# CIS 4190/5190: Lec 13 Mon Oct 21, 2024

# Convolutional Neural Networks Part 1/2

# Course Progress

Till now: (mostly) foundational algorithms applicable to large classes of machine learning problems.

Going forward: (mostly) applications to specific types of data and specific types of problems.

- New Types of Data: Grids (e.g. Images), Sequences (e.g. Language)
- New Types of Problems: Making Sequences of Decisions (e.g. Robotics), Recommendation Systems

# Types of Data

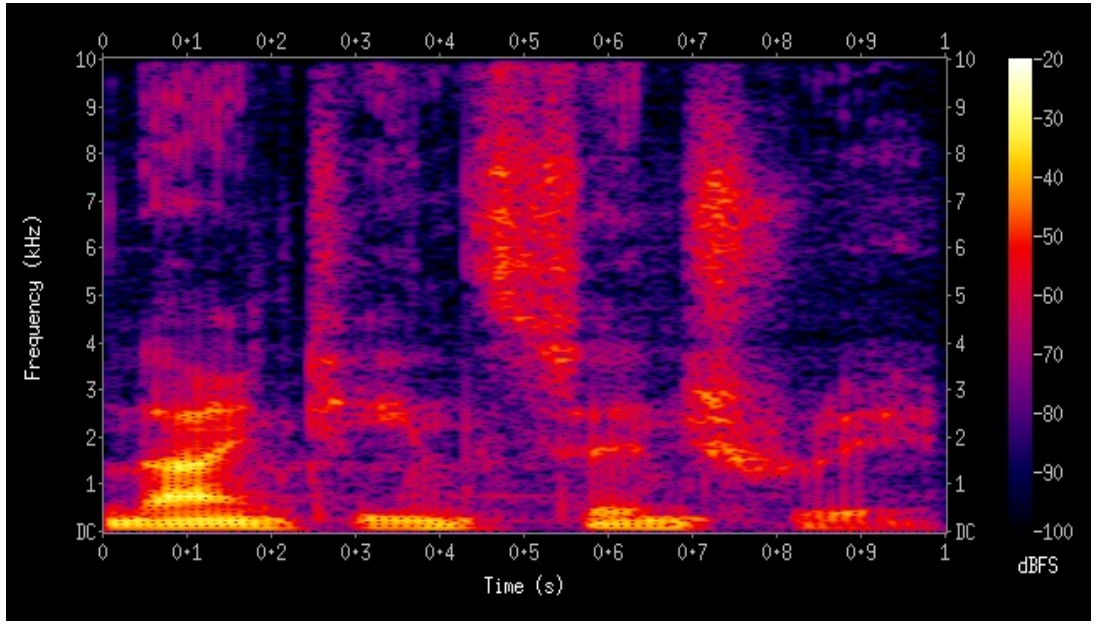
- Until now, the $i^{\text{th}}$ sample in our dataset was either naturally a **vector** $x_i$ or we converted it into one.

- What if our data samples were more naturally expressed in a different structure?
  - $\boldsymbol{x}_i$ is a "grid": e.g. images
  - $\boldsymbol{x}_i$ is a "sequence": e.g. text
  - $\boldsymbol{x}_i$ is a "graph": e.g. protein structure

# Neural Networks Specialized to Grid Data

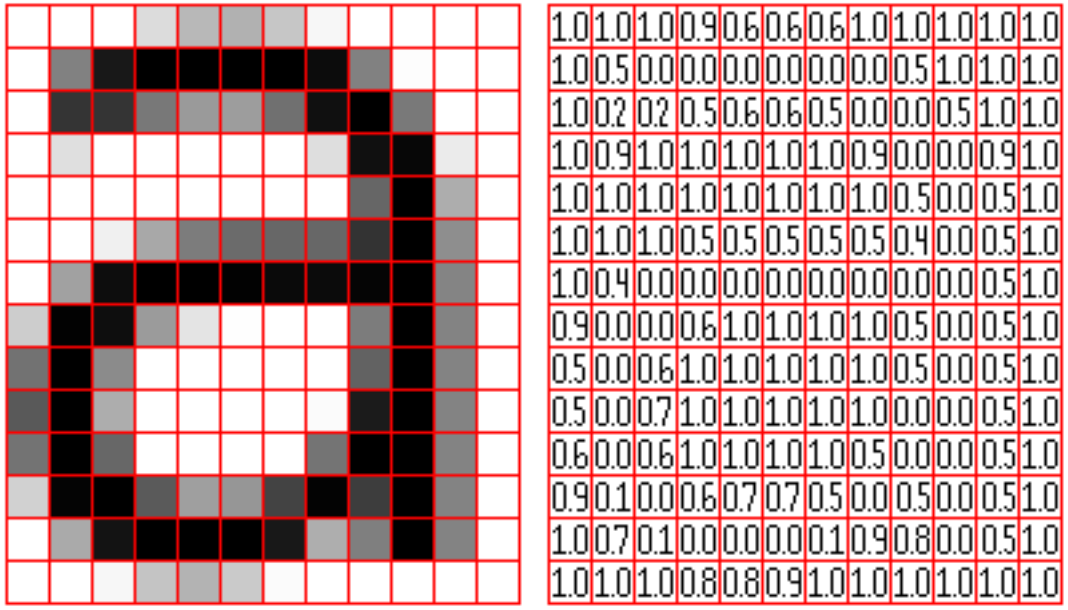
- We will study a class of neural networks called convolutions that specialize to properties often present in *grid* data, particularly images.

https://en.wikipedia.org/wiki/Spectrogram




Spectrogram encoding of audio

https://towardsdatascience.com/convolutional-neural-networks-explained-9cc5188c4939




Digital image

# Images as 2D Arrays

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 3 | 2 | 5 | 4 | 7 | 6 | 9 | 8 |
| 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 5 | 2 | 3 | 0 | 1 | 2 | 3 | 4 | 5 |
| 4 | 3 | 2 | 1 | 0 | 3 | 2 | 5 | 4 |
| 7 | 4 | 5 | 2 | 3 | 0 | 1 | 2 | 3 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 | 3 | 2 |
| 9 | 6 | 7 | 4 | 5 | 2 | 3 | 0 | 1 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

