Lecture 21

CIS 341: COMPILERS

Announcements

- HW6: Dataflow Analysis
  - Available soon
- Talk: Sumit Gulwani of Microsoft
  "Data Manipulation using Programming By Examples and Natural Language"
  - 3:00-4:00 in Wu & Chen
- My office hours: 4:00 – 5:15 today

CODE ANALYSIS

Iterative Dataflow Analysis

- Find a solution to those constraints by starting from a rough guess.
- Start with: in[n] = Ø and out[n] = Ø
- They don't satisfy the constraints:
  - in[n] ⊇ use[n]
  - in[n] ⊇ out[n] - def[n]
  - out[n] ⊇ in[n′] if n′ ∈ succ[n]

- Idea: iteratively re-compute in[n] and out[n] where forced to by the constraints.
  - Each iteration will add variables to the sets in[n] and out[n] (i.e. the live variable sets will increase monotonically)
- We stop when in[n] and out[n] satisfy these equations: (which are derived from the constraints above)
  - $\mathrm{in}[n] = \mathrm{use}[n] \cup (\mathrm{out}[n] - \mathrm{def}[n])$
  - $\mathrm{out}[n] = \bigcup_{n' \in \mathrm{succ}[n]} \mathrm{in}[n']$

A Worklist Algorithm

- Use a FIFO queue of nodes that might need to be updated.

```
for all n, in[n] := Ø, out[n] := Ø
w = new queue with all nodes
repeat until w is empty
    let n = w.pop()                          // pull a node off the queue
      old_in = in[n]                         // remember old in[n]
      out[n] := ∪ n'∈succ[n] in[n']
      in[n] := use[n] ∪ (out[n] - def[n])
      if (old_in != in[n]),                  // if in[n] has changed
        for all m in pred[n], w.push(m)      // add to worklist
end
```

OTHER DATAFLOW ANALYSES

Generalizing Dataflow Analyses

- The kind of iterative constraint solving used for liveness analysis applies to other kinds of analyses as well.
  - Reaching definitions analysis
  - Available expressions analysis
  - Alias Analysis
  - Constant Propagation
  - These analyses follow the same 3-step approach as for liveness.

- To see these as an instance of the same kind of algorithm, the next few examples to work over a canonical intermediate instruction representation called *quadruples*
  - Allows easy definition of def[n] and use[n]
  - A "looser" variant of LLVM's IR that doesn't require the "static single assignment" – i.e. it has *mutable* local variables

Quadruple Format

- A Quadruple sequence is just a control-flow graph (flowgraph) where each node is a quadruple:
- Quadruple forms n:

Quadruple forms n:	def[n]	use[n]	description
a = b op c	{a}	{b,c}	arithmetic
a = [b]	{a}	{b}	load
[a] = b	Ø	{b}	store
$a = f(b_1,\ldots,b_n)$	{a}	$\{b_1,\ldots,b_n\}$	call w/return
$f(b_1,\ldots,b_n)$	Ø	$\{b_1,\ldots,b_n\}$	call no return
jump L	Ø	Ø	jump
if a goto L1 else L2	Ø	{a}	branch
return a	Ø	{a}	return

REACHING DEFINITIONS

Reaching Definition Analysis

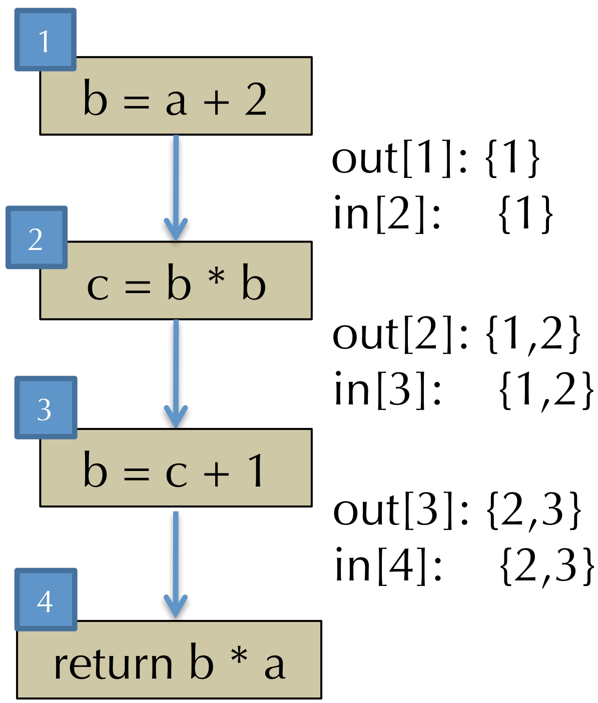
- Question: what uses in a program does a given variable definition reach?

