Lecture 16

CIS 341: COMPILERS

Announcements

- Reminder: HW4 Compiling OAT v.1
- DUE: Thursday, March 26th
- ***START TODAY! (IF YOU HAVEN'T ALREADY)***

See fun.ml Eval2 and Eval3

dynamic scoping vs. static scoping

ENVIRONMENT-BASED INTERPRETERS

Compiling lambda calculus to straight-line code.
Representing evaluation environments at runtime.

CLOSURE CONVERSION

Compiling First-class Functions

- To implement first-class functions on a processor, there are two problems:
  - First: we must implement substitution of free variables
  - Second: we must separate 'code' from 'data'

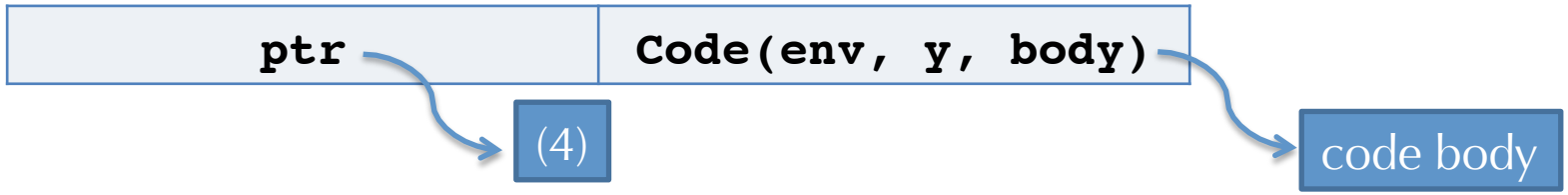
- Reify the substitution:
  - Move substitution from the meta language to the object language by making the data structure & lookup operation explicit
  - The environment-based interpreter is one step in this direction
- Closure Conversion:
  - Eliminates free variables by packaging up the needed environment in the data structure.
- Hoisting:
  - Separates code from data, pulling closed code to the top level.

Example of closure creation

- Recall the "add" function:
  `let add = fun x -> fun y -> x + y`

- Consider the inner function: `fun y -> x + y`

- When run the function application: `add 4`
  the program builds a closure and returns it.
  - The closure is a pair of the environment and a code pointer.



- The code pointer takes a pair of parameters: env and y
  - The function code is (essentially):
    `fun (env, y) -> let x = nth env 0 in x + y`

