Announcements

- **Homework 1:** Due in **one week (next Wednesday at 8pm)!**
	- Should only take you a few hours

• **Waitlist**

- Admitted to capacity
- Only considering additional applications if students drop or do not enroll
- If you have been accepted off the waitlist, **please enroll by Thursday**
- We may make second round of decisions on Friday

TA Team

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Project: Goals

- Apply algorithms you learn in this class to realistic datasets
- Understand strengths and weaknesses of different machine learning approaches in different domains

Project: Details

- **Dataset:** We provide two datasets, one NLP and one computer vision
- **Algorithms:** Evaluate two algorithms on each dataset
- **Analysis:** Implement and evaluate one perturbation on each dataset

Project: Algorithms

• **NLP dataset**

• Feature engineering + traditional model vs. RNN or transformer

• **Computer vision**

- Traditional model vs. CNN or transformer
- You must evaluate **some** nontrivial architecture variation for each one
	- **Example:** Kinds of layers used, kind of data augmentation used, etc.
	- **Non-examples:** Number of hidden units or layers, nonlinearity

Project: Analysis

• **NLP dataset**

• Short vs. long text, omit sentences with certain words from training, etc.

• **Computer vision**

- Rotations/translations/scaling, color/brightness shifts, etc.
- More details on this step in the future

Project: Logistics (Tentative)

• **Teams of 3 students**

• Find teammates on your own

• **Project milestones**

- **Team Selection (due 9/20)**
- **Milestone 1 (1 page, due 10/18):** Project proposal
- **Milestone 2 (2 pages, due 11/15):** Preliminary results (half of algorithms)
- **Milestone 3 (4 pages, due 12/6):** Final report

Lecture 2: Linear Regression

CIS 4190/5190 Fall 2023

Recap: Types of Learning

• **Supervised learning**

- **Input:** Examples of inputs and outputs
- **Output:** Model that predicts unknown output given a new input

• **Unsupervised learning**

- **Input:** Examples of some data (no "outputs")
- **Output:** Representation of structure in the data

• **Reinforcement learning**

- **Input:** Sequence of interactions with an environment
- **Output:** Policy that performs a desired task

Today

- Deep dive into **linear regression**
	- Basic example of a **supervised learning algorithm**
- Captures many fundamental machine learning concepts
	- Function approximation view of machine learning
	- Bias-variance tradeoff
	- Regularization
	- Training/validation/test split
	- Optimization and gradient descent

Agenda

• **Function approximation view of machine learning**

- Modern strategy for designing machine learning algorithms
- **By example:** Linear regression, a simple machine learning algorithm

• **Bias-variance tradeoff**

- Fundamental challenge in machine learning
- **By example:** Linear regression with feature maps

Supervised Learning Data $Z = \{ (x_i, y_i) \}$ Machine learning algorithm Model f New input x Predicted output y

Question: What **model family** (a.k.a. **hypothesis class**) to consider?

Linear Functions

• Consider the space of linear functions $f_\beta(x)$ defined by

$$
f_{\beta}(x) = \beta^{\top} x
$$

Linear Functions

• Consider the space of linear functions $f_\beta(x)$ defined by

$$
f_{\beta}(x) = \beta^{\top} x = [\beta_1 \quad \cdots \quad \beta_d] \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix} = \beta_1 x_1 + \cdots + \beta_d x_d
$$

- $x \in \mathbb{R}^d$ is called an **input** (a.k.a. **features** or **covariates**)
- $\beta \in \mathbb{R}^d$ is called the **parameters** (a.k.a. **parameter vector**)
- $y = f_{\beta}(x)$ is called the **label** (a.k.a. **output** or **response**)

- **Input:** Dataset $Z = \{(x_1, y_1), ..., (x_n, y_n)\}\)$, where $x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{R}$
- **Output:** A linear function $f_{\beta}(x) = \beta^{\top}x$ such that $y_i \approx \beta^{\top}x_i$

• **Typical notation**

- Use *i* to index examples (x_i, y_i) in data Z
- Use j to index components x_j of $x \in \mathbb{R}^d$
- x_{ij} is component *j* of input example *i*
- **Goal:** Estimate $\beta \in \mathbb{R}^d$

- **Input:** Dataset $Z = \{ (x_1, y_1), ..., (x_n, y_n) \}$, where
- **Output:** A linear function $f_{\beta}(x) = \beta^{\top}x$ such the