- This analysis is used for constant propagation & copy prop.
  - If only one definition reaches a particular use, can replace use by the definition (for constant propagation).
  - Copy propagation additionally requires that the copied value still has its same value – computed using an *available expressions* analysis (next)

- Input: Quadruple CFG
- Output: in[n] (resp. out[n]) is the set of nodes defining some variable such that the definition may reach the beginning (resp. end) of node n

Example of Reaching Definitions

- Results of computing reaching definitions on this simple CFG:



Reaching Definitions Step 1

- Define the sets of interest for the analysis
- Let defs[a] be the set of *nodes* that define the variable a
- Define gen[n] and kill[n] as follows:
- Quadruple forms n:

Quadruple forms n:	gen[n]	kill[n]
a = b op c	{n}	defs[a] - {n}
a = load b	{n}	defs[a] - {n}
[a] = b	Ø	Ø
a = f(b_1,…,b_n)	{n}	defs[a] - {n}
f(b_1,…,b_n)	Ø	Ø
jump L	Ø	Ø
if a goto L1 else L2	Ø	Ø
L:	Ø	Ø
return a	Ø	Ø

Reaching Definitions Step 2

- Define the constraints that a reaching definitions solution must satisfy.
- $out[n] \supseteq gen[n]$
  "The definitions that reach the end of a node at least include the definitions generated by the node"
- $in[n] \supseteq out[n']$ if n' is in $pred[n]$
  "The definitions that reach the beginning of a node include those that reach the exit of *any* predecessor"
- $out[n] \cup kill[n] \supseteq in[n]$
  "The definitions that come in to a node either reach the end of the node or are killed by it."
  - Equivalently: $out[n] \supseteq in[n] - kill[n]$

Reaching Definitions Step 3

- Convert constraints to iterated update equations:
- $\text{in}[n] := \bigcup_{n' \in \text{pred}[n]} \text{out}[n']$
- $\text{out}[n] := \text{gen}[n] \cup (\text{in}[n] - \text{kill}[n])$

- Algorithm: initialize in[n] and out[n] to Ø
  - Iterate the update equations until a fixed point is reached

- The algorithm terminates because in[n] and out[n] increase only *monotonically*
  - At most to a maximum set that includes all variables in the program
- The algorithm is precise because it finds the *smallest* sets that satisfy the constraints.

AVAILABLE EXPRESSIONS

Available Expressions

- Idea: want to perform common subexpression elimination:
  - a = x + 1 ⇒ a = x + 1
    … …
    b = x + 1 b = a
- This transformation is safe if x+1 means computes the same value at both places (i.e. x hasn't been assigned).
  - "x+1" is an *available expression*

- Dataflow values:
  - in[n] = set of nodes whose values are available on entry to n
  - out[n] = set of nodes whose values are available on exit of n

Available Expressions Step 1

- Define the sets of values
- Define gen[n] and kill[n] as follows:
- Quadruple forms n:

Quadruple forms n:	gen[n]	kill[n]
a = b op c	{n} - kill[n]	uses[a]
a = [b]	{n} - kill[n]	uses[a]
[a] = b	Ø	uses[[x]] (for all x that may equal a)
jump L	Ø	Ø
if a goto L1 else L2	Ø	Ø
L:	Ø	Ø
$a = f(b_1,\ldots,b_n)$	Ø	uses[a] ∪ uses[[x]] (for all x)
$f(b_1,\ldots,b_n)$	Ø	uses[[x]] (for all x)
return a	Ø	Ø

Note the need for "may alias" information…

Note that functions are assumed to be impure…

Available Expressions Step 2

- Define the constraints that an available expressions solution must satisfy.
- out[n] ⊇ gen[n]
  "The expressions made available by n that reach the end of the node"
- in[n] ⊆ out[n′] if n′ is in pred[n]
  "The expressions available at the beginning of a node include those that reach the exit of *every* predecessor"
- out[n] ∪ kill[n] ⊇ in[n]
  "The expressions available on entry either reach the end of the node or are killed by it."
  - Equivalently: out[n] ⊇ in[n] - kill[n]

Note similarities and differences with constraints for "reaching definitions".

