

Last time, we talked about encoding objects in the typed lambda calculus with records, recursion, references and subtyping.
 We have a little more to talk about this topic, but let's work through an example to see where we are.

Object Encodings

CIS 500
 Software Foundations
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```
\heading{Example from last time}
class SetCounter {
  protected int x = 1;
  int get() { return x; }
  void set(int i) { x = i; return; }
  void inc() { this.set(this.get() + 1); return; }
}

class InstrCounter extends SetCounter {
  protected int a = 0;
  void set(int i) { a++; super.set(i); return; }
  int accesses() { return a; }
}
```

- ◆ My office hours this week, Friday 1:30-3:00 PM
- ◆ Last homework out today, due Dec 8th.
- ◆ Regrades for midterm II in by Dec 8th.

Announcements

One more refinement...

The implementation we have given for instrumented counters is not very useful because calling the object creation function

```
newInstrCounter =
  λ_:Unit. let r = {x=ref 1, a=ref 0} in
  fix (InstrCounterClass r);
```

will cause the evaluator to diverge!

Intuitively (see TAPL for details), the problem is the “unprotected” use of `self` in the call to `setCounterClass` in `InstrCounterClass`:

```
InstrCounterClass =
  λr:InstrCounterRep.
  λself:InstrCounter.
  let super = setCounterClass r self in
  ...
```

A small fly in the ointment

```
SetCounter = {get:Unit→Nat, set:Nat→Unit, inc:Unit→Unit};
CounterRep = { x:Ref Nat };
setCounterClass =
  λr:CounterRep.
  λself:SetCounter.
  {get = λ_:Unit. ! (r.x),
  set = λi:Nat. r.x:=i,
  inc = λ_:Unit. self.set (succ(self.get unit))};
```

```
InstrCounterRep = {x:Ref Nat, a:Ref Nat};
InstrCounter = {get:Unit→Nat, set:Nat→Unit,
  inc:Unit→Unit, accesses:Unit→Nat};
InstrCounterClass =
  λr:InstrCounterRep.
  λself:InstrCounter.
  let super = setCounterClass r self in
  {get = super.get,
  set = λi:Nat. (r.a:=succ(! (r.a))); super.set i,
  inc = super.inc,
  accesses = λ_:Unit. ! (r.a)};
newInstrCounter =
  λ_:Unit. let r = {x=ref 1, a=ref 0} in
  fix (InstrCounterClass r);
```

```

Similarly:
InstrCounterClass =
  Ar:InstrCounterRep.
  Aself: Unit → InstrCounter.
  λ_:Unit.
    let super = setCounterClass r self in
    {get = super.get,
      set = λi:Nat. (r.a:=succ(i(r.a))); super.set i},
    inc = super.inc,
    accesses = λ_:Unit. i(r.a)};
newInstrCounter =
  λ_:Unit. let r = {x=ref 1, a=ref 0} in
  fix (InstrCounterClass r) unit;
  
```

```

To see why this diverges, consider a simpler example:
ff = λf:Nat → Nat.
  let f' = f in
  λn:Nat. 0
⇔ ff : (Nat → Nat) → (Nat → Nat)
Now:
  fix ff
  → ff (fix ff)
  → let f' = (fix ff) in λn:Nat. 0
  → let f' = ff (fix ff) in λn:Nat. 0
  → uh oh...
  
```

Success

This works, in the sense that we can now instantiate `InstrCounterClass` (without diverging!), and its instances behave in the way we intended.