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• **Output:** A linear function $f_{\beta}(x) = \beta^{\top}x$ such the

Choice of Loss Function

- $y_i \approx \beta^{\top} x_i$ if $(y_i \beta^{\top} x_i)^2$ small
- **Mean squared error (MSE):**

$$
L(\beta; Z) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \beta^\top x_i)^2
$$

• Computationally convenient and works well in practice

- **Input:** Data $Z = \{(x_1, y_1), ..., (x_n, y_n)\}\)$, where $x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{R}$
- **Output:** A linear function $f_{\beta}(x) = \beta^{\top}x$ such that $y_i \approx \beta^{\top}x_i$

- **Input:** Data $Z = \{(x_1, y_1), ..., (x_n, y_n)\}\)$, where $x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{R}$
- **Output:** A linear function $f_{\beta}(x) = \beta^T x$ that minimizes the MSE:

$$
L(\beta; Z) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \beta^\top x_i)^2
$$

Linear Regression Algorithm

- **Input:** Dataset $Z = \{(x_1, y_1), ..., (x_n, y_n)\}\$
- Compute

$$
\hat{\beta}(Z) = \underset{\beta \in \mathbb{R}^d}{\arg \min} L(\beta; Z)
$$

$$
= \underset{\beta \in \mathbb{R}^d}{\arg \min} \frac{1}{n} \sum_{i=1}^n (y_i - \beta^\top x_i)^2
$$

- **Output:** $f_{\widehat{\beta}(Z)}(x) = \widehat{\beta}(Z)^{\top}x$
- Discuss algorithm for computing the minimal β later

• **Convex** ("bowl shaped") in general

"Good" Mean Squared Error?

- Need to compare to baseline!
	- Constant prediction
	- Handcrafted model
	- …
- **Later:** Training vs. test MSE

Alternative Loss Functions

• Mean absolute error:

$$
\frac{1}{n}\sum_{i=1}^{n}|\hat{y}_i - y_i|
$$

• Mean relative error:

$$
\frac{1}{n} \sum_{i=1}^{n} \frac{|\widehat{y}_i - y_i|}{|y_i|}
$$

• R^2 score: 1 1

$$
1 - \frac{MSE}{Variance}
$$

- "Coefficient of determination"
- Higher is better, $R^2 = 1$ is perfect

Alternative Loss Functions

• **Pearson correlation:**

$$
\frac{1}{n} \sum_{i=1}^{n} \frac{(\hat{y}_i - \hat{\mu})(y_i - \mu)}{\hat{\sigma} \sigma}
$$

• Usually estimated from some sampled measurements of those variables, and denoted as R (related to R^2 on the last slide!)

• **Rank-order correlation:**

- First rank the measurements of \hat{y}_i and y separately, then replace each value in y by its rank, and ditto for \hat{y}
- Then measure the linear correlation between those ranks

Taking a Step Up…

Function Approximation View of ML

ML algorithm outputs a model f that best "approximates" the given data Z

Function Approximation View of ML

• Framework for designing machine learning algorithms

• **Two design decisions**

- What is the family of candidate models f ? (E.g., linear functions)
- How to define "approximating"? (E.g., MSE loss)
- Why is called "function approximation"?

Aside: "True Function"

- **Input: Dataset Z**
	- Presume there is an unknown function f^* that generates Z
- Goal: Find an approximation $f_\beta \approx f^*$ in our model family $f_\beta \in F$
	- Often, f^* not in our model family F

Function Approximation View of ML

• Framework for designing machine learning algorithms

• **Two design decisions**

- What is the family F of candidate models f ? (E.g., linear functions)
- How to define "approximating" (i.e., the loss $L(f; Z)$)? (E.g., MSE loss)
- How do we specialize to linear regression?

Function Approximation View of ML

Data Z Machine learning algorithm

Model f

Loss Minimization

Data Z Machine learning algorithm

Model f
Loss Minimization

Loss Minimization

ML algorithm minimizes loss of parameters β over data Z

Loss Minimization for Supervised Learning

 $\hat{\beta}(Z) = \arg \min_{\beta} L(\beta; Z)$ Data Z

Model $f_{\widehat{\beta}(Z)}$

Loss Minimization for Supervised Learning

Data $Z = \{ (x_i, y_i) \}_{i=1}^n$ $\hat{\beta}(Z) = \arg \min_{\beta} L(\beta; Z)$ *L* encodes $y_i \approx f_\beta(x_i)$