What we see

What a computer sees

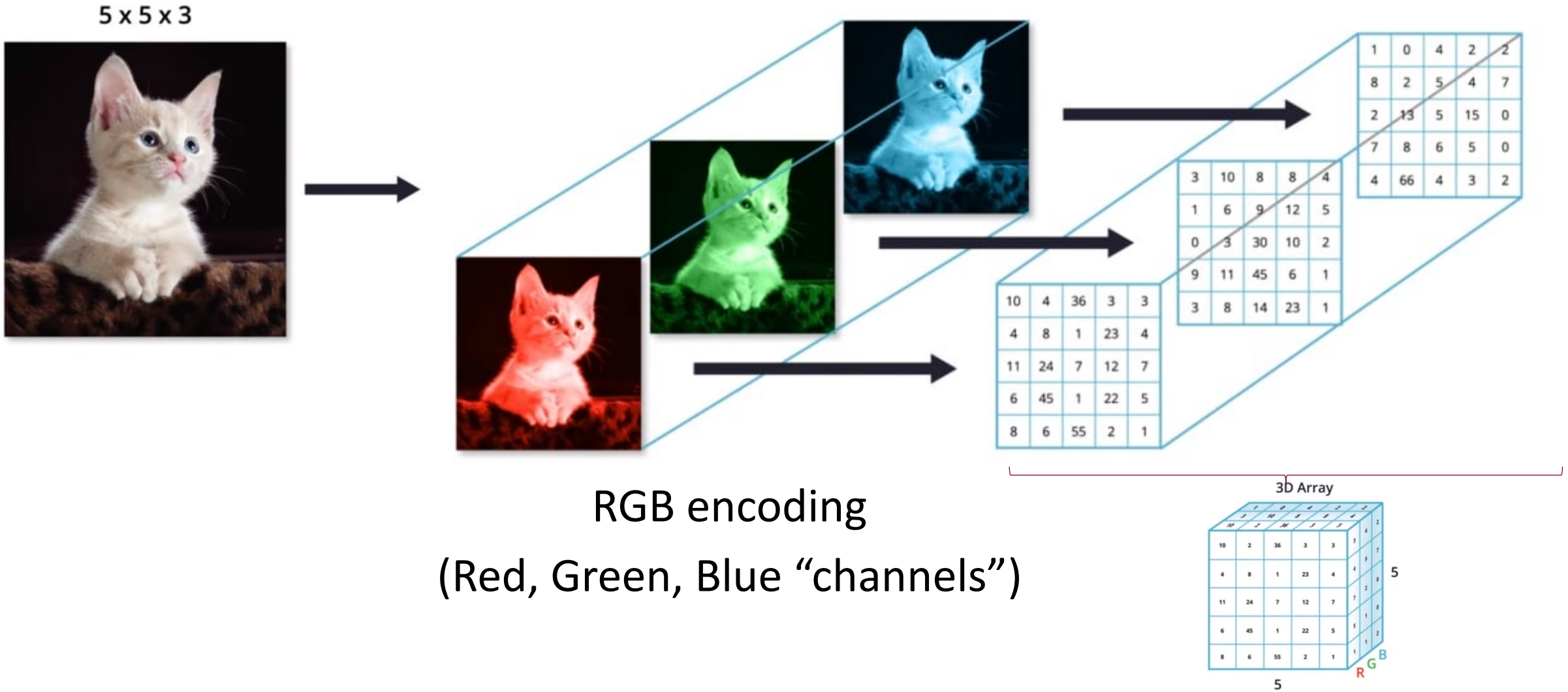
**Computer vision:**

How to extract meaning out of these 2D arrays?

Note: for color images, a stack of (typically 3) 2D arrays, each called a “channel”.

Source: S. Narasimhan, S. Lazebnik

# Color Images Are 3D Arrays with 2 Spatial Dimensions




RGB encoding

(Red, Green, Blue “channels”)

We will see: convenient to deal with the ***spatial*** *dimensions* separately, and there are still only two of those.

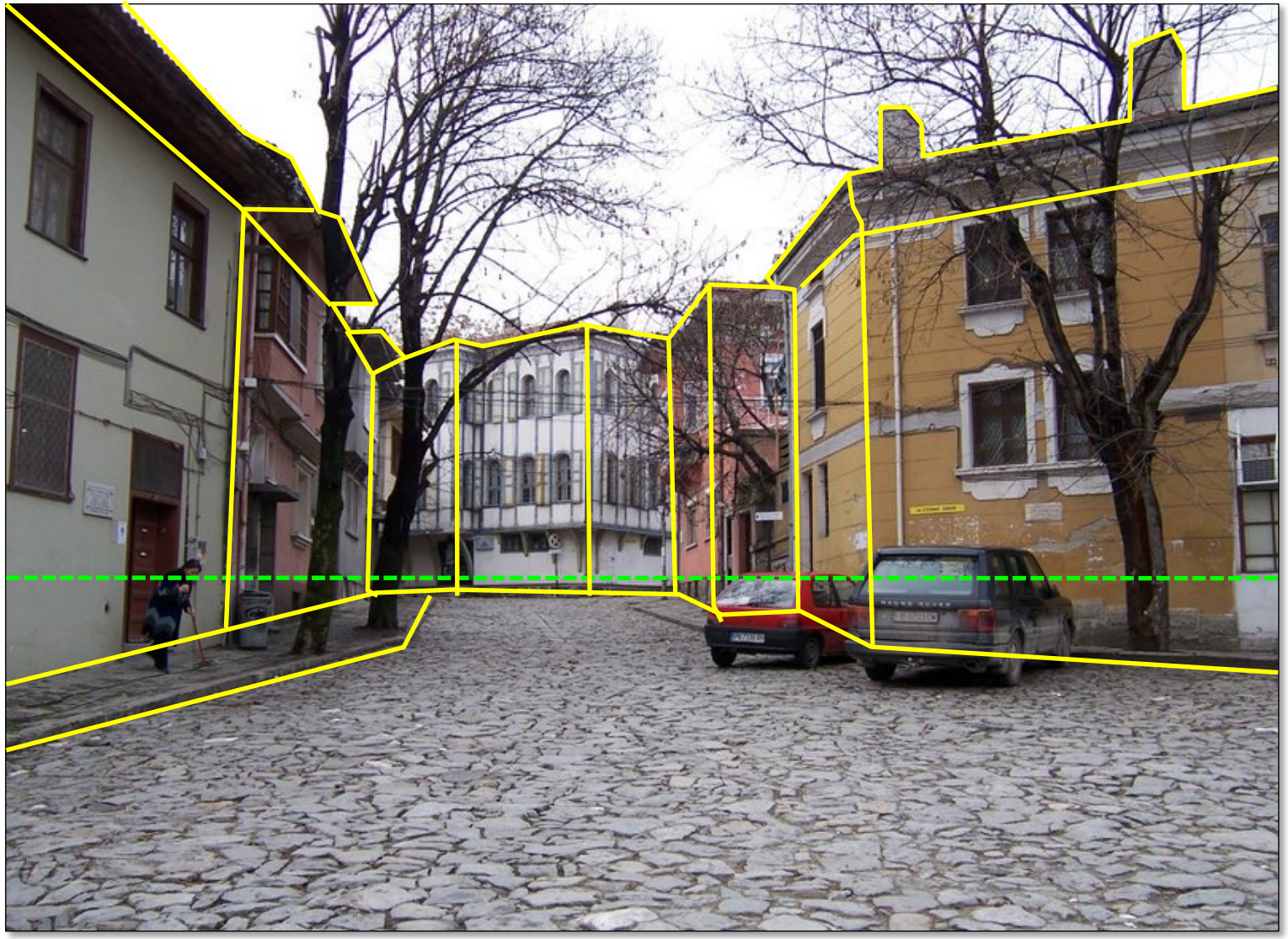
https://dev.to/sandeepbalachandran/machine-learning-going-furthur-with-cnn-part-2-41km