Available Expressions Step 3

- Convert constraints to iterated update equations:
- $\text{in}[n] := \bigcap_{n' \in \text{pred}[n]} \text{out}[n']$
- $\text{out}[n] := \text{gen}[n] \cup (\text{in}[n] - \text{kill}[n])$

- Algorithm: initialize in[n] and out[n] to {set of all nodes}
  - Iterate the update equations until a fixed point is reached

- The algorithm terminates because in[n] and out[n] *decrease* only *monotonically*
  - At most to a minimum of the empty set
- The algorithm is precise because it finds the *largest* sets that satisfy the constraints.

GENERAL DATAFLOW ANALYSIS

Comparing Dataflow Analyses

- Look at the update equations in the inner loop of the analyses
- Liveness: (backward)
  - Let gen[n] = use[n] and kill[n] = def[n]
  - out[n] := = $\bigcup_{n' \in succ[n]} in[n']$
  - in[n] := gen[n] ∪ (out[n] - kill[n])
- Reaching Definitions: (forward)
  - in[n] := $\bigcup_{n' \in pred[n]} out[n']$
  - out[n] := gen[n] ∪ (in[n] - kill[n])
- Available Expressions: (forward)
  - in[n] := $\bigcap_{n' \in pred[n]} out[n']$
  - out[n] := gen[n] ∪ (in[n] - kill[n])

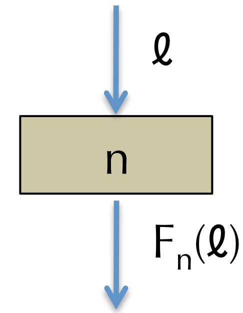
Common Features

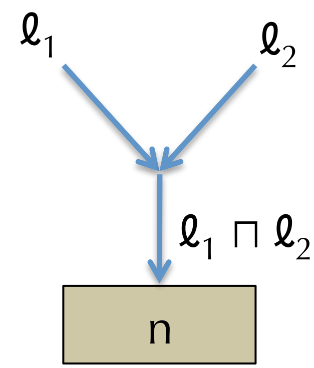
- All of these analyses have a *domain* over which they solve constraints.
  - Liveness, the domain is sets of variables
  - Reaching defns., Available exprs. the domain is sets of nodes
- Each analysis has a notion of gen[n] and kill[n]
  - Used to explain how information propagates across a node.
- Each analysis is propagates information either *forward* or *backward*
  - Forward: in[n] defined in terms of predecessor nodes' out[]
  - Backward: out[n] defined in terms of successor nodes' in[]
- Each analysis has a way of aggregating information
  - Liveness & reaching definitions take union ($\cup$)
  - Available expressions uses intersection ($\cap$)
  - Union expresses a property that holds for *some* path (existential)
  - Intersection expresses a property that holds for *all* paths (universal)

(Forward) Dataflow Analysis Framework

A forward dataflow analysis can be characterized by:

1. A domain of dataflow values $\mathcal{L}$
   - e.g. $\mathcal{L}$ = the powerset of all variables
   - Think of $\ell \in \mathcal{L}$ as a property, then "$x \in \ell$" means "x has the property"
2. For each node n, a flow function $F_n : \mathcal{L} \rightarrow \mathcal{L}$
   - So far we've seen $F_n(\ell) = gen[n] \cup (\ell - kill[n])$
   - So: $out[n] = F_n(in[n])$
   - "If ℓ is a property that holds before the node n, then $F_n(\ell)$ holds after n"
3. A combining operator $\sqcap$
   - "If we know *either* ℓ_1 *or* ℓ_2 holds on entry to node n, we know at most $\ell_1 \sqcap \ell_2$"
   - $in[n] := \bigsqcap_{n' \in pred[n]} out[n']$





Generic Iterative (Forward) Analysis

for all n, in[n] := $\top$, out[n] := $\top$
repeat until no change
    for all n