Model $f_{\widehat{\mathcal{B}}(Z)}$

Goal is for function to approximate label y given input x

Loss Minimization for Regression

Model $f_{\widehat{\beta}(Z)}$

Data $Z = \{ (x_i, y_i) \}_{i=1}^n$ $\hat{\beta}(Z) = \arg \min_{\beta} L(\beta; Z)$ *L* encodes $y_i \approx f_\beta(x_i)$

Label is a real number $y_i \in \mathbb{R}$

Linear Regression

Data
$$
Z = \{(x_i, y_i)\}_{i=1}^n
$$
 $\hat{\beta}(Z) = \arg\min_{\beta} L(\beta; Z)$ Model $f_{\hat{\beta}(Z)}$
\n L encodes $y_i \approx f_{\beta}(x_i)$
\nMSE loss Model is a linear function $f_{\beta}(x) = \beta^{\top}x$

Linear Regression

General strategy

- Model family $F = \{f_\beta\}_{\beta}$
- Loss function $L(\beta; Z)$

Linear regression strategy

• Linear functions $F = \{f_{\beta}(x) = \beta^{\top}x\}$

• MSE
$$
L(\beta; Z) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \beta^\top x_i)^2
$$

Linear regression algorithm

$$
\hat{\beta}(Z) = \argmin_{\beta} L(\beta; Z)
$$

Agenda

• **Function approximation view of machine learning**

- Modern strategy for designing machine learning algorithms
- **By example:** Linear regression, a simple machine learning algorithm

• **Bias-variance tradeoff**

- Fundamental challenge in machine learning
- **By example:** Linear regression with feature maps

Example: Quadratic Function

Example: Quadratic Function

Can we get a better fit?

Feature Maps

General strategy

- Model family $F = \{f_\beta\}_{\beta}$
- Loss function $L(\beta; Z)$

Linear regression with feature map

• Linear functions over a given feature map $\phi: X \to \mathbb{R}^d$

$$
F = \{ f_{\beta}(x) = \beta^{\top} \phi(x) \}
$$

• MSE
$$
L(\beta; Z) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \beta^\top \phi(x_i))^2
$$

Quadratic Feature Map

• Consider the feature map $\phi: \mathbb{R} \to \mathbb{R}^2$ given by

$$
\phi(x) = \begin{bmatrix} x \\ x^2 \end{bmatrix}
$$

• Then, the model family is

$$
f_{\beta}(x) = \beta_1 x + \beta_2 x^2
$$

Quadratic Feature Map

Feature Maps

• Powerful strategy for encoding prior knowledge

• **Terminology**

- x is the **input** and $\phi(x)$ are the **features**
- Often used interchangeably

Examples of Feature Maps

• **Polynomial features**

- $\phi(x) = \beta_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_1^2 + \beta_5 x_1 x_2 + \beta_6 x_2^2 + \cdots$
- Quadratic features are very common; capture "feature interactions"
- Can use other nonlinearities (exponential, logarithm, square root, etc.)

• **Intercept term**

- $\phi(x) = [1 \quad x_1 \quad ... \quad x_d]^\top$
- Almost always used; captures constant effect

• **Encoding non-real inputs**

- E.g., $x =$ "the food was good" and $y = 4$ stars
- $\phi(x) = [1("good" \in x) \quad 1("bad" \in x) \quad ...]^{T}$

Algorithm

- Reduces to linear regression
- **Step 1:** Compute $\phi_i = \phi(x_i)$ for each x_i in Z
- Step 2: Run linear regression with $Z' = \{(\phi_1, y_1), ..., (\phi_n, y_n)\}$

Question

• **Why not throw in lots of features?**

•
$$
\phi(x) = \beta_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_1^2 + \beta_5 x_1 x_2 + \beta_6 x_2^2 + \cdots
$$

• Can fit any n points using a polynomial of degree n

Prediction

- **Issue:** The goal in machine learning is **prediction**
	- Given a **new** input x, predict the label $\hat{y} = f_{\beta}(x)$

The errors on new inputs is very large!

Prediction

- **Issue:** The goal in machine learning is **prediction**
	- Given a **new** input x, predict the label $\hat{y} = f_{\beta}(x)$

Vanilla linear regression actually works better!