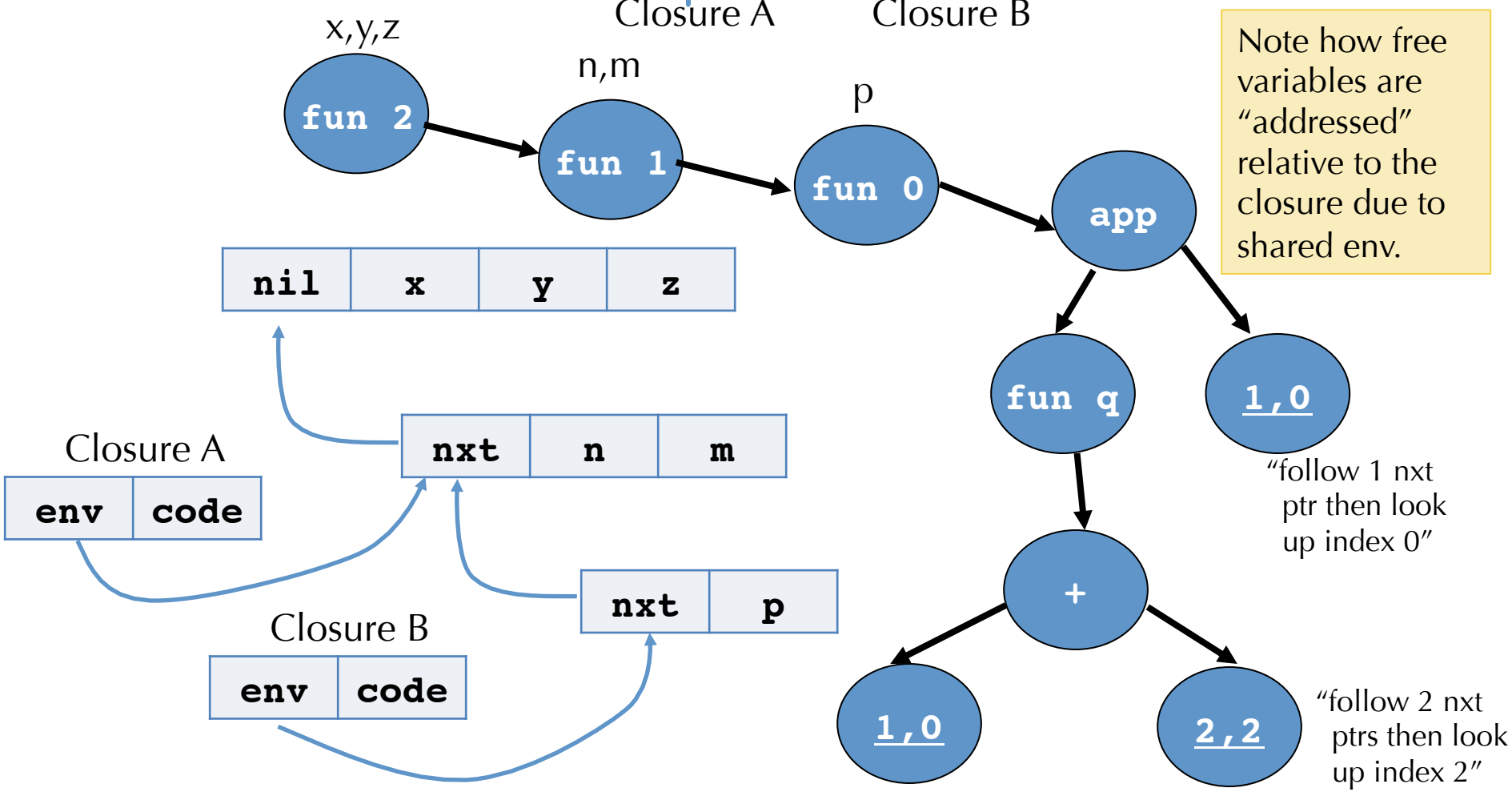
Representing Closures

- As we saw, the simple closure conversion algorithm doesn't generate very efficient code.
  - It stores all the values for variables in the environment, even if they aren't needed by the function body.
  - It copies the environment values each time a nested closure is created.
  - It uses a linked-list datastructure for tuples.

- There are many options:
  - Store only the values for free variables in the body of the closure.
  - Share subcomponents of the environment to avoid copying
  - Use vectors or arrays rather than linked structures

Array-based Closures with N-ary Functions

```
(fun (x y z) ->
   (fun (n m) -> (fun p -> (fun q -> n + z) x)
```



BACK TO TYPECHECKING

Type Checking / Static Analysis

- Recall the interpreter from the Eval3 module:

```
let rec eval env e =
  match e with
   | …
   | Add (e1, e2) ->
     (match (eval env e1, eval env e2) with
        | (IntV i1, IntV i2) -> IntV (i1 + i2)
        | _ -> failwith "tried to add non-integers")
   | …
```

- The interpreter might fail at runtime.
  - Not all operations are defined for all values (e.g. 3/0, 3 + true, …)
- A compiler can't generate sensible code for this case.
  - A naïve implementation might "add" an integer and a pointer

Notes about this Typechecker

- In the interpreter, we only evaluate the body of a function when it's applied.
- In the typechecker, we always check the body of the function (even if it's never applied.)
  - Because of this, we must *assume* the input has some type (say t_1) and reflect this in the type of the function (t_1 `->` t_2).
- Dually, at a call site (e_1 e_2), we don't know what *closure* we're going to get.
  - But we can calculate e_1's type, check that e_2 is an argument of the right type, and also determine what type e_1 will return.

- Question: Why is this an approximation?
- Question: What if `well_typed` always returns `false`?

Type Judgments

- In the judgment: $E \vdash e : t$
  - E is a *typing environment* or a *type context*
  - E maps variables to types. It is just a set of bindings of the form: $x_1 : t_1, x_2 : t_2, \ldots, x_n : t_n$

- For example: $x : int, b : bool \vdash$ if (b) 3 else x : int

- What do we need to know to decide whether "if (b) 3 else x" has type int in the environment x : int, b : bool?
  - b must be a bool i.e. $x : int, b : bool \vdash b : bool$
  - 3 must be an int i.e. $x : int, b : bool \vdash 3 : int$
  - x must be an int i.e. $x : int, b : bool \vdash x : int$

Simply-typed Lambda Calculus

- For the language in "tc.ml" we have five inference rules:

INT

$$\frac{}{E \vdash i : \text{int}}$$

VAR

$$\frac{x : T \in E}{E \vdash x : T}$$

ADD

$$\frac{E \vdash e_1 : \text{int} \qquad E \vdash e_2 : \text{int}}{E \vdash e_1 + e_2 : \text{int}}$$

FUN

$$\frac{E, x : T \vdash e : S}{E \vdash \text{fun } (x{:}T) \text{ -> } e \ : T \text{ -> } S}$$