# What Info can be Extracted from Images?

Source: S. Lazebnik

# What Info can be Extracted from Images?

geometric information



Source: S. Lazebnik

# What Info can be Extracted from Images?

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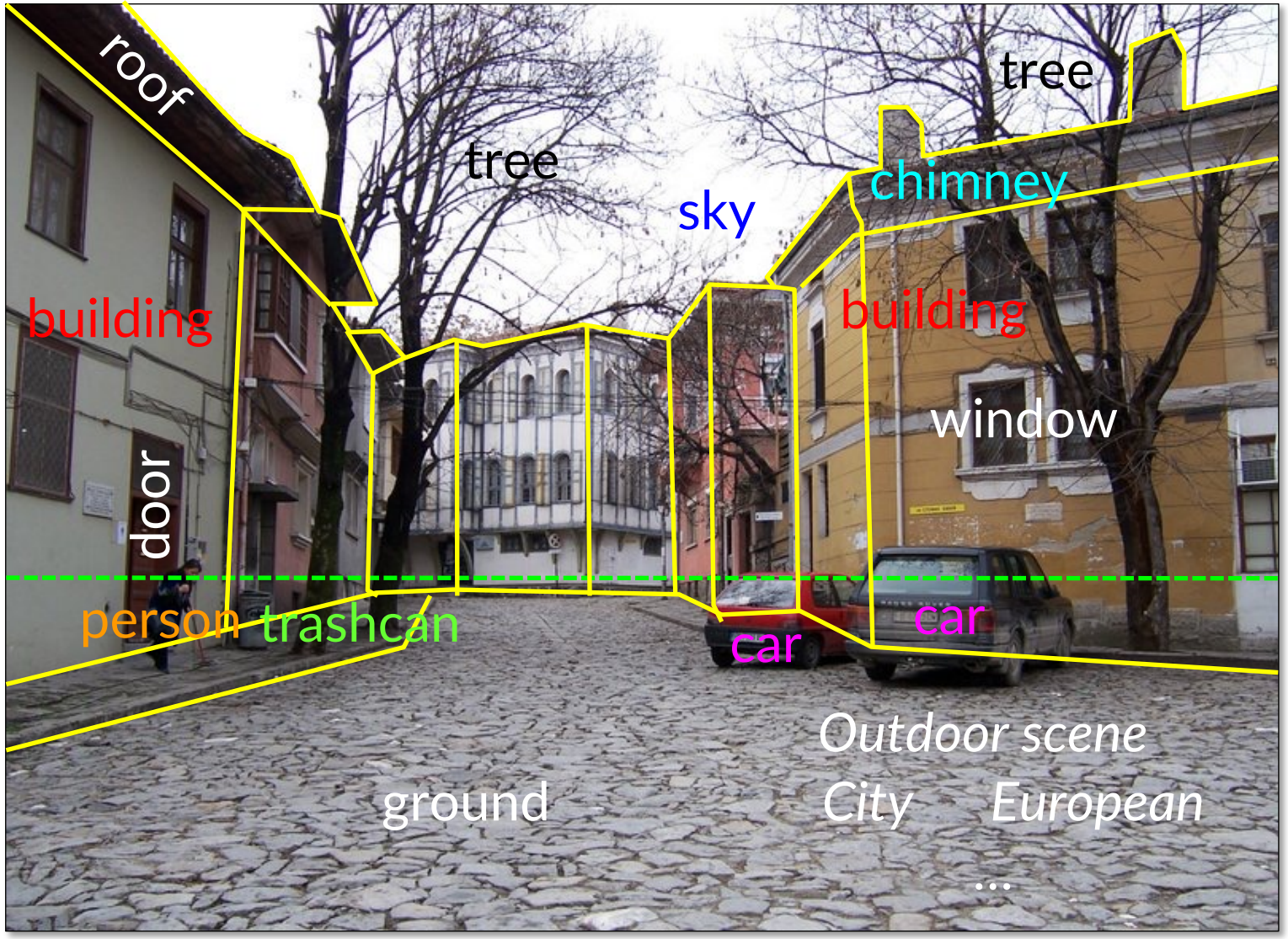