```

setCounterClass =
  Ar:CounterRep.
  Aself: Unit → SetCounter.
  λ_:Unit.
    {get = λ_:Unit. i(r.x),
      set = λi:Nat. r.x:=i,
      inc = λ_:Unit. (self unit).set(succ((self unit).get unit))};
  ⇔
  setCounterClass : CounterRep → (Unit → SetCounter) → (Unit → SetCounter)
  newSetCounter =
    λ_:Unit. let r = {x=ref 1} in
    fix (setCounterClass r) unit;
  
```

Idea: “delay” `self` by putting a dummy abstraction in front of it...

One possible solution

Multiple representations

All the objects we have built in this series of examples have type `Counter`.
But their internal representations vary widely.

Encapsulation

An object is a record of functions, which maintain common internal state via a shared reference to a record of mutable instance variables.
This state is inaccessible outside of the object because there is no way to name it. (Instance variables can only be named from inside the methods.)

Success (?)

This works, in the sense that we can now instantiate `InstCounterClass` (without diverging!), and its instances behave in the way we intended. However, all the “delaying” we added has an unfortunate side effect: instead of computing the “method table” just once, when an object is created, we will now re-compute it every time we invoke a method!
Section 18.12 in TAPL shows how this can be repaired by using references instead of `fix` to “tie the knot” in the method table.

Recap

Where we are...

The (an) essence of objects

- ◆ Multiple representations
- ◆ Encapsulation of state with behavior
- ◆ Subtyping
- ◆ Inheritance (incremental definition of behaviors)
- ◆ “Open recursion” through `self`

Subtyping

Subtyping between object types is just ordinary subtyping between types of records of functions.
 Functions like `inc3` that expect `Counter` objects as parameters can (safely) be called with objects belonging to any subtype of `Counter`.

Inheritance

Classes are data structures that can be both extended and instantiated. We modeled inheritance by copying implementations of methods from superclasses to subclasses.
 Each class

- ◆ waits to be told a record `r` of instance variables and an object `self` (which should have the same interface and be based on the same record of instance variables)
- ◆ uses `r` and `self` to instantiate its superclass
- ◆ constructs a record of method implementations, copying some directly from `super` and implementing others in terms of `self` and `super`.

The `self` parameter is “resolved” at object creation time using `fix`.

No such thing as a “perfect model” — “The nature of a model is to abstract away from details!
 So models are never just “good”: they are always “good for some specific set of purposes.”

Models in General

The peculiar status of **classes** (which are both run-time and compile-time things)
Named types with **declared** subtyping
 Recursive types
 Run-time type analysis (casting, etc.)
 (...lots of other stuff)

What's missing

- ◆ Lots of different purposes → lots of different kinds of models
- ◆ Source-level vs. bytecode level
- ◆ Large (inclusive) vs. small (simple) models
- ◆ Models of type system vs. models of run-time features (not entirely separate issues)
- ◆ Models of specific features (exceptions, concurrency, reflection, class loading, ...)
- ◆ Models designed for extension

Models of Java

Modeling Java

- ◆ Reflection, concurrency, class loading, inner classes, ...
- ◆ Exceptions, loops, ...

Things left out

- ◆ Reflection, concurrency, class loading, inner classes, ...
- ◆ Exceptions, loops, ...
- ◆ Interfaces, overloading, ...

Things left out

- ◆ Originally proposed by a Penn PhD student (Atsushi Igarashi) as a tool for analyzing GJ (“Java plus generics”)
- ◆ Since used by many others for studying a wide variety of Java features and proposed extensions

History:

Purpose: model the “core OO features” and their types and *nothing else*.

Featherweight Java

- ◆ Reflection, concurrency, class loading, inner classes, ...

Things left out

```

class A extends Object { A() { super(); } }
class B extends Object { B() { super(); } }
class Pair extends Object {
    Object fst;
    Object snd;
    Pair(Object fst, Object snd) {
        super(); this.fst=fst; this.snd=snd; }
    Pair setfst(Object newfst) {
        return new Pair(newfst, this.snd); }
}
    