Training vs. Test Data

- **Training data:** Examples $Z = \{(x, y)\}$ used to fit our model
- **Test data:** New inputs x whose labels y we want to predict

Overfitting vs. Underfitting

• **Overfitting**

- Fit the **training data** Z well
- Fit new **test data** (x, y) poorly

• **Underfitting**

- Fit the **training data** Z poorly
- (Necessarily fit new **test data** (x, y) poorly)

Aside: Why Does Overfitting Happen?

• Overfitting typically due to fitting noise in the data

• Noise in labels y_i

- True data generating process is more complex than we can capture
- May depend on unobserved features

• **Noise in features** &

- Measurement error in the feature values
- Errors due to preprocessing
- Some features might be irrelevant to the decision function

Training/Test Split

- **Issue:** How to detect overfitting vs. underfitting?
- **Solution:** Use **held-out test data** to estimate loss on new data
	- Typically, randomly shuffle data first

• **Step 1:** Split Z into Z_{train} and Z_{test}

Training data Z_{train} Test data Z_{test}

- **Step 2:** Run linear regression with Z_{train} to obtain $\hat{\beta}(Z_{train})$
- **Step 3:** Evaluate
	- **Training loss:** $L_{\text{train}} = L(\hat{\beta}(Z_{\text{train}}); Z_{\text{train}})$
	- **Test (or generalization) loss:** $L_{\text{test}} = L(\hat{\beta}(Z_{\text{train}}); Z_{\text{test}})$

• **Overfitting**

- Fit the **training data** Z well
- Fit new **test data** (x, y) poorly

• **Underfitting**

- Fit the **training data** Z poorly
- (Necessarily fit new **test data** (x, y) poorly)

 $\boldsymbol{\mathcal{X}}$

• **Overfitting**

- L_{train} is small
- L_{test} is large

• **Underfitting**

- Fit the **training data** Z poorly
- (Necessarily fit new **test data** (x, y) poorly)

- **Overfitting**
	- L_{train} is small
	- L_{test} is large

• **Underfitting**

- L_{train} is large
- L_{test} is large

Aside: IID Assumption

• **Underlying IID assumption**

- Future data are drawn IID from same data distribution $P(x, y)$ as Z_{test}
- IID = independent and identically distributed
- This is a strong (but common) assumption!

• **Time series data**

- Particularly important failure case since data distribution may shift over time
- **Solution:** Split along time (e.g., data before 9/1/20 vs. data after 9/1/20)

How to Fix Underfitting/Overfitting?

• Choose the right model family!

Role of Capacity

- **Capacity** of a model family captures "complexity" of data it can fit
	- Higher capacity \rightarrow more likely to overfit (model family has high **variance**)
	- Lower capacity \rightarrow more likely to underfit (model family has high **bias**)
- For linear regression, capacity roughly corresponds to feature dimension d
	- I.e., number of features in $\phi(x)$

• **Overfitting (high variance)**

- High capacity model capable of fitting complex data
- Insufficient data to constrain it

• **Underfitting (high bias)**

- Low capacity model that can only fit simple data
- Sufficient data but poor fit

 χ

Warning: Very stylized plot! Solide by Padhraic Smyth, UCIrvine

- For linear regression, increasing feature dimension $d...$
	- Tends to **increase capacity**
	- Tends to **decrease bias** but **increase variance**
- Need to construct ϕ to balance tradeoff between bias and variance
	- **Rule of thumb:** $n \approx d \log d$
	- Large fraction of data science work is data cleaning + feature engineering

- Increasing number of examples n in the data...
	- Tends to **keep bias fixed** and **decrease variance**
- **General strategy**
	- **High bias:** Increase model capacity d
	- **High variance:** Increase data size n (i.e., gather more labeled data)

Housing Dataset

- Sales of residential property in Ames, Iowa from 2006 to 2010
	- **Examples:** 1,022
	- **Features:** 79 total (real-valued + categorical), some are missing!
	- **Label**: Sales price

Housing Dataset

· dataframe.describe()

Feature Correlation Matrix

Features Most Correlated with Label

Missing Values

- Possible ways to handle missing values
	- **Numerical:** Impute with mean
	- **Categorical:** Impute with mode

Other Preprocessing

- **Categorical:** Featurize using one-hot encoding
- **Ordinal**
	- Convert to integer (e.g., low, medium, high \rightarrow 1, 2, 3)
	- Does not fully capture relationships (try different featurizations!)

Evaluation

- 438 test examples, **preprocessed same as training data**
- Sorted by prediction error