        $\text{in}[n] := \sqcap_{n' \in \text{pred}[n]} \text{out}[n']$
        $\text{out}[n] := F_n(\text{in}[n])$
    end
end

- Here, $\top \in \mathcal{L}$ ("top") represents having the "maximum" amount of information.
  - Having "more" information enables more optimizations
  - "Maximum" amount could be inconsistent with the constraints.
  - Iteration refines the answer, eliminating inconsistencies

Structure of $\mathcal{L}$

- The domain has structure that reflects the "amount" of information contained in each dataflow value.
- Some dataflow values are more informative than others:
  - Write $\ell_1 \sqsubseteq \ell_2$ whenever ℓ_2 provides at least as much information as ℓ_1.
  - The dataflow value ℓ_2 is "better" for enabling optimizations.

- Example 1: for liveness analysis, *smaller* sets of variables are more informative.
  - Having smaller sets of variables live across an edge means that there are fewer conflicts for register allocation assignments.
  - So: $\ell_1 \sqsubseteq \ell_2$ if and only if $\ell_1 \supseteq \ell_2$

- Example 2: for available expressions analysis, larger sets of nodes are more informative.
  - Having a larger set of nodes (equivalently, expressions) available means that there is more opportunity for common subexpression elimination.
  - So: $\ell_1 \sqsubseteq \ell_2$ if and only if $\ell_1 \subseteq \ell_2$

$\mathcal{L}$ as a Partial Order

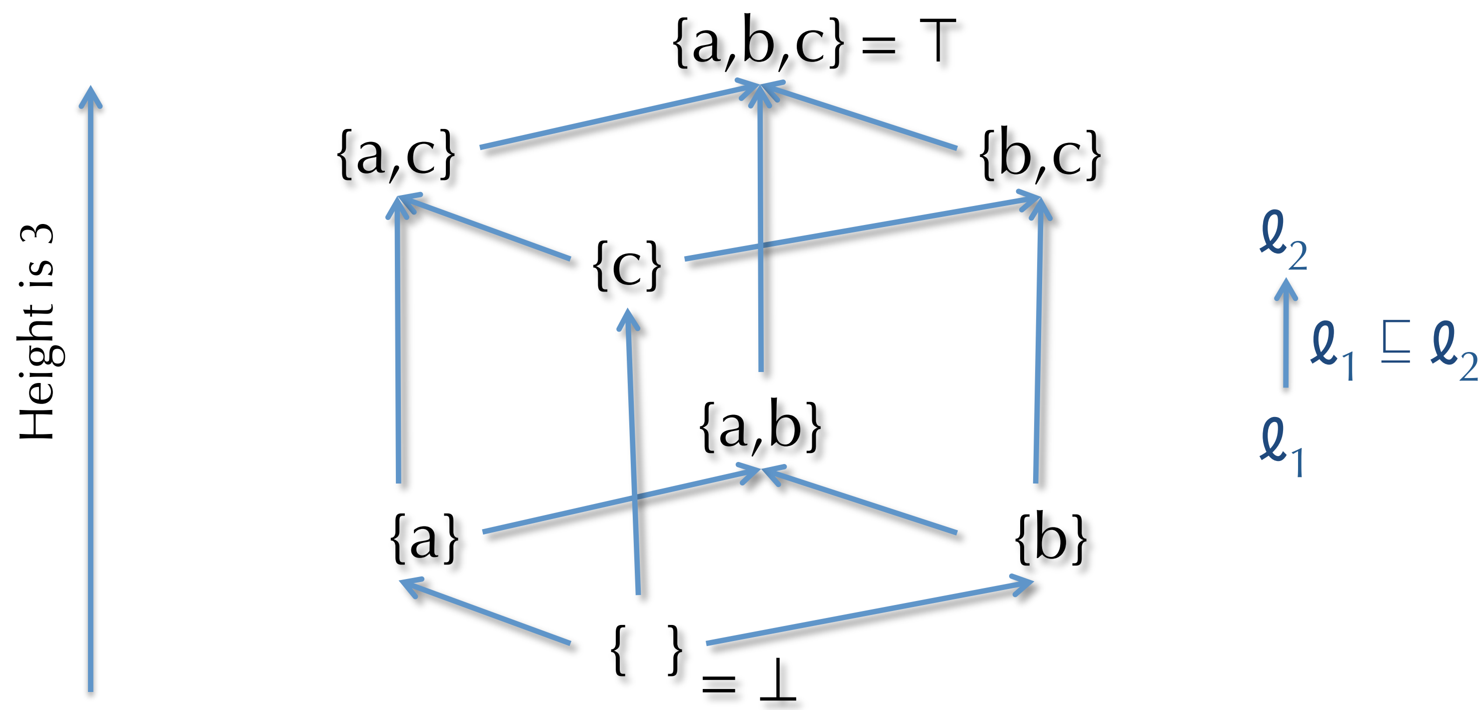
- $\mathcal{L}$ is a *partial order* defined by the ordering relation $\sqsubseteq$.
- A partial order is an ordered set.
- Some of the elements might be *incomparable*.
  - That is, there might be $\ell_1, \ell_2 \in \mathcal{L}$ such that neither $\ell_1 \sqsubseteq \ell_2$ nor $\ell_2 \sqsubseteq \ell_1$