```

Example

- ◆ Reflection, concurrency, class loading, inner classes, ...
- ◆ Exceptions, loops, ...
- ◆ Interfaces, overloading, ...
- ◆ Assignment (ii)

Things left out

- For syntactic regularity...
- ◆ Always include superclass (even when it is **Object**)
 - ◆ Always write out constructor (even when trivial)
 - ◆ Always call **super** from constructor (even when no arguments are passed)
 - ◆ Always explicitly name receiver object in method invocation or field access (even when it is **this**)
 - ◆ Methods always consist of a single **return** expression
 - ◆ Constructors always
 - ◆ Take same number (and types) of parameters as fields of the class
 - ◆ Assign constructor parameters to "local fields"
 - ◆ Call **super** constructor to assign remaining fields
 - ◆ Do nothing else

Conventions

- ◆ Classes and objects
- ◆ Methods and method invocation
- ◆ Fields and field access
- ◆ Inheritance (including open recursion through **this**)
- ◆ Casting

Things left in

Advantages of Structural Systems

Somewhat simpler, cleaner, and more elegant (no need to always work wrt. a set of “name definitions”)
 Easier to extend (e.g. with parametric polymorphism)
 Caveat: when recursive types are considered, some of this simplicity and elegance slips away...

Advantages of Nominal Systems

Recursive types fall out easily
 Using names everywhere makes typechecking (and subtyping, etc.) easy and efficient
 Type names are also useful at run-time (for casting, type testing, reflection, ...).
 Java (like most other mainstream languages) is a nominal system.

Formalizing FJ

Nominal type systems

- Big dichotomy in the world of programming languages:
- ◆ **Structural** type systems:
 - ◆ What matters about a type (for typing, subtyping, etc.) is just its structure.
 - ◆ Names are just convenient (but inessential) abbreviations.
 - ◆ **Nominal** type systems:
 - ◆ Types are always named.
 - ◆ Typechecker mostly manipulates names, not structures.
 - ◆ Subtyping is declared explicitly by programmer (and checked for consistency by compiler).

Syntax (terms and values)

$t ::=$ x variable
 $t ::=$ $t.f$ field access
 $t ::=$ $t.m(t)$ method invocation
 $t ::=$ $\text{new } C(t)$ object creation
 $t ::=$ $(C) t$ cast
 $v ::=$ $\text{new } C(\Delta)$ values
 $v ::=$ object creation

Representing objects

Our decision to omit assignment has a nice side effect...
 The only ways in which two objects can differ are (1) their classes and (2) the parameters passed to their constructor when they were created.
 All this information is available in the `new` expression that creates an object. So we can **identify** the created object with the `new` expression.
 Formally: object values have the form `new C(Δ)`

Syntax (methods and classes)

$K ::=$ $C(\underline{F}) \{ \text{super}(\underline{F}); \text{this.}\underline{f}=\underline{f}; \}$ constructor declarations
 $M ::=$ $C m(\underline{C} \underline{x}) \{ \text{return } t; \}$ method declarations
 $CL ::=$ $\text{class } C \text{ extends } C \{ \underline{C} \underline{F}; K \underline{M} \}$ class declarations

FJ Syntax

From the class table, we can read off a number of other useful properties of the definitions (which we will need later for typechecking and operational semantics)...

More auxiliary definitions

$$\begin{array}{c}
 \text{fields(Object)} = \emptyset \\
 \hline
 CT(C) = \text{class } C \text{ extends } D \{ \underline{C} \ \underline{F}; \ K \ \underline{M} \} \\
 \text{fields}(D) = \underline{D} \ \underline{g} \\
 \hline
 \text{fields}(C) = \underline{D} \ \underline{g}, \ \underline{C} \ \underline{F}
 \end{array}$$

Fields lookup

Subtyping

As in Java, subtyping in FJ is declared.
 Assume we have a (global, fixed) class table CT mapping class names to definitions.

$$\begin{array}{c}
 \text{C} <: \text{D} \\
 \hline
 \text{C} <: \text{D} \quad \text{D} <: \text{E} \\
 \text{C} <: \text{C} \\
 \text{C} <: \text{E}
 \end{array}$$

Subtyping

Valid method overriding

$$\frac{\text{override}(m, D, \bar{c} \rightarrow c_0)}{\text{type}(m, D) = \bar{D} \rightarrow D_0 \text{ implies } \bar{c} = \bar{D} \text{ and } c_0 = D_0}$$

Method type lookup

$$\frac{\text{type}(m, C) = B \rightarrow B}{\text{class } C \text{ extends } D \{ \bar{c} \ \bar{f}; \ K \ \bar{M} \} \text{ return } t; \} \in \bar{M}}{\text{type}(m, C) = m \text{ is not defined in } \bar{M}}$$

Evaluation

Method body lookup

$$\frac{\text{body}(m, C) = (\bar{x}, t)}{\text{class } C \text{ extends } D \{ \bar{c} \ \bar{f}; \ K \ \bar{M} \} \text{ return } t; \} \in \bar{M}}{\text{body}(m, C) = m \text{ is not defined in } \bar{M}}$$

```
(Pair)new Pair(new A(), new B()) → new Pair(new A(), new B())
```

Casting:

Evaluation

