHS is a *string* of terminals and nonterminals

- Example: The balanced parentheses language:

$$S \longmapsto (S)S$$

$$S \longmapsto \varepsilon$$

- How many terminals? How many nonterminals? Productions?

Another Example: Sum Grammar

- A grammar that accepts parenthesized sums of numbers:

$$
\begin{array}{lcll}
S & \longmapsto & E + S & \mid \; E \\
E & \longmapsto & \text{number} & \mid \; (S)
\end{array}
$$

e.g.: (1 + 2 + (3 + 4)) + 5

- Note the vertical bar '|' is shorthand for multiple productions:

$$
\left.
\begin{array}{l}
S \longmapsto E + S \\
S \longmapsto E \\
E \longmapsto \text{number} \\
E \longmapsto (S)
\end{array}
\right\}
\begin{array}{l}
\text{4 productions} \\
\text{2 nonterminals: S, E} \\
\text{4 terminals: (,), +, number} \\
\text{Start symbol: S}
\end{array}
$$

Derivations in CFGs

$S \longmapsto E + S \mid E$
$E \longmapsto \text{number} \mid (S)$

- Example: derive (1 + 2 + (3 + 4)) + 5
- $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{E}} + S$
  $\longmapsto (\underline{\mathbf{S}}) + S$
  $\longmapsto (\underline{\mathbf{E}} + S) + S$
  $\longmapsto (1 + \underline{\mathbf{S}}) + S$
  $\longmapsto (1 + \underline{\mathbf{E}} + S) + S$
  $\longmapsto (1 + 2 + \underline{\mathbf{S}}) + S$
  $\longmapsto (1 + 2 + \underline{\mathbf{E}}) + S$
  $\longmapsto (1 + 2 + (\underline{\mathbf{S}})) + S$
  $\longmapsto (1 + 2 + (\underline{\mathbf{E}} + S)) + S$
  $\longmapsto (1 + 2 + (3 + \underline{\mathbf{S}})) + S$
  $\longmapsto (1 + 2 + (3 + \underline{\mathbf{E}})) + S$
  $\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{S}}$
  $\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{E}}$
  $\longmapsto (1 + 2 + (3 + 4)) + 5$

For arbitrary strings α, β, γ and production rule $A \longmapsto \beta$
a single step of the derivation is:

$$\alpha A \gamma \longmapsto \alpha\beta\gamma$$

(*substitute* β for an occurrence of A)

In general, there are many possible derivations for a given string

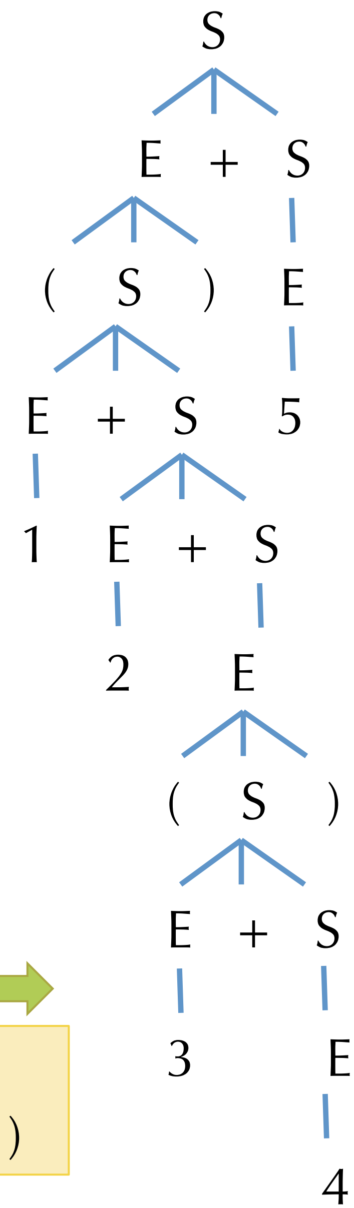
Note: Underline indicates symbol being expanded.