- Properties of a partial order:
  - *Reflexivity:* $\ell \sqsubseteq \ell$
  - *Transitivity:* $\ell_1 \sqsubseteq \ell_2$ and $\ell_2 \sqsubseteq \ell_3$ implies $\ell_1 \sqsubseteq \ell_2$
  - *Anti-symmetry:* $\ell_1 \sqsubseteq \ell_2$ and $\ell_2 \sqsubseteq \ell_1$ implies $\ell_1 = \ell_2$

- Examples:
  - Integers ordered by $\leq$
  - Types ordered by <:
  - Sets ordered by $\subseteq$ or $\supseteq$

Subsets of {a,b,c} ordered by $\subseteq$

Partial order presented as a Hasse diagram.



order $\sqsubseteq$ is $\subseteq$ meet $\sqcap$ is $\cap$ join $\sqcup$ is $\cup$

Meets and Joins

- The combining operator $\sqcap$ is called the "meet" operation.
- It constructs the *greatest lower bound*:
  - $\ell_1 \sqcap \ell_2 \sqsubseteq \ell_1$ and $\ell_1 \sqcap \ell_2 \sqsubseteq \ell_2$
    "the meet is a lower bound"
  - If $\ell \sqsubseteq \ell_1$ and $\ell \sqsubseteq \ell_2$ then $\ell \sqsubseteq \ell_1 \sqcap \ell_2$
    "there is no greater lower bound"

- Dually, the $\sqcup$ operator is called the "join" operation.
- It constructs the *least upper bound*:
  - $\ell_1 \sqsubseteq \ell_1 \sqcup \ell_2$ and $\ell_2 \sqsubseteq \ell_1 \sqcup \ell_2$
    "the join is an upper bound"
  - If $\ell_1 \sqsubseteq \ell$ and $\ell_2 \sqsubseteq \ell$ then $\ell_1 \sqcup \ell_2 \sqsubseteq \ell$
    "there is no smaller upper bound"

- A partial order that has all meets and joins is called a *lattice*.
  - If it has just meets, it's called a *meet semi-lattice*.

Another Way to Describe the Algorithm

- Algorithm repeatedly computes (for each node n):
- $\text{out}[n] := F_n(\text{in}[n])$

- Equivalently: $\text{out}[n] := F_n(\sqcap_{n' \in \text{pred}[n]} \text{out}[n'])$
  - By definition of in[n]
- We can write this as a simultaneous update of the vector of out[n] values:
  - let $x_n = \text{out}[n]$
  - Let $\mathbf{X} = (x_1, x_2, \ldots, x_n)$ it's a vector of points in $\mathcal{L}$
  - $\mathbf{F}(\mathbf{X}) = (F_1(\sqcap_{j \in \text{pred}[1]} \text{out}[j]), F_2(\sqcap_{j \in \text{pred}[2]} \text{out}[j]), \ldots, F_n(\sqcap_{j \in \text{pred}[n]} \text{out}[j]))$

- Any solution to the constraints is a *fixpoint* **X** of **F**
  - i.e. $\mathbf{F}(\mathbf{X}) = \mathbf{X}$