```
new Pair(new A(), new B()).setfst(new B())
    newfst ↦ new B(),
    this ↦ new Pair(new A(), new B())
```

Method invocation:

```
new Pair(newfst, this.snd)
```

```
i.e., new Pair(new B(), new Pair(new A(), new B()).snd)
```

Evaluation

```
class A extends Object { A() { super(); } }
class B extends Object { B() { super(); } }
class Pair extends Object {
    Object fst;
    Object snd;
    Pair(Object fst, Object snd) {
        super(); this.fst=fst; this.snd=snd; }
    Pair setfst(Object newfst) {
        return new Pair(newfst, this.snd); }
}
```

The example again

```
new Pair(new A(), new B()).snd → new B()
```

Projection:

Evaluation

$$\begin{array}{l}
 \text{(E-FIELD)} \quad \frac{t_0 \rightarrow t'_0 \quad t_0.f \rightarrow t'_0.f}{t_0 \rightarrow t'_0} \\
 \text{(E-INVK-RECV)} \quad \frac{t_0 \rightarrow t'_0 \quad t_0.m(t) \rightarrow t'_0.m(t)}{t_0 \rightarrow t'_0} \\
 \text{(E-INVK-ARG)} \quad \frac{t_1 \rightarrow t'_1 \quad v_0.m(\underline{v}, t_1, \underline{t}) \rightarrow v_0.m(\underline{v}, t'_1, \underline{t})}{t_1 \rightarrow t'_1} \\
 \text{(E-NEW-ARG)} \quad \frac{t_1 \rightarrow t'_1 \quad \text{new } C(\underline{v}, t_1, \underline{t}) \rightarrow \text{new } C(\underline{v}, t'_1, \underline{t})}{t_1 \rightarrow t'_1} \\
 \text{(E-CAST)} \quad \frac{t_0 \rightarrow t'_0 \quad (C)t_0 \rightarrow (C)t'_0}{t_0 \rightarrow t'_0}
 \end{array}$$

Typing

$$\begin{array}{l}
 \rightarrow \text{new } B() \\
 \rightarrow \frac{\text{new Pair}(\text{new } A(), \text{new } B()).snd}{\text{new Pair}(\text{new } A(), \text{new } B())} \\
 \rightarrow \frac{\text{((Pair) new Pair}(\text{new } A(), \text{new } B()))}.snd}{\text{((Pair) new Pair}(\text{new } A(), \text{new } B()))}
 \end{array}$$

$$\begin{array}{l}
 \text{(E-PROJNEW)} \quad \frac{\text{fields}(C) = \underline{c} \ \underline{f} \quad (\text{new } C(\underline{v})).f_1 \rightarrow v_1}{\text{body}(m, C) = (\underline{x}, t_0)} \\
 \text{(E-INVKNEW)} \quad \frac{(\text{new } C(\underline{v})).m(\underline{u}) \rightarrow [\underline{x} \mapsto \underline{u}, \text{this} \mapsto \text{new } C(\underline{v})]t_0}{C <: D \quad (D)(\text{new } C(\underline{v})) \rightarrow \text{new } C(\underline{v})} \\
 \text{(E-CASTNEW)} \quad \text{plus some congruence rules...}
 \end{array}$$

Evaluation rules

Typing rules

(T-FIELD)

$$\frac{\Gamma \vdash t_0.f_i : C_i}{\Gamma \vdash t_0 : C_0 \quad \text{fields}(C_0) = \{i\}}$$

FJ has no rule of substitution (because we want to follow Java). The typing rules are algorithmic.

(Where would this make a difference?..)

Notes

Typing rules

(T-UCAST)