APP

$$\frac{E \vdash e_1 : T \text{ -> } S \qquad E \vdash e_2 : T}{E \vdash e_1\ e_2 : S}$$

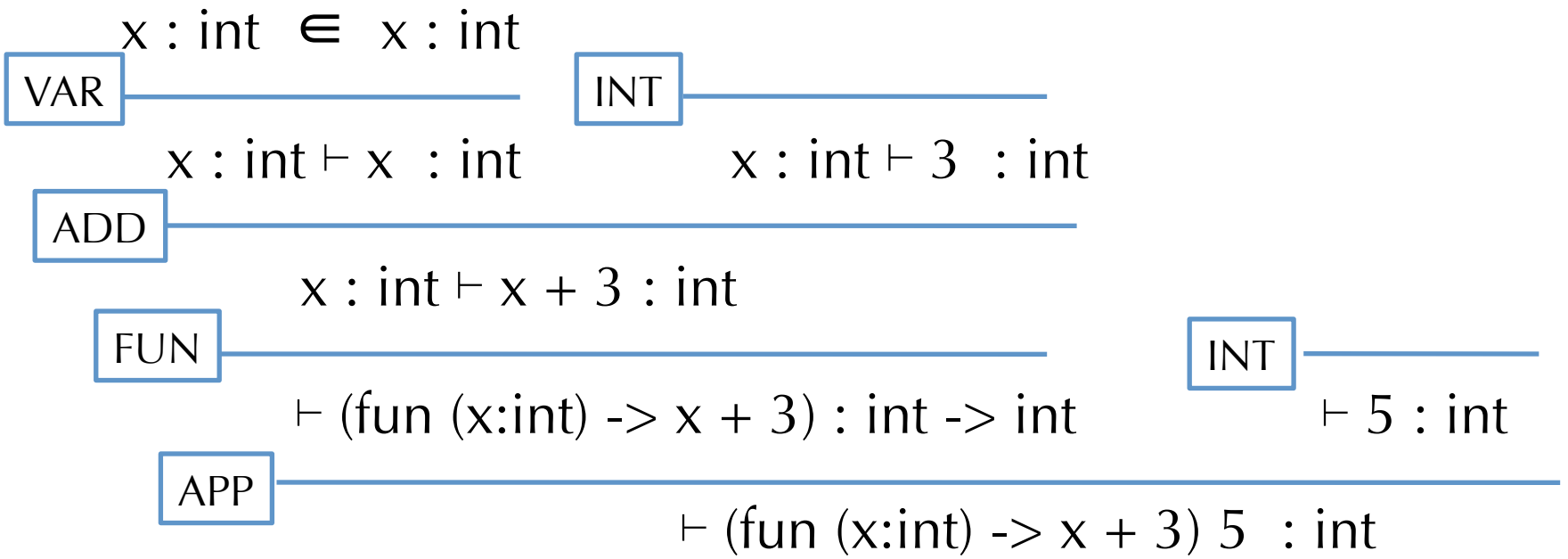
- Note how these rules correspond to the code.

Type Checking Derivations

- A *derivation* or *proof tree* has (instances of) judgments as its nodes and edges that connect premises to a conclusion according to an inference rule.
- Leaves of the tree are *axioms* (i.e. rules with no premises)
  - Example: the INT rule is an axiom
- Goal of the typechecker: verify that such a tree exists.
- Example: Find a tree for the following program using the inference rules on the previous slide:

$$\vdash \text{(fun (x:int) -> x + 3) 5 : int}$$

Example Derivation Tree



- Note: the OCaml function `typecheck` verifies the existence of this tree. The structure of the recursive calls when running `typecheck` is the same shape as this tree!
- Note that x : int ∈ E is implemented by the function `lookup`

Arrays

- Array constructs are not hard either, here is one possibility
- First: add a new type constructor: T[]

NEW

$$\frac{E \vdash e_1 : \text{int} \qquad E \vdash e_2 : T}{E \vdash \text{new } T[e_1](e_2) \; : T[]}$$

e_1 is the size of the newly allocated array. e_2 is initializes the elements of the array.

INDEX

$$\frac{E \vdash e_1 : T[] \qquad E \vdash e_2 : \text{int}}{E \vdash e_1[e_2] \; : T}$$

UPDATE

$$\frac{E \vdash e_1 : T[] \quad E \vdash e_2 : \text{int} \quad E \vdash e_3 : T}{E \vdash e_1[e_2] = e_3 \text{ ok}}$$

Note: These rules don't ensure that the array index is in bounds – that should be checked dynamically.