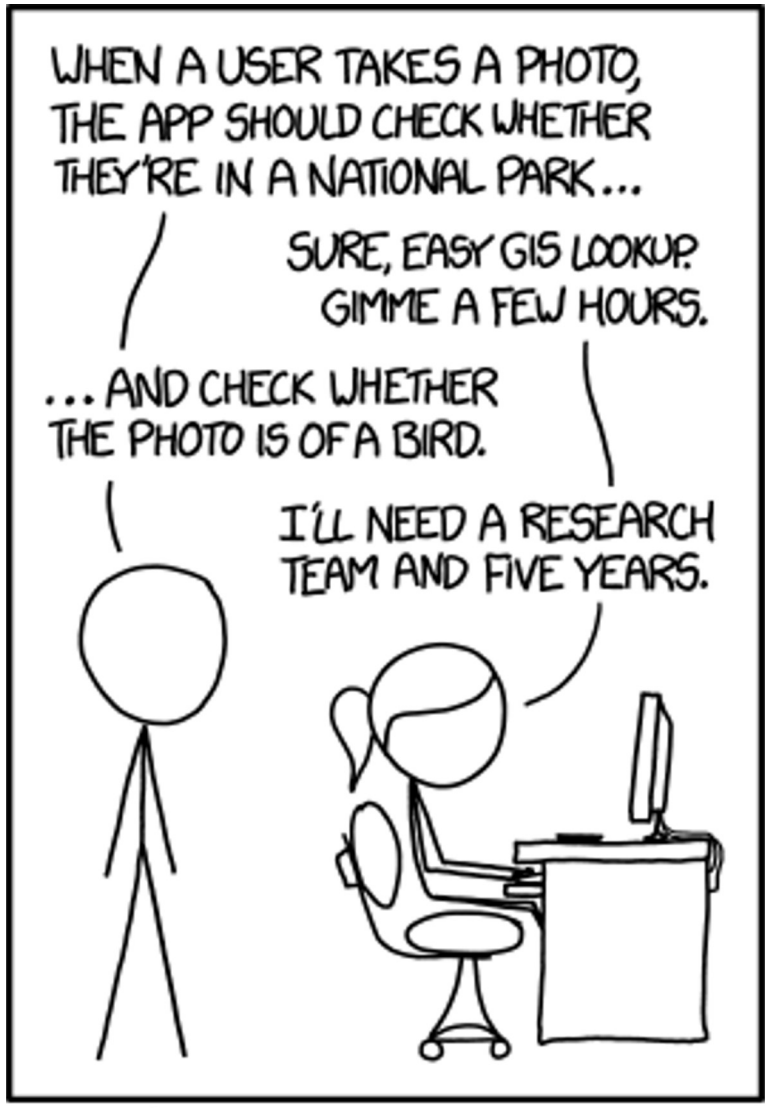

semantic information

Source: S. Lazebnik

# Vision is Deceptively Hard!

In the 1960s, Marvin Minsky assigned a couple of undergrads to spend the summer programming a computer to use a camera to identify objects in a scene. He figured they'd have the problem solved by the end of the summer.

Half a century later, we're still working on it.



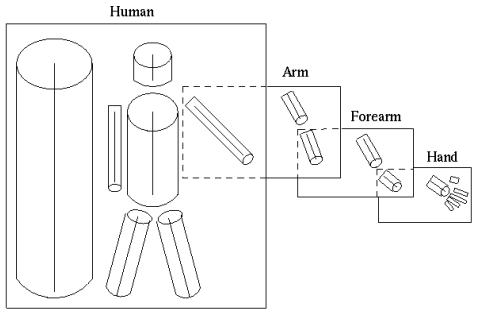

Source: XKCD

# The Treachery of Images – Rene Magritte


"This is not a pipe"

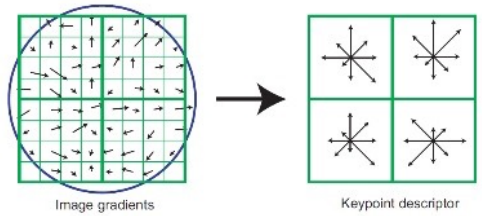
Vision often involves making educated guesses.

# ML in Computer Vision




**The *very* old: 1960's - Mid 1990's**

Image → hand-def. features → hand-def. classifier



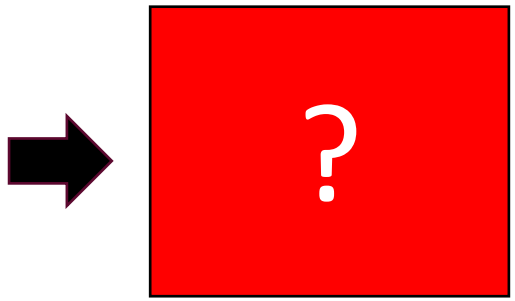

**The old: Mid 1990's – 2012**

Image → hand-def. features → learned classifier

# What Should Good Visual Representations Do?

Image

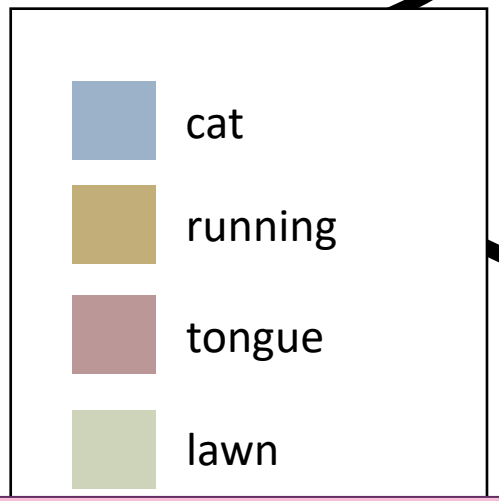
$D$-length feature $\boldsymbol{x}$




# What Should Good Visual Representations Do?

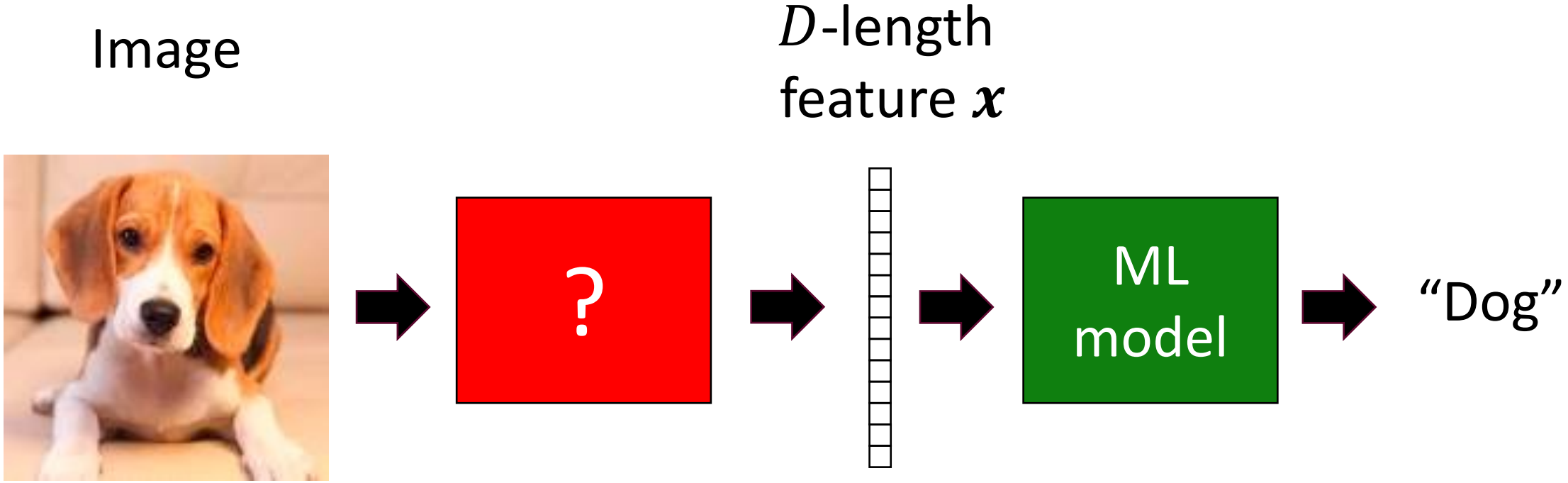
What is a “good” feature space?

