CIS 500 Software Foundations

Homework Assignment 8

Subtyping

Due: Wednesday, November 30, 2005, by noon

Submission instructions:

You must submit your solutions electronically (in ascii, postscript, or PDF format). Electronic solutions should be submitted following the same instructions as last time; these can be found at `http://www.seas.upenn.edu/~cis500/homework.html`. Do **not** email your solutions to us.

1 Exercise Consider the addition of downcasting as described in TAPL Section 15.5:

$$\frac{\Gamma \vdash \mathtt{t}_1 : \mathtt{S}}{\Gamma \vdash \mathtt{t}_1 \ \mathtt{as}\ \mathtt{T} : \mathtt{T}}\ \text{T-DOWNCAST}$$

$$\frac{\vdash \mathtt{v}_1 : \mathtt{T}}{\mathtt{v}_1 \ \mathtt{as}\ \mathtt{T} \longrightarrow \mathtt{v}_1}\ \text{E-DOWNCAST}$$

$$\frac{\mathtt{t}_1 \longrightarrow \mathtt{t}_1'}{\mathtt{t}_1 \ \mathtt{as}\ \mathtt{T} \longrightarrow \mathtt{t}_1' \ \mathtt{as}\ \mathtt{T}}\ \text{E-DOWNCAST2}$$

1. Show that the preservation theorem holds for closed terms of the language in Figure 15-1, extended with these rules and new syntactic form `t as T` for downcasts.
2. Does the preservation theorem hold for open terms? Why or why not?
3. Give an example of how progress breaks with downcasting.
4. What if we replaced T-DOWNCAST with the following rule:

$$\frac{}{\Gamma \vdash \mathtt{t}_1 \ \mathtt{as}\ \mathtt{T} : \mathtt{T}}\ \text{T-DOWNCAST2}$$

   Does preservation still hold? Discuss why we might or might not want this rule in a programming language.

2 Exercise State and prove an extension of Lemma 15.3.2 for subtyping references.

3 Exercise 15.5.3 in TAPL.

4 Exercise For each of the following questions answer YES or NO. If the answer is YES, show the subtyping derivation. If the answer is NO, give either a term that demonstrates how type safety breaks if we allow the two types in the subtype relation, or a short explanation of why type safety is preserved even if we allow the two types in the subtype relation.

1. Is $\{\mathtt{x} : \mathtt{Top} \to \mathtt{Ref\ Top}\}$ a subtype of $\{\mathtt{x} : \mathtt{Top} \to \mathtt{Top}\}$?
2. Is $\{\mathtt{x} : \mathtt{Top} \to \mathtt{Ref\ Top}\}$ a subtype of $\{\mathtt{x} : \mathtt{Ref\ Top} \to \mathtt{Ref}\ \{\mathtt{y} : \mathtt{Top}\}\}$?
3. Is $\{\mathtt{x} : \mathtt{Ref}\ \{\mathtt{y} : \mathtt{Top}\}\}$ a subtype of $\{\mathtt{x} : \mathtt{Ref\ Top}\}$?
4. Is $\{\mathtt{x} : \mathtt{Top}\}$ a subtype of $\{\mathtt{x} : \{\}\}$?

5 Exercise 16.1.2 in TAPL. (Even though the answer in the back of the book says "straightforward induction" for part 1, please show the details of the proof for part 1).

6 Exercise The algorithmic subtyping judgement replaces three rules for subtyping records (S-RcdWidth, S-RcdDepth, S-RcdPerm) in the declarative system with one rule (SA-Rcd). What new algorithmic rule for variants should replace the three rules S-VariantWidth, S-VariantDepth, and S-VariantPerm ?

7 Debriefing

1. How many hours did each person in your group spend on this assignment, including time taken to read TAPL?
2. Would you rate it as easy, moderate, or difficult?
3. Did everyone in your study group participate?
4. How deeply do you feel you understand the material it covers (0%–100%)?

If you have any other comments, we would like to hear them; please send them to `cis500@cis.upenn.edu`.