Tuples

- ML-style tuples with statically known number of products:
- First: add a new type constructor: $T_1 * \ldots * T_n$

TUPLE

$$\frac{E \vdash e_1 : T_1 \quad \ldots \quad E \vdash e_n : T_n}{E \vdash (e_1, \ldots, e_n) : T_1 * \ldots * T_n}$$

PROJ

$$\frac{E \vdash e : T_1 * \ldots * T_n \quad 1 \leq i \leq n}{E \vdash \#i\ e\ :\ T_i}$$

References

- ML-style references (note that ML uses only expressions)
- First, add a new type constructor: T ref

REF

$$\frac{E \vdash e : T}{E \vdash \text{ref } e : T \text{ ref}}$$

DEREF

$$\frac{E \vdash e : T \text{ ref}}{E \vdash !e : T}$$

ASSIGN

$$\frac{E \vdash e_1 : T \text{ ref} \qquad E \vdash e_2 : T}{E \vdash e_1 := e_2 : \text{unit}}$$

Note the similarity with the rules for arrays…

Recursive Definitions

- Consider the ML factorial function:

```
let rec fact (x:int) : int =
 if (x == 0) 1 else x * fact(x-1)
```

- Note that the function name `fact` appears inside the body of `fact`'s definition!
- To typecheck the body of fact, we must assume that the type of fact is already known.

$$\frac{E, \text{fact} : \text{int} \to \text{int}, x : \text{int} \vdash e_{body} : \text{int}}{E \vdash \text{int fact(int x) (} e_{body}\text{)} : \text{int} \to \text{int}}$$

- In general: Collect the names and types of all mutually recursive definitions, add them all to the context E before checking any of the definition bodies.
- Often useful to separate the "global context" from the "local context"

Beyond describing “structure”… describing “properties”
Types as sets
Subsumption

TYPES, MORE GENERALLY

What are types, anyway?

- A *type* is just a predicate on the set of values in a system.
  - For example, the type "int" can be thought of as a boolean function that returns "true" on integers and "false" otherwise.
  - Equivalently, we can think of a type as just a *subset* of all values.

- For efficiency and tractability, the predicates are usually taken to be very simple.
  - Types are an *abstraction* mechanism

- We can easily add new types that distinguish different subsets of values:

```
type tp =
     | IntT                    (* type of integers *)
     | PosT | NegT | ZeroT     (* refinements of ints *)
     | BoolT                   (* type of booleans *)
     | TrueT | FalseT          (* subsets of booleans *)
     | AnyT                    (* any value *)
```

Modifying the typing rules

- We need to refine the typing rules too…
- Some easy cases:
  - Just split up the integers into their more refined cases:

P-INT

$$\frac{i > 0}{E \vdash i : Pos}$$

N-INT

$$\frac{i < 0}{E \vdash i : Neg}$$

ZERO

$$\frac{}{E \vdash 0 : Zero}$$

- Same for booleans:

TRUE

$$\frac{}{E \vdash true : True}$$

FALSE

$$\frac{}{E \vdash false : False}$$

What about "if"?

- Two cases are easy:

IF-T
$$\frac{E \vdash e_1 : \text{True} \quad E \vdash e_2 : T}{E \vdash \text{if } (e_1)\ e_2 \text{ else } e_3 : T}$$

IF-F
$$\frac{E \vdash e_1 : \text{False} \quad E \vdash e_3 : T}{E \vdash \text{if } (e_1)\ e_2 \text{ else } e_3 : T}$$

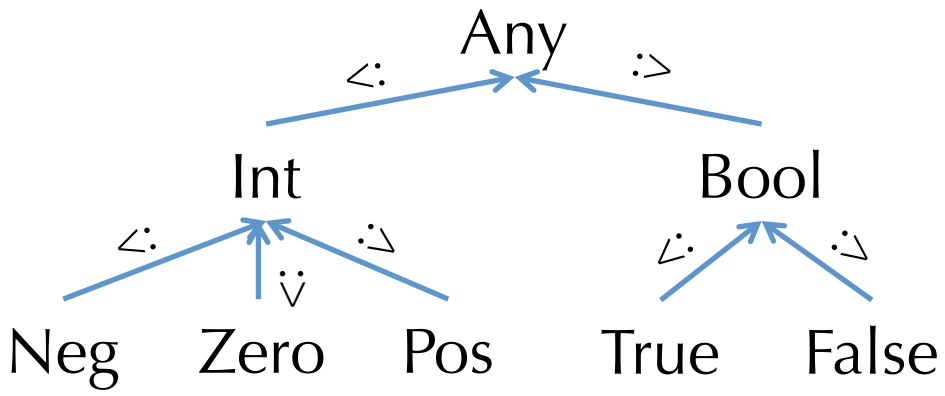
- What happens when we don't know statically which branch will be taken?
- Consider the typechecking problem:

$$x{:}\text{bool} \vdash \text{if } (x)\ 3 \text{ else } -1 : ?$$

- The true branch has type Pos and the false branch has type Neg.
  - What should be the result type of the whole if?