?




Good features make useful tasks easy to perform.

# What Should Good Visual Representations Do?

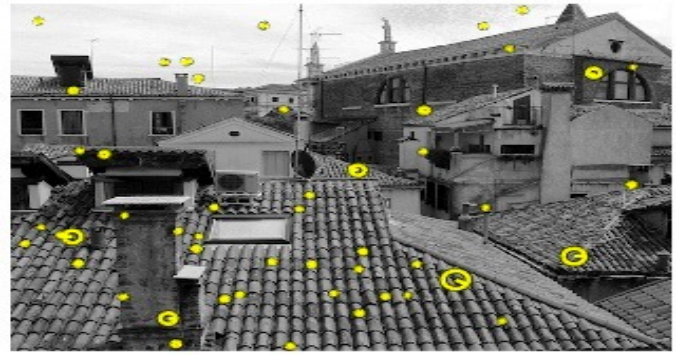


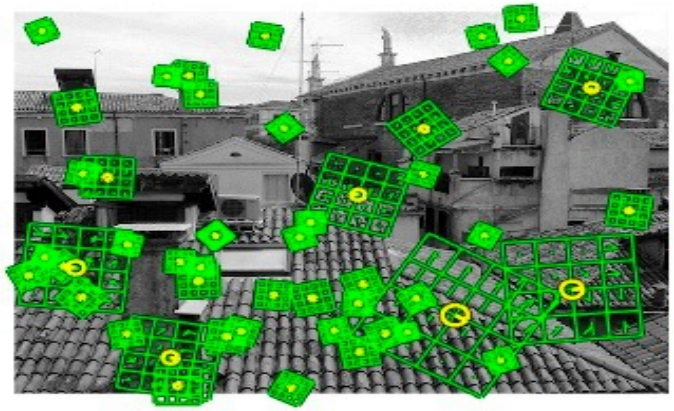

How should we produce such good features?

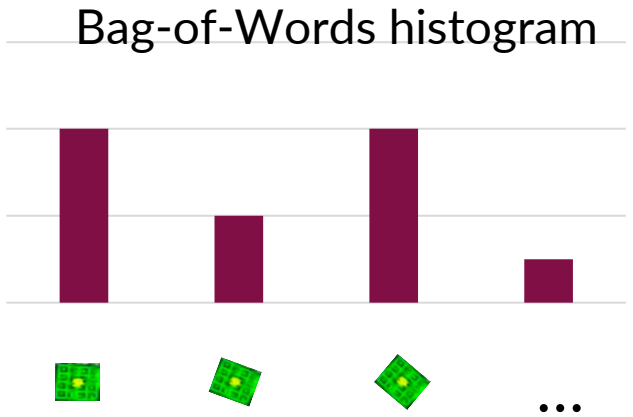
# Visual Features Before Deep Learning

# Most Feature Extraction Frameworks Pre-2012

- Step 1: Focus on “interest points” rather than all pixels
  - E.g. corner points, “difference of gaussians”, or even a uniform grid
- Step 2: Compute features at interest points.
  - E.g. “SIFT”, “HOG”, “SURF”, “GIST”, etc.
- Step 3: Convert to fixed-dimensional feature vector by measuring statistics of the features such as histograms
  - E.g. “Bag of Words”, “Spatial Pyramids”, etc.



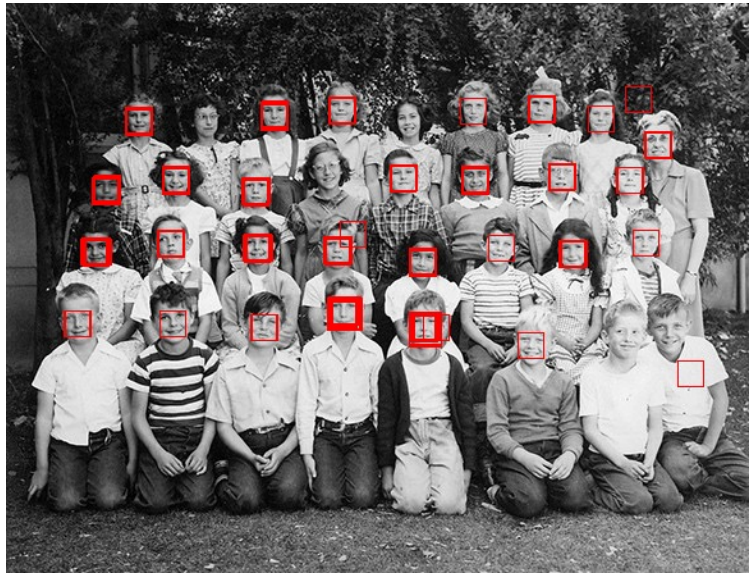





See libraries like VLFeat and OpenCV

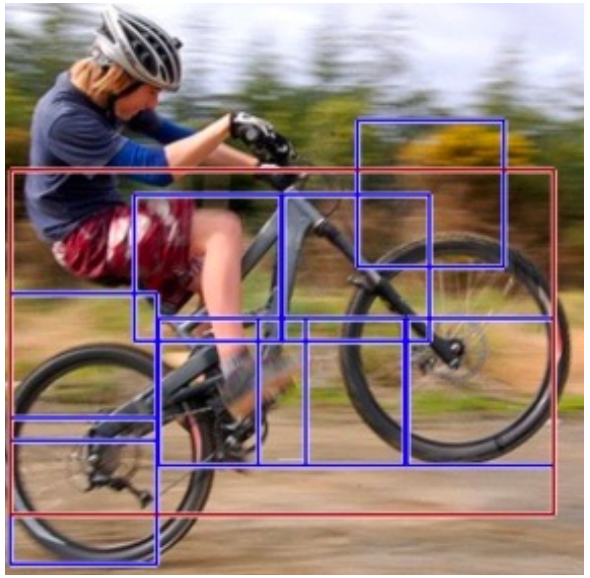
Use your favorite ML model now!