From Derivations to Parse Trees

- Tree representation of the derivation
- Leaves of the tree are terminals
  - In-order traversal yields the input sequence of tokens
- Internal nodes: nonterminals
- No information about the *order* of the derivation steps

- (1 + 2 + (3 + 4)) + 5

$$S \longmapsto E + S \;|\; E$$
$$E \longmapsto \text{number} \;|\; (\,S\,)$$

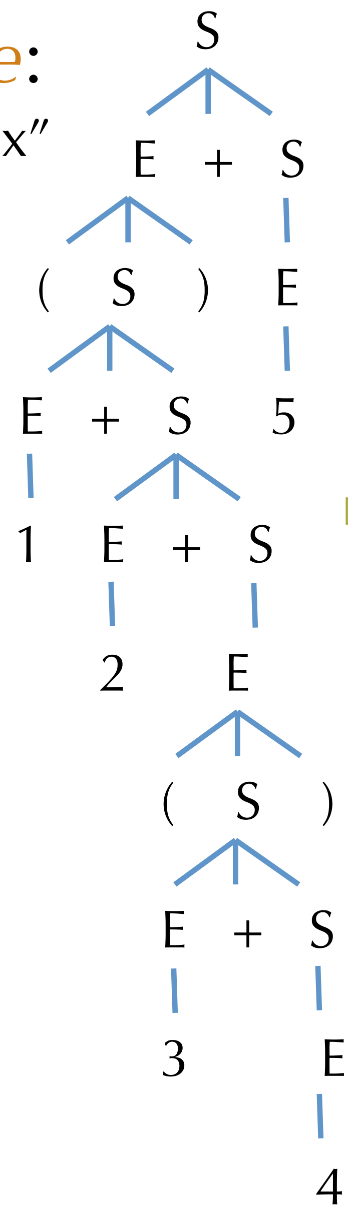


Parse Tree

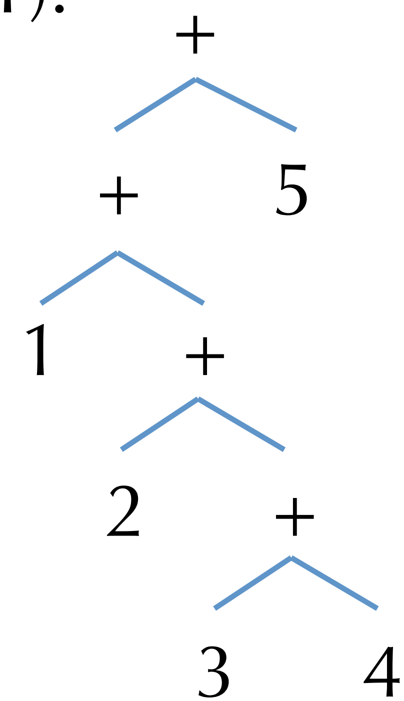
From Parse Trees to Abstract Syntax

- *Parse tree*:

"concrete syntax"



- *Abstract syntax tree* (AST):



- Hides, or *abstracts*, unneeded information.

Derivation Orders

- Productions of the grammar can be applied in any order.
- There are two standard orders:
  - *Leftmost derivation*: Find the left-most nonterminal and apply a production to it.
  - *Rightmost derivation*: Find the right-most nonterminal and apply a production there.
- Note that both strategies (and any other) yield the same parse tree!
  - Parse tree doesn't contain the information about what order the productions were applied.

Example: Left- and rightmost derivations

$$S \longmapsto E + S \mid E$$
$$E \longmapsto \text{number} \mid (\ S\ )$$

- Leftmost derivation:
- $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{E}} + S$

$$
\begin{aligned}
&\longmapsto (\underline{\mathbf{S}}) + S \\
&\longmapsto (\underline{\mathbf{E}} + S) + S \\
&\longmapsto (1 + \underline{\mathbf{S}}) + S \\
&\longmapsto (1 + \underline{\mathbf{E}} + S) + S \\
&\longmapsto (1 + 2 + \underline{\mathbf{S}}) + S \\
&\longmapsto (1 + 2 + \underline{\mathbf{E}}) + S \\
&\longmapsto (1 + 2 + (\underline{\mathbf{S}})) + S \\
&\longmapsto (1 + 2 + (\underline{\mathbf{E}} + S)) + S \\
&\longmapsto (1 + 2 + (3 + \underline{\mathbf{S}})) + S \\
&\longmapsto (1 + 2 + (3 + \underline{\mathbf{E}})) + S \\
&\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{S}} \\
&\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{E}} \\
&\longmapsto (1 + 2 + (3 + 4)) + 5
\end{aligned}
$$

Rightmost derivation:

$\underline{\mathbf{S}} \longmapsto E + \underline{\mathbf{S}}$

$$
\begin{aligned}
&\longmapsto E + \underline{\mathbf{E}} \\
&\longmapsto \underline{\mathbf{E}} + 5 \\
&\longmapsto (\underline{\mathbf{S}}) + 5 \\
&\longmapsto (E + \underline{\mathbf{S}}) + 5 \\
&\longmapsto (E + E + \underline{\mathbf{S}}) + 5 \\
&\longmapsto (E + E + \underline{\mathbf{E}}) + 5 \\
&\longmapsto (E + E + (\underline{\mathbf{S}})) + 5 \\
&\longmapsto (E + E + (E + \underline{\mathbf{S}})) + 5 \\
&\longmapsto (E + E + (E + \underline{\mathbf{E}})) + 5 \\
&\longmapsto (E + E + (\underline{\mathbf{E}} + 4)) + 5 \\
&\longmapsto (E + \underline{\mathbf{E}} + (3 + 4)) + 5 \\
&\longmapsto (\underline{\mathbf{E}} + 2 + (3 + 4)) + 5 \\
&\longmapsto (1 + 2 + (3 + 4)) + 5
\end{aligned}
$$

Loops and Termination

- Some care is needed when defining CFGs
- Consider:

$$\begin{aligned} S &\mapsto E \\ E &\mapsto S \end{aligned}$$

  - This grammar has nonterminal definitions that are "nonproductive". (i.e. they don't mention any terminal symbols)
  - There is no finite derivation starting from S, so the language is empty.

- Consider: $S \mapsto (\ S\ )$

  - This grammar is productive, but again there is no finite derivation starting from S, so the language is empty

- Easily generalize these examples to a "chain" of many nonterminals, which can be harder to find in a large grammar

- Upshot: be aware of "vacuously empty" CFG grammars.
  - Every nonterminal should eventually rewrite to an alternative that contains only terminal symbols.

Associativity, ambiguity, and precedence.

GRAMMARS FOR PROGRAMMING LANGUAGES

Associativity

$S \longmapsto E + S \mid E$
$E \longmapsto \text{number} \mid (\ S\ )$

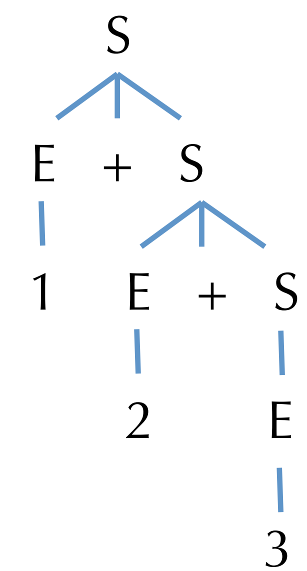
Consider the input: $1 + 2 + 3$

Leftmost derivation:

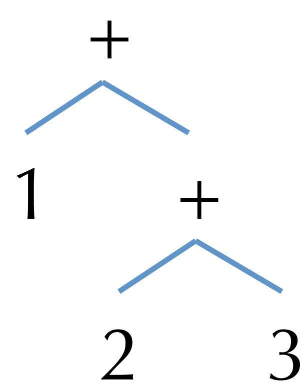
$\underline{\mathbf{S}} \longmapsto \underline{\mathbf{E}} + S$
$\longmapsto 1 + \underline{\mathbf{S}}$
$\longmapsto 1 + \underline{\mathbf{E}} + S$
$\longmapsto 1 + 2 + \underline{\mathbf{S}}$
$\longmapsto 1 + 2 + \underline{\mathbf{E}}$
$\longmapsto 1 + 2 + 3$

Rightmost derivation:

$\underline{\mathbf{S}} \longmapsto E + \underline{\mathbf{S}}$
$\longmapsto E + E + \underline{\mathbf{S}}$
$\longmapsto E + E + \underline{\mathbf{E}}$
$\longmapsto E + \underline{\mathbf{E}} + 3$
$\longmapsto \underline{\mathbf{E}} + 2 + 3$
$\longmapsto 1 + 2 + 3$



Parse Tree



AST

Associativity

- This grammar makes '+' *right associative*…
- The abstract syntax tree is the same for both 1 + 2 + 3 and 1 + (2 + 3)
- Note that the grammar is *right recursive*…

$$
\begin{aligned}
S &\longmapsto E + S \mid E \\
E &\longmapsto \text{number} \mid (S)
\end{aligned}
$$

- How would you make '+' left associative?
- What are the trees for "1 + 2 + 3"?