Subtyping and Upper Bounds

- If we think of types as sets of values, we have a natural inclusion relation: Pos ⊆ Int
- This subset relation gives rise to a *subtype* relation: Pos <: Int
- Such inclusions give rise to a *subtyping hierarchy*:



- Given any two types T_1 and T_2, we can calculate their *least upper bound* (LUB) according to the hierarchy.
  - Example: LUB(True, False) = Bool, LUB(Int, Bool) = Any
  - Note: might want to add types for "NonZero", "NonNegative", and "NonPositive" so that set union on values corresponds to taking LUBs on types.

"If" Typing Rule Revisited

- For statically unknown conditionals, we want the return value to be the LUB of the types of the branches:

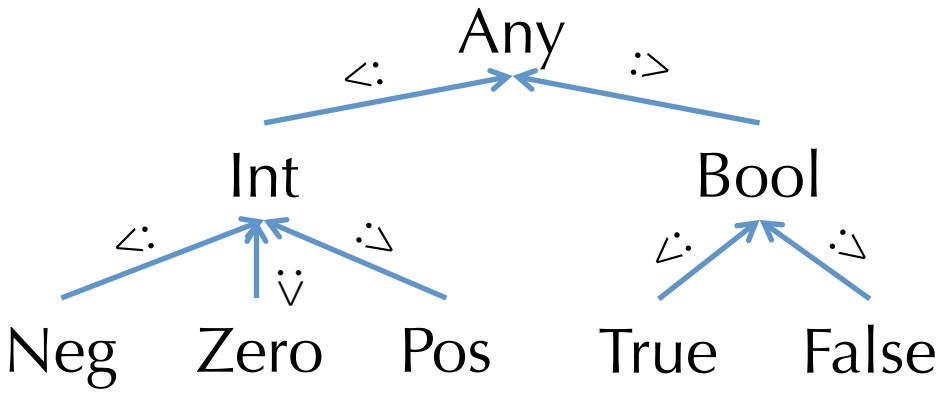
IF-BOOL

$$\frac{E \vdash e_1 : \text{bool} \quad E \vdash e_2 : T_1 \quad E \vdash e_3 : T_2}{E \vdash \text{if } (e_1)\ e_2 \text{ else } e_3 : \text{LUB}(T_1, T_2)}$$

- Note that LUB(T_1, T_2) is the most precise type (according to the hierarchy) that is able to describe any value that has either type T_1 or type T_2.
- In math notation, LUB(T1, T2) is sometimes written $T_1 \vee T_2$
- LUB is also called the *join* operation.

Subtyping Hierarchy

- A *subtyping hierarchy*:



- The subtyping relation is a *partial order*:
  - Reflexive: $T <: T$ for any type T
  - Transitive: $T_1 <: T_2$ and $T_2 <: T_3$ then $T_1 <: T_3$
  - Antisymmetric: If $T_1 <: T_2$ and $T_2 <: T_1$ then $T_1 = T_2$

Soundness of Subtyping Relations

- We don't have to treat *every* subset of the integers as a type.
  - e.g., we left out the type NonNeg

- A subtyping relation $T_1 <: T_2$ is *sound* if it approximates the underlying semantic subset relation.
- Formally: write $[\![T]\!]$ for the subset of (closed) values of type T
  - i.e. $[\![T]\!] = \{v \mid \vdash v : T\}$
  - e.g. $[\![\text{Zero}]\!] = \{0\}$, $[\![\text{Pos}]\!] = \{1, 2, 3, \ldots\}$

- If $T_1 <: T_2$ implies $[\![T_1]\!] \subseteq [\![T_2]\!]$, then $T_1 <: T_2$ is sound.
  - e.g. Pos <: Int is sound, since $\{1,2,3,\ldots\} \subseteq \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$
  - e.g. Int <: Pos is not sound, since it is *not* the case that $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\} \subseteq \{1,2,3,\ldots\}$

Soundness of LUBs

- Whenever you have a sound subtyping relation, it follows that:

$$[\![\text{LUB}(T_1, T_2)]\!] \supseteq [\![T_1]\!] \cup [\![T_2]\!]$$

  - Note that the LUB is an over approximation of the "semantic union"
  - Example: $[\![\text{LUB}(\text{Zero}, \text{Pos})]\!] = [\![\text{Int}]\!] = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\} \supseteq \{0,1,2,3,\ldots\} = \{0\} \cup \{1,2,3,\ldots\} = [\![\text{Zero}]\!] \cup [\![\text{Pos}]\!]$