$$\frac{\Gamma \vdash (C)t_0 : C}{\Gamma \vdash t_0 : D \quad C <: C}$$

(T-DCAST)

$$\frac{\Gamma \vdash (C)t_0 : C}{\Gamma \vdash t_0 : D \quad C \neq D}$$

Why two cast rules?

Typing rules

(T-VAR)

$$\frac{\Gamma \vdash x : C}{x : C \in \Gamma}$$

Why? Because Java does it this way!
style of chapter 15.

Note that this rule “has subassumption built in” — i.e., the typing relation in FJ is written in the **algorithmic** style of TAPL chapter 16, not the declarative

$$\frac{\Gamma \vdash t_0 : c_0 \quad \text{type}(m, c_0) = \underline{d} \rightarrow c}{\Gamma \vdash \underline{t} : \underline{c} \quad \underline{c} <: \underline{d}}$$

(T-INVK)

Typing rules

Why? Because Java does it this way!
But why does Java do it this way??

Note that this rule “has subassumption built in” — i.e., the typing relation in FJ is written in the **algorithmic** style of TAPL chapter 16, not the declarative style of chapter 15.

$$\frac{\Gamma \vdash t_0 : c_0 \quad \text{type}(m, c_0) = \underline{d} \rightarrow c}{\Gamma \vdash \underline{t} : \underline{c} \quad \underline{c} <: \underline{d}}$$

(T-INVK)

Typing rules

Why two cast rules? Because that's how Java does it!

$$\frac{\Gamma \vdash t_0 : D \quad C <: D \quad C \neq D}{\Gamma \vdash (C) t_0 : C}$$

(T-DCAST)

(T-UCAST)

Typing rules

Note that this rule “has subassumption built in” — i.e., the typing relation in FJ is written in the **algorithmic** style of TAPL chapter 16, not the declarative style of chapter 15.

$$\frac{\Gamma \vdash t_0 : c_0 \quad \text{type}(m, c_0) = \underline{d} \rightarrow c}{\Gamma \vdash \underline{t} : \underline{c} \quad \underline{c} <: \underline{d}}$$

(T-INVK)

Typing rules

$$\frac{t_1 \in \text{bool} \quad t_2 \in T_2 \quad t_3 \in T_3}{t_1 \ ? \ t_2 : t_3 \in ?}$$

Java conditionals

Java typing is algorithmic

The Java typing relation is defined in the algorithmic style, for (at least) two reasons:

1. In order to perform static **overloading resolution**, we need to be able to speak of “the type” of an expression
2. We would otherwise run into trouble with typing of conditional expressions

Let's look at the second in more detail...

$$\frac{t_1 \in \text{bool} \quad t_2 \in T_2 \quad t_3 \in T_3}{t_1 \ ? \ t_2 : t_3 \in \text{min}(T_2, T_3)}$$

Actual Java rule (algorithmic):

$$\frac{t_1 \in \text{bool} \quad t_2 \in T_2 \quad t_3 \in T_3}{t_1 \ ? \ t_2 : t_3 \in ?}$$

Java conditionals

Java typing must be algorithmic

We haven't included them in FJ, but full Java has both **interfaces** and **conditional expressions**.

The two together actually make the declarative style of typing rules unworkable!