# Successes of ML for Vision Pre-2012



https://github.com/alexdemartos/ViolaAndJones

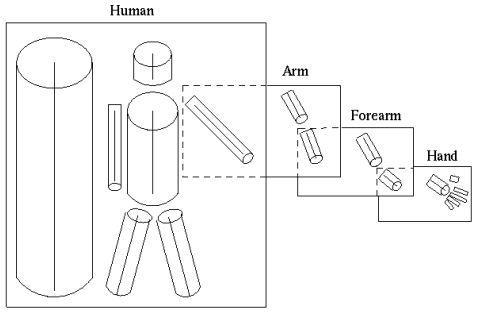
Viola-Jones face detector
(with AdaBoost!)
~2000



Deformable Parts Model
object detection
(with SVMs!)
~2010

GIST
Scene retrieval
(with nearest neighbors!)
~2006

# ML in Computer Vision

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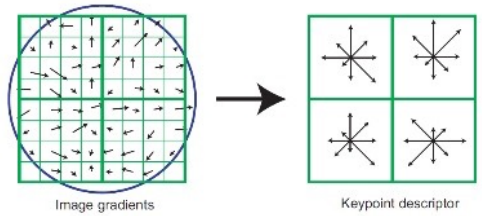
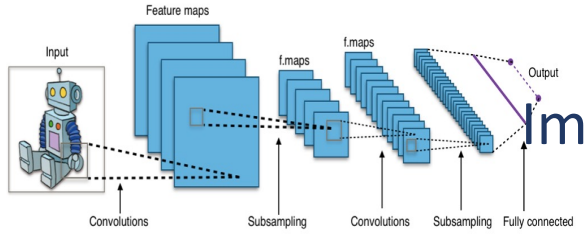



**The old: Mid 1990's – 2012**

Image → hand-def. features → learned classifier



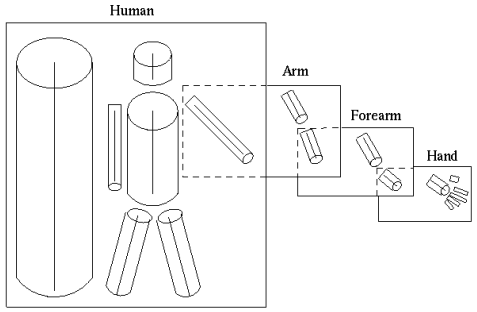

**The new: 2012 – ?**

Image → jointly learned features + classifier with "deep" multi-layer neural networks

# Representation Learning for Images

Convolutional Neural Networks

# What is Different Now?

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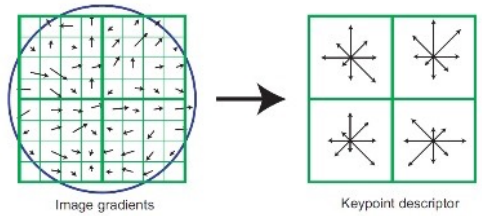
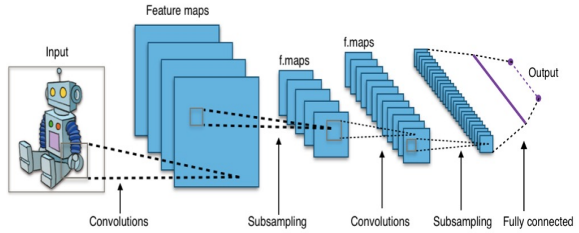



**The old: Mid 90's – 2012**

Image → hand-def. features → learned classifier




**The new: 2012 – ?**

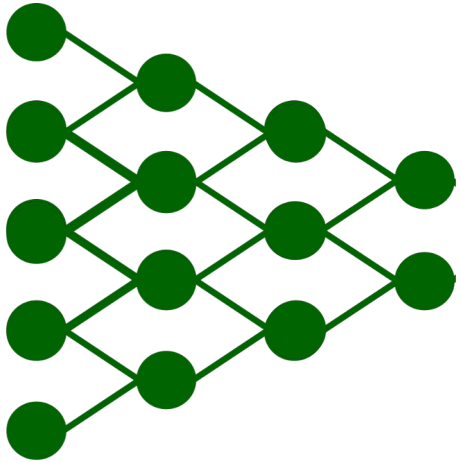
Image → jointly learned features + classifier

Answer: Representation learning

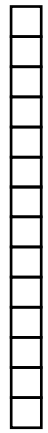
# “Deep” Learning

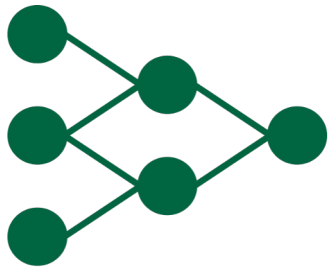
- “Deep” multi-layer neural networks are **representation learners**.
- Every layer improves upon its preceding layer, tailoring the representation to the task.