- Using LUBs in the typing rules yields sound approximations of the program behavior (as if the IF-B rule).
- It just so happens that LUBs on types <: Int correspond to +

ADD

$$\frac{E \vdash e_1 : T_1 \quad E \vdash e_2 : T_2 \quad T_1 <: \text{Int} \quad T_2 <: \text{Int}}{E \vdash e_1 + e_2 : T_1 \vee T_2}$$

Subsumption Rule

- When we add subtyping judgments of the form T <: S we can uniformly integrate it into the type system generically:

SUBSUMPTION

$$\frac{E \vdash e : T \qquad T <: S}{E \vdash e : S}$$

- Subsumption allows any value of type T to be treated as an S whenever T <: S.
- Adding this rule makes the search for typing derivations more difficult – this rule can be applied anywhere, since T <: T.
  - But careful engineering of the typing system can incorporate the subsumption rule into a deterministic algorithm.

Downcasting

- What happens if we have an Int but need something of type Pos?
  - At compile time, we don't know whether the Int is greater than zero.
  - At run time, we do.

- Add a "checked downcast"

$$\frac{E \vdash e_1 : \text{Int} \qquad E, x : \text{Pos} \vdash e_2 : T_2 \qquad E \vdash e_3 : T_3}{E \vdash \text{ifPos } (x = e_1)\ e_2 \text{ else } e_3 : T_2 \vee T_3}$$

- At runtime, ifPos checks whether e_1 is > 0. If so, branches to e_2 and otherwise branches to e_3.
- Inside the expression e_2, x is the name for e_1's value, which is known to be strictly positive because of the dynamic check.
- Note that such rules force the programmer to add the appropriate checks
  - We could give integer division the type: Int -> NonZero -> Int

SUBTYPING OTHER TYPES

Extending Subtyping to Other Types

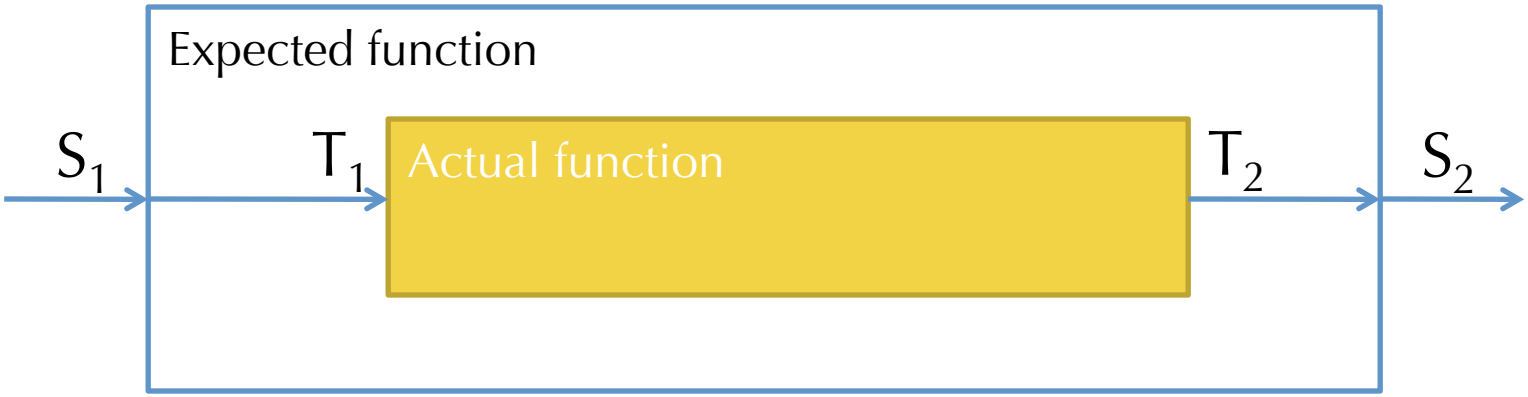
- What about subtyping for tuples?
  - Intuition: whenever a program expects something of type $S_1 * S_2$, it is sound to give it a $T_1 * T_2$.
  - Example: (Pos * Neg) <: (Int * Int)

$$\frac{T_1 <: S_1 \quad T_2 <: S_2}{(T_1 * T_2) <: (S_1 * S_2)}$$

- What about functions?
- When is $T_1 \to T_2 <: S_1 \to S_2$?

Subtyping for Function Types

- One way to see it:



- Need to convert an S1 to a T1 and T2 to S2, so the argument type is *contravariant* and the output type is *covariant*.

$$\frac{S_1 <: T_1 \quad T_2 <: S_2}{(T_1 \to T_2) <: (S_1 \to S_2)}$$

Immutable Records

- Record type: $\{lab_1{:}T_1;\ lab_2{:}T_2;\ \ldots\ ;\ lab_n{:}T_n\}$
  - Each lab_i is a label drawn from a set of identifiers.