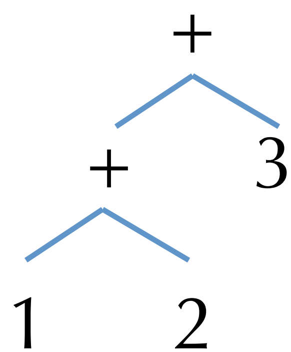
Ambiguity

- Consider this grammar:

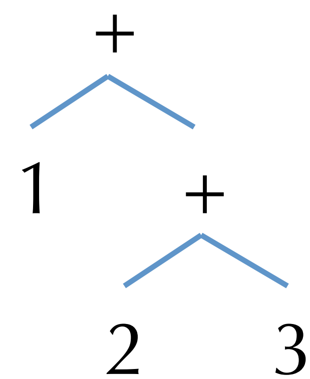
$$S \longmapsto S + S \;\;|\;\; (S) \;\;|\;\; \text{number}$$

- Claim: it accepts the *same* set of strings as the previous one.
- What's the difference?
- Consider these *two* leftmost derivations:
  - $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{S}} + S \longmapsto 1 + \underline{\mathbf{S}} \longmapsto 1 + \underline{\mathbf{S}} + S \longmapsto 1 + 2 + \underline{\mathbf{S}} \longmapsto 1 + 2 + 3$
  - $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{S}} + S \longmapsto \underline{\mathbf{S}} + S + S \longmapsto 1 + \underline{\mathbf{S}} + S \longmapsto 1 + 2 + \underline{\mathbf{S}} \longmapsto 1 + 2 + 3$

- One derivation gives left associativity, the other gives right associativity to '+'
  - Which is which?



AST 1



AST 2

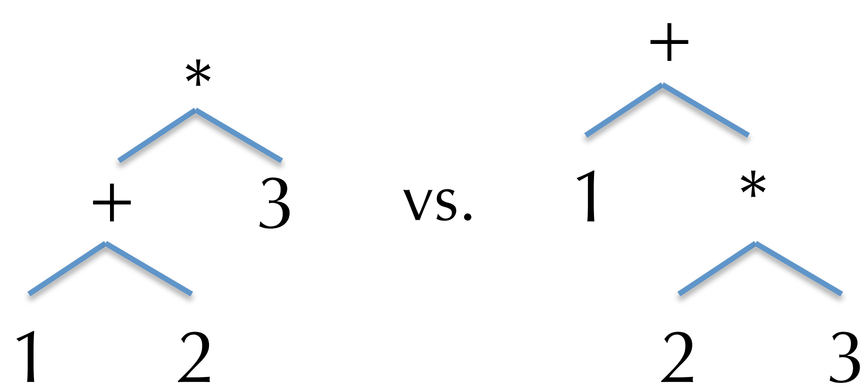
Why do we care about ambiguity?

- The '+' operation is associative, so it doesn't matter which tree we pick. Mathematically, $x + (y + z) = (x + y) + z$
  - But, some operations aren't associative. Examples?
  - Some operations are only left (or right) associative. Examples?

- Moreover, if there are multiple operations, ambiguity in the grammar leads to ambiguity in their *precedence*
- Consider:

$$S \longmapsto S + S \mid S * S \mid (S) \mid \text{number}$$

- Input: 1 + 2 * 3
  - One parse = (1 + 2) * 3 = 9
  - The other = 1 + (2 * 3) = 7



Eliminating Ambiguity

- We can often eliminate ambiguity by adding nonterminals and allowing recursion only on the left (or right) .
- Higher-precedence operators go *farther* from the start symbol.
- Example:

$$S \mapsto S + S \mid S * S \mid (S) \mid \text{number}$$

- To disambiguate:
  - Decide (following math) to make '*' higher precedence than '+'
  - Make '+' left associative
  - Make '*' right associative
- Note:
  - S_2 corresponds to 'atomic' expressions

$$\begin{array}{lcll} S_0 & \mapsto & S_0 + S_1 & \mid S_1 \\ S_1 & \mapsto & S_2 * S_1 & \mid S_2 \\ S_2 & \mapsto & \text{number} & \mid (S_0) \end{array}$$

CFGs Summary

- Context-free grammars allow concise specifications of programming languages.
  - An unambiguous CFG specifies how to parse: convert a token stream to a (parse tree)
  - Ambiguity can (often) be removed by encoding precedence and associativity in the grammar.

- Even with an unambiguous CFG, there may be more than one derivation
  - Though all derivations correspond to the same abstract syntax tree.

- Still to come: finding a derivation
  - But first: yacc

parser.mly, lexer.mll, range.ml, ast.ml, main.ml

DEMO: BOOLEAN LOGIC