Image



$D$-length feature $\boldsymbol{x}$

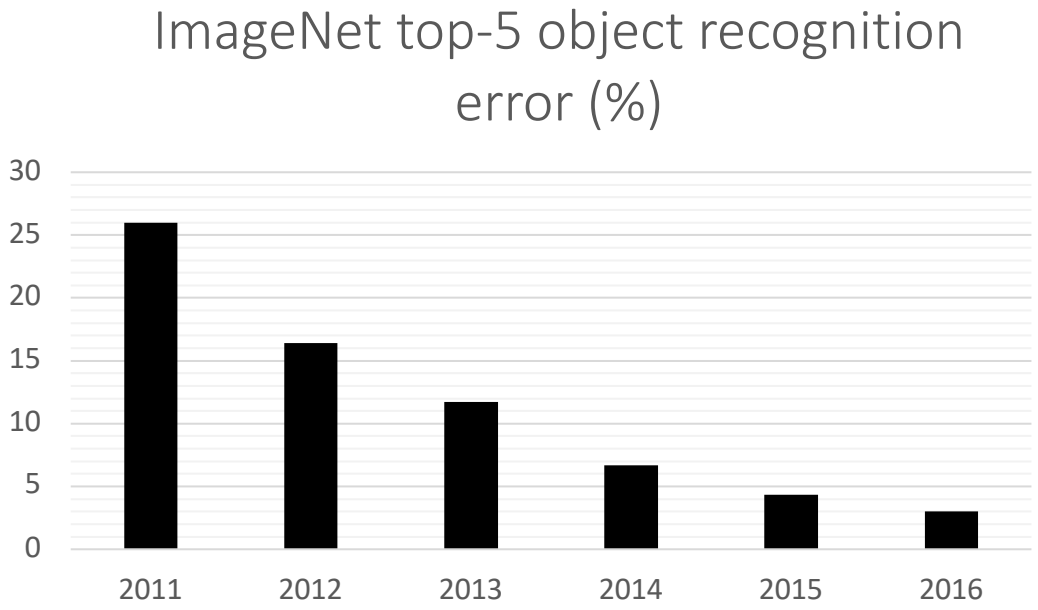




“dog”

# Impact of Deep Learning in Computer Vision

**ImageNet 1000-object category recognition challenge**



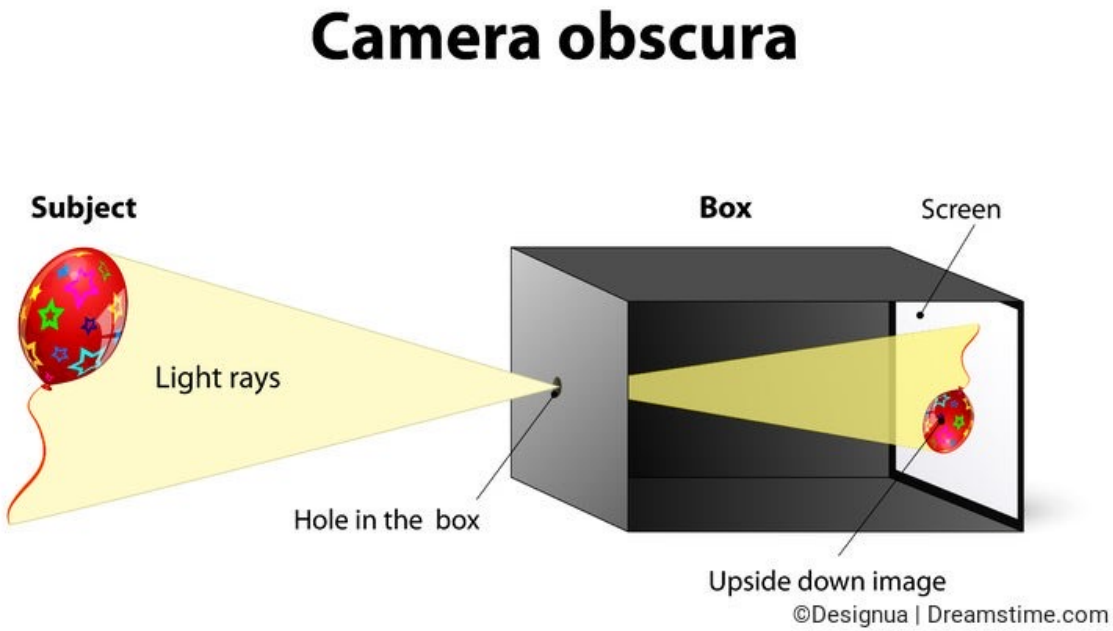

But the neural networks you have seen so far won't work well on images!

# What’s special about images?

- Images are special. Why?
- Bad news: They are very high-dimensional, which makes all ML harder.
- Good news: We don’t have to treat images as just vectors of pixels. We know more about them, and can exploit that knowledge.

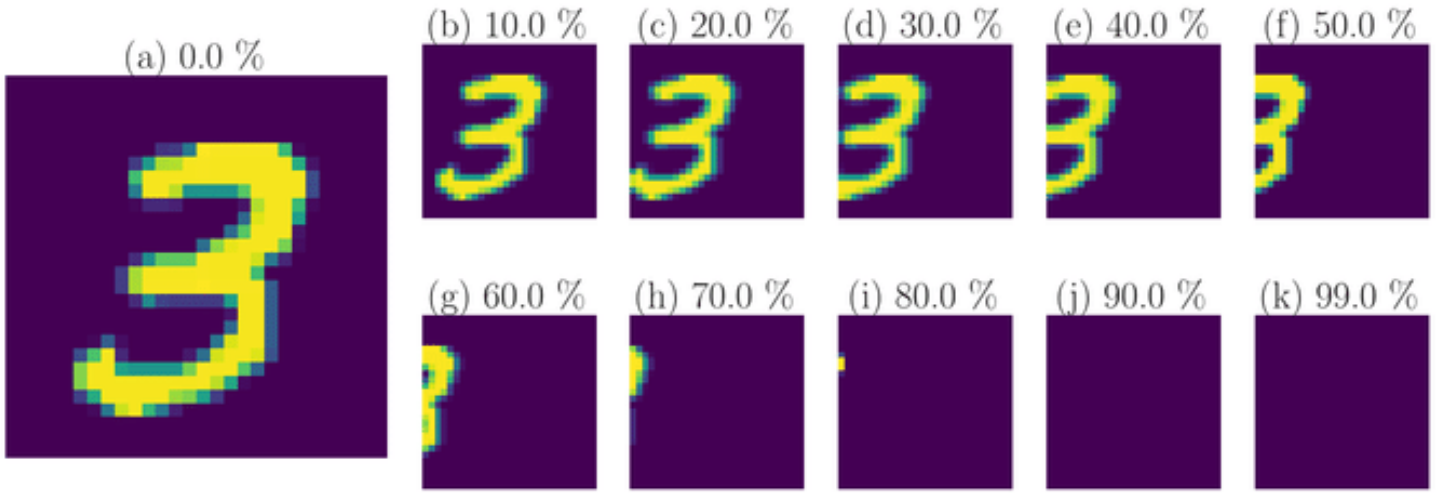
# Structure in Images

- **2D image structure**
  - So far, we could shuffle features without changing the problem (e.g., $\beta^\top x$)
  - Not true for images! Location associations and spatial neighborhoods are meaningful



# Structure in Images

- **Translation invariance**
  - Consider image classification (e.g., labels are cat, dog, etc.)
  - **Invariance:** If we translate an image, it does not change the category label




**Source:** Ott et al., Learning in the machine: To share or not to share?

# Structure in Images

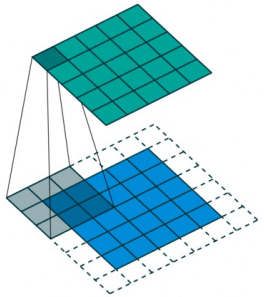
- **Translation equivariance**
  - Consider object detection (e.g., find the position of the cat in an image)
  - **Equivariance:** If we translate an image, the object is translated similarly


We will exploit this through image-specific operations in neural networks.