RECORD

$$\frac{E \vdash e_1 : T_1 \qquad E \vdash e_2 : T_2 \quad \ldots \quad E \vdash e_n : T_n}{E \vdash \{lab_1 = e_1;\ lab_2 = e_2;\ \ldots\ ;\ lab_n = e_n\} : \{lab_1{:}T_1;\ lab_2{:}T_2;\ \ldots\ ;\ lab_n{:}T_n\}}$$

PROJECTION

$$\frac{E \vdash e : \{lab_1{:}T_1;\ lab_2{:}T_2;\ \ldots\ ;\ lab_n{:}T_n\}}{E \vdash e.lab_i : T_i}$$

Immutable Record Subtyping

- Depth subtyping:
  - Corresponding fields may be subtypes

DEPTH

$$\frac{T_1 <: U_1 \quad T_2 <: U_2 \quad \ldots \quad T_n <: U_n}{\{lab_1:T_1;\ lab_2:T_2;\ \ldots\ ;\ lab_n:T_n\} <: \{lab_1:U_1;\ lab_2:U_2;\ \ldots\ ;\ lab_n:U_n\}}$$

- Width subtyping:
  - Subtype record may have *more* fields:

WIDTH

$$\frac{m \leq n}{\{lab_1:T_1;\ lab_2:T_2;\ \ldots\ ;\ lab_n:T_n\} <: \{lab_1:T_1;\ lab_2:T_2;\ \ldots\ ;\ lab_m:T_m\}}$$

Immutable Record Subtyping (cont'd)

- Width subtyping assumes an implementation in which order of fields in a record matters:

  $\{x{:}int;\ y{:}int\} \neq \{y{:}int;\ x{:}int\}$

- But: $\{x{:}int;\ y{:}int;\ z{:}int\} <: \{x{:}int;\ y{:}int\}$
  - Implementation: a record is a struct, subtypes just add fields at the *end* of the struct.

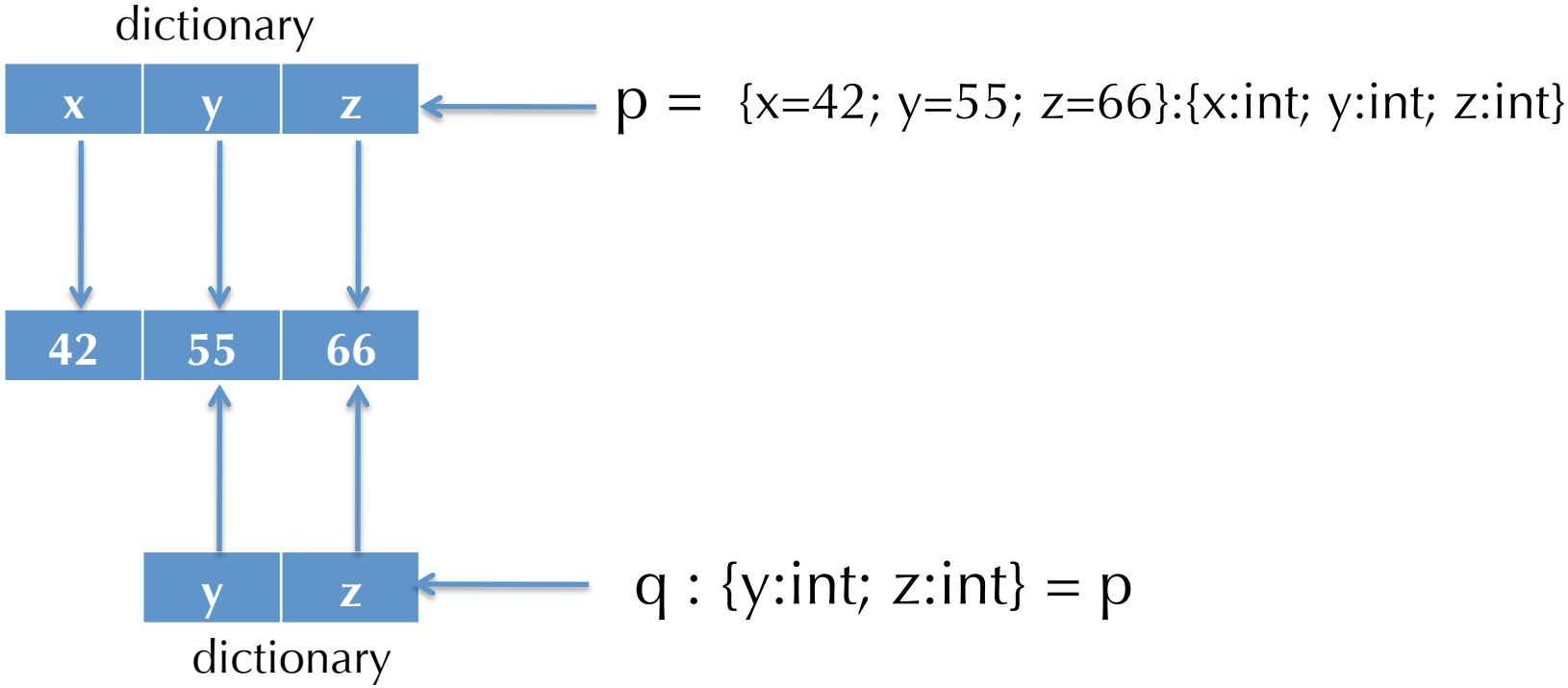
- Alternative: allow permutation of record fields:

  $\{x{:}int;\ y{:}int\} = \{y{:}int;\ x{:}int\}$
  - Implementation: compiler sorts the fields before code generation.
  - Need to know *all* of the fields to generate the code
- Permutation is not directly compatible with width subtyping:

  $\{x{:}int;\ z{:}int;\ y{:}int\} = \{x{:}int;\ y{:}int;\ z{:}int\} \not<: \{y{:}int;\ z{:}int\}$

If you want both:

- If you want permutability & dropping, you need to either copy (to rearrange the fields) or use a dictionary like this:



Subtyping and References

- What is the proper subtyping relationship for references and arrays?

- Suppose we have NonZero as a type and the division operation has type: Int -> NonZero -> Int
  - Recall that NonZero <: Int
- Should (NonZero ref) <: (Int ref) ?
- Consider this program:

```
Int bad(NonZero ref r) {
  Int ref a = r;    (* OK because (NonZero ref <: Int ref*)
  a := 0;           (* OK because 0 : Zero <: Int *)
  return (42 / !r) (* OK because !r has type NonZero *)
}
```

Mutable Structures are Invariant

- Covariant reference types are unsound
  - As demonstrated in the previous example
- Contravariant reference types are also unsound
  - i.e. If $T_1 <: T_2$ then ref $T_2 <:$ ref T_1 is also unsound
  - Exercise: construct a program that breaks contravariant references.

- Moral: Mutable structures are invariant:

  T_1 ref $<: T_2$ ref implies $T_1 = T_2$

- Same holds for arrays, OCaml-style mutable records, object fields, etc.
  - Note: Java and C# get this wrong. They allows covariant array subtyping, but then compensate by adding a dynamic check on *every* array update!

Another Way to See It

- We can think of a reference cell as an immutable record (object) with two functions (methods) and some hidden state:

  T ref ≃ {get: unit -> T; set: T -> unit}

  - get returns the value hidden in the state.
  - set updates the value hidden in the state.

- When is T ref <: S ref?
- Records are like tuples: subtyping extends pointwise over each component.
- {get: unit -> T; set: T -> unit} <: {get: unit -> S; set: S -> unit}
  - get components are subtypes: unit -> T <: unit -> S
    set components are subtypes: T -> unit <: S -> unit
- From get, we must have T <: S (covariant return)
- From set, we must have S <: T (contravariant arg.)
- From T <: S and S <: T we conclude T = S.

STRUCTURAL VS. NOMINAL TYPES

Structural vs. Nominal Typing

- Is type equality / subsumption defined by the *structure* of the data or the *name* of the data?
- Example 1: type abbreviations (OCaml) vs. "newtypes" (a la Haskell)

```
(* OCaml: *)
type cents = int    (* cents = int in this scope *)
type age = int

let foo (x:cents) (y:age) = x + y
```

```
(* Haskell: *)
newtype Cents = Cents Integer  (* Integer and Cents arr
                                  isomorphic, not identical. *)
newtype Age = Age Integer

foo :: Cents -> Age -> Int
foo x y = x + y                (* Ill typed! *)
```

- Type abbreviations are treated "structurally"
  Newtypes are treated "by name"

Nominal Subtyping in Java

- In Java, Classes and Interfaces must be named and their relationships *explicitly* declared:

```
(* Java: *)
interface Foo {
  int foo();
}

class C {       /* Does not implement the Foo interface */
  int foo() {return 2;}
}

class D implements Foo {
  int foo() {return 341;}
}
```

- Similarly for inheritance: programmers must declare the subclass relation via the "`extends`" keyword.
  - Typechecker still checks that the classes are structurally compatible