```
interface I {...}
interface J {...}
interface K extends I, J {...}
interface L extends I, J {...}
```

E.g.:

But, in full Java (with interfaces), there are types that have no join!

Java has no joins

K and L have no join (least upper bound) — both I and J are common upper bounds, but neither of these is less than the other.

So: algorithmic typing rules are really our only option.

$$\frac{t_1 \in \text{bool} \quad t_2 \in T \quad t_3 \in T}{t_1 \ ? \ t_2 : t_3 \in T}$$

More standard (declarative) rule:

$$\frac{t_1 \in \text{bool} \quad t_2 \in T \quad t_3 \in T}{t_1 \ ? \ t_2 : t_3 \in T}$$

More standard (declarative) rule:

Algorithmic version:

$$\frac{t_1 \ ? \ t_2 : t_3 \in T_2 \ \vee \ T_3}{t_1 \in \text{bool} \quad t_2 \in T_2 \quad t_3 \in T_3}$$

Requires joins!

$$\frac{\Gamma \vdash_{\text{new}} c(\underline{f}) : C}{\Gamma \vdash \underline{f} : \underline{C} \quad \underline{C} <: \underline{D} \quad \text{fields}(C) = \underline{D} \ \underline{f}}$$

FJ Typing rules

(T-NEW)

Theorem [Preservation]: If $\Gamma \vdash t : C$ and $t \mapsto t'$, then $\Gamma \vdash t' : C'$ for some $C' \triangleleft C$.

Proof: Straightforward induction.

Preservation

Typing rules (methods, classes)

$$\begin{array}{c} \bar{x} : \bar{c}, \text{this} : C \vdash t_0 : E_0 \quad E_0 \triangleleft C_0 \\ \text{class } C \text{ extends } D \{ \dots \} \\ \hline CT(C) = \text{class } C \text{ extends } D \{ \dots \} \\ \text{override}(m, D, \bar{c} \rightarrow C_0) \\ \hline C_0 \text{ m } (\bar{c} \ \bar{x}) \{ \text{return } t_0; \} \text{ OK in } C \\ \hline K = C(D \ \bar{g}, C \ \bar{f}) \{ \text{super}(\bar{g}); \text{this}.\bar{f} = \bar{f}; \} \\ \text{fields}(D) = D \ \bar{g} \quad M \text{ OK in } C \\ \hline \text{class } C \text{ extends } D \{ C \ \bar{f}; K \ M \} \text{ OK} \end{array}$$

Theorem [Preservation]: If $\Gamma \vdash t : C$ and $t \mapsto t'$, then $\Gamma \vdash t' : C'$ for some $C' \triangleleft C$.

Proof: Straightforward induction. ???

Preservation

Properties

$(A) \text{ (Object)new } B() \rightarrow (A)\text{new } B()$

Surprise: well-typed programs **can** step to ill-typed ones!
(How?)

Preservation?

Preservation?

Surprise: well-typed programs **can** step to ill-typed ones!
(How?)

Preservation?

Solution: "Stupid Cast" typing rule

Add another typing rule, marked "stupid" to

$\Gamma \vdash t_0 : D \quad C \not\leq D \quad D \not\leq C$

stupid warning

$\Gamma \vdash (C)t_0 : C$

(T-SCAST)

- ◆ Loosen preservation theorem
- ◆ Use big-step semantics

Alternative approaches to casting

This is an example of a modeling technicality; not very interesting or deep, but we have to get it right if we're going to claim that the model is an accurate representation of (this fragment of) Java.

$$\frac{\Gamma \vdash (C) t_0 : C}{\Gamma \vdash t_0 : D \quad C \not\leq D \quad D \not\leq C} \text{ (T-SCAST)}$$

stupid warning

Add another typing rule, marked "stupid" to

Solution: "Stupid Cast" typing rule

Progress

Claim:

Let's try to state precisely what we mean by "FJ corresponds to Java":

1. Every syntactically well-formed FJ program is also a syntactically well-formed Java program.
 2. A syntactically well-formed FJ program is typable in FJ (without using the T-SCAST rule.) iff it is typable in Java.
 3. A well-typed FJ program behaves the same in FJ as in Java. (E.g., evaluating it in FJ diverges iff compiling and running it in Java diverges.)
- Of course, without a formalization of full Java, we cannot **prove** this claim. But it's still very useful to say precisely what we are trying to accomplish—in particular, it provides a rigorous way of judging counterexamples.
- (Cf. "conservative extension" between logics.)

Correspondence with Java

Formalizing Progress

Solution: Weaken the statement of the progress theorem to
 A well-typed FJ term is either a value or can reduce one step or is
 stuck at a failing cast.

Formalizing this takes a little more work...

Progress

Problem: well-typed programs **can** get stuck.
 How?

Evaluation Contexts

E ::= []
 E.f
 E.m(t)
 v.m(v, E, t)
 new C(v, E, t)
 (C)E

evaluation contexts
 hole
 field access
 method invocation (receiver)
 method invocation (arg)
 object creation (arg)
 cast

Evaluation contexts capture the notion of the “next subterm to be reduced,” in the sense that, if $t \rightarrow t'$, then we can express t and t' as $t = E[r]$ and $t' = E[r']$ for a unique E , r , and r' , with $r \rightarrow r'$ by one of the computation rules E-PROJNEW, E-INVKNEW, or E-CASTNEW.

Progress

Problem: well-typed programs **can** get stuck.
 How?
 Cast failure:
 (A)new Object()

Progress

Theorem [Progress]: Suppose t is a closed, well-typed normal form. Then either (1) t is a value, or (2) $t \rightarrow t'$ for some t' , or (3) for some evaluation context E , we can express t as $t = E[(C) (\text{new } D(\Delta))]$, with $D \not\leq C$.