Iteration Computes Fixpoints

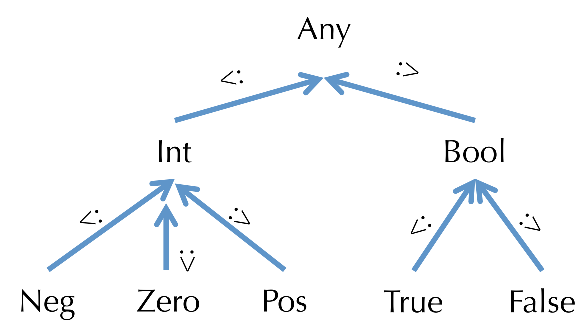
- Let $\mathbf{X}_0 = (\top, \top, \ldots, \top)$
- Each loop through the algorithm apply F to the old vector:
  $\mathbf{X}_1 = \mathbf{F}(\mathbf{X}_0)$
  $\mathbf{X}_2 = \mathbf{F}(\mathbf{X}_1)$
  …
- $\mathbf{F}^{k+1}(\mathbf{X}) = \mathbf{F}(\mathbf{F}^{k}(\mathbf{X}))$
- A fixpoint is reached when $\mathbf{F}^{k}(\mathbf{X}) = \mathbf{F}^{k+1}(\mathbf{X})$
  - That's when the algorithm stops.
- Wanted: a maximal fixpoint
  - Because that one is more informative/useful for performing optimizations

Monotonicity & Termination

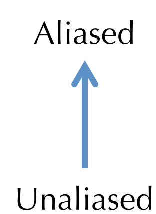
- Each flow function F_n maps lattice elements to lattice elements; to be sensible is should be *monotonic*:
- $F : \mathcal{L} \rightarrow \mathcal{L}$ is *monotonic* iff:
  $\ell_1 \sqsubseteq \ell_2$ implies that $F(\ell_1) \sqsubseteq F(\ell_2)$
  - Intuitively: "If you have more information entering a node, then you have more information leaving the node."
- Monotonicity lifts point-wise to the function: $\mathbf{F} : \mathcal{L}^n \rightarrow \mathcal{L}^n$
  - vector $(x_1, x_2, \ldots, x_n) \sqsubseteq (y_1, y_2, \ldots, y_n)$ iff $x_i \sqsubseteq y_i$ for each i
- Note that **F** is consistent: $\mathbf{F}(\mathbf{X}_0) \sqsubseteq \mathbf{X}_0$
  - So each iteration moves at least one step down the lattice (for some component of the vector)
  - $\ldots \sqsubseteq \mathbf{F}(\mathbf{F}(\mathbf{X}_0)) \sqsubseteq \mathbf{F}(\mathbf{X}_0) \sqsubseteq \mathbf{X}_0$
- Therefore, # steps needed to reach a fixpoint is at most the height H of $\mathcal{L}$ times the number of nodes: O(Hn)

Building Lattices?

- Information about individual nodes or variables can be lifted *pointwise:*
  - If $\mathcal{L}$ is a lattice, then so is $\{ f : X \to \mathcal{L} \}$ where $f \sqsubseteq g$ if and only if $f(x) \sqsubseteq g(x)$ for all $x \in X$.
- Like *types*, the dataflow lattices are *static approximations* to the dynamic behavior:
  - Could pick a lattice based on subtyping:



  - Or other information:



- Points in the lattice are sometimes called dataflow "facts"

See HW6: Dataflow Analysis

IMPLEMENTATION

Def / Use for SSA

- Instructions n:

Instructions n:	def[n]	use[n]	description
a = b op c	{a}	{b,c}	arithmetic
a = load b	{a}	{b}	load
store a, b	Ø	{b}	store
a = alloca t	{a}	Ø	alloca
a = bitcast b to u	{a}	{b}	bitcast
a = gep b [c,d, …]	{a}	{b,c,d,…}	getelementptr
a = f(b_1,…,b_n)	{a}	{b_1,…,b_n}	call w/return
f(b_1,…,b_n)	Ø	{b_1,…,b_n}	void call (no return)

- Terminators

br L	Ø	Ø	jump
br a L1 L2	Ø	{a}	conditional branch
return a	Ø	{a}	return