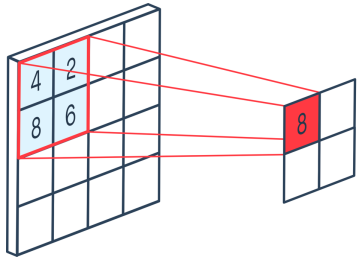
# “Image”-Specific Operators/Layers

- We want to retain useful **location associations,** and exploit **translation invariance** and **equivariance**.

- **Two key operations in neural networks for images:**



Convolution layers
(capture equivariance)




Pooling layers
(capture invariance)

https://towardsdatascience.com/types-of-convolutions-in-deep-learning-717013397f4d
https://peltarion.com/static/2d_max_pooling_pa1.png

# Convolutions Beyond Images

- Recall: convolutions try to gather useful **location associations,** and exploit **translation invariance** and **equivariance**.

- These properties are useful beyond just images. Need not even be 2-D grids.
  - E.g. detecting spikes in a time series of stock prices, or an audio stream. (1-D)
    - Also important to retain location associations
    - Local operations, invariance, equivariance.

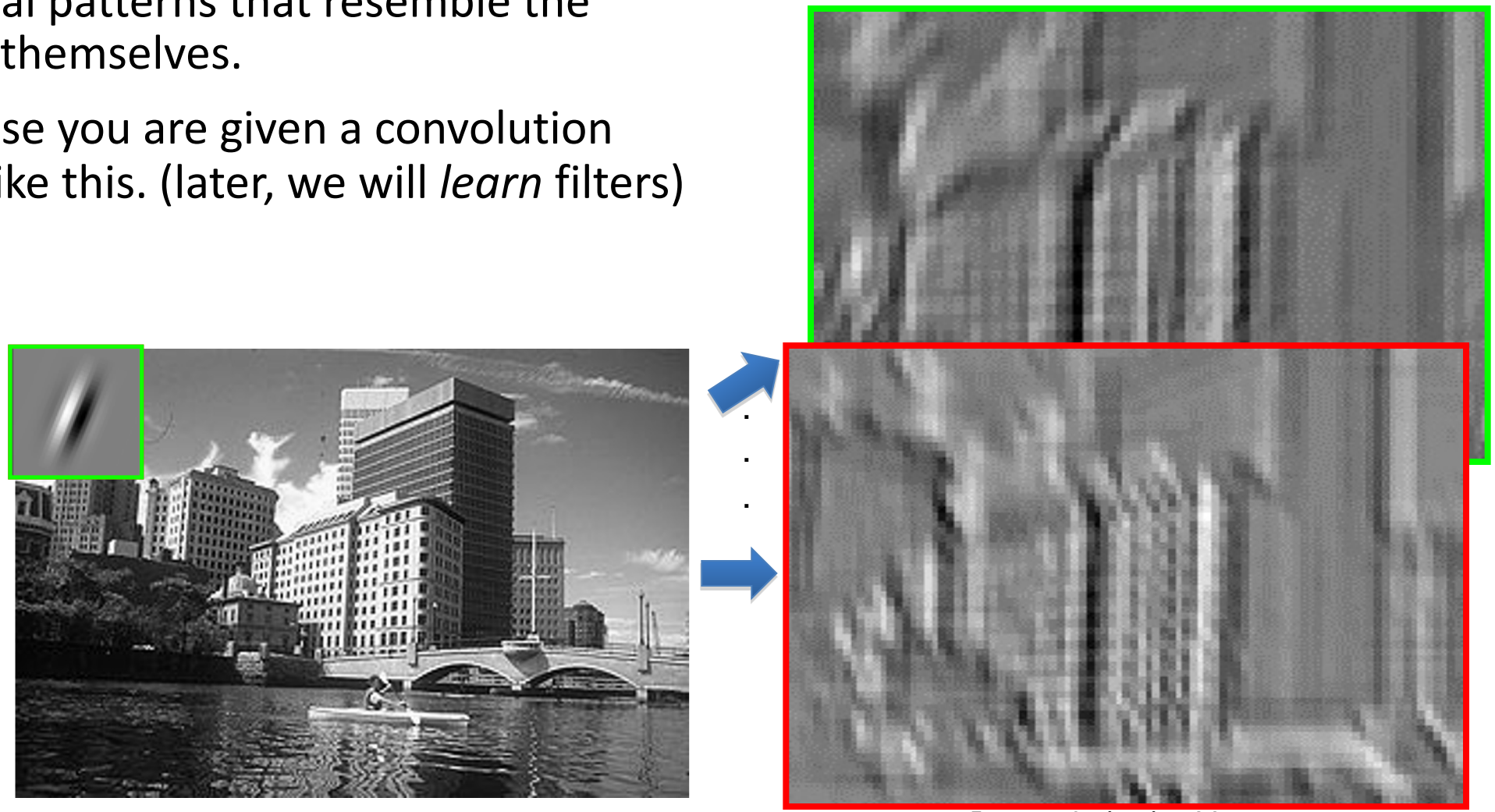
  - Can also apply in higher dimensions. E.g. convolving over a 3D “grid” of voxels to detect objects.

https://towardsdatascience.com/types-of-convolutions-in-deep-learning-717013397f4d
https://peltarion.com/static/2d_max_pooling_pa1.png

# Convolution

# Convolution Filters: Template Matching over an Image

- Intuitively, convolutional filters search for local patterns that resemble the filters themselves.
- Suppose you are given a convolution filter like this. (later, we will *learn* filters)



Input

Feature Activation Map

graphic credit: S. Lazebnik

# Convolutional filtering in 1D

- Suppose your input $x$ is a 1-D sequence, such as a time sequence, e.g. the stock market: $\boldsymbol{x} = [25000, 28000, 30000, 21000, 18000, \dots]$
- Given a "kernel" sequence, e.g. $\boldsymbol{k} = [-1, 1, -1]$
- Convolution is defined by the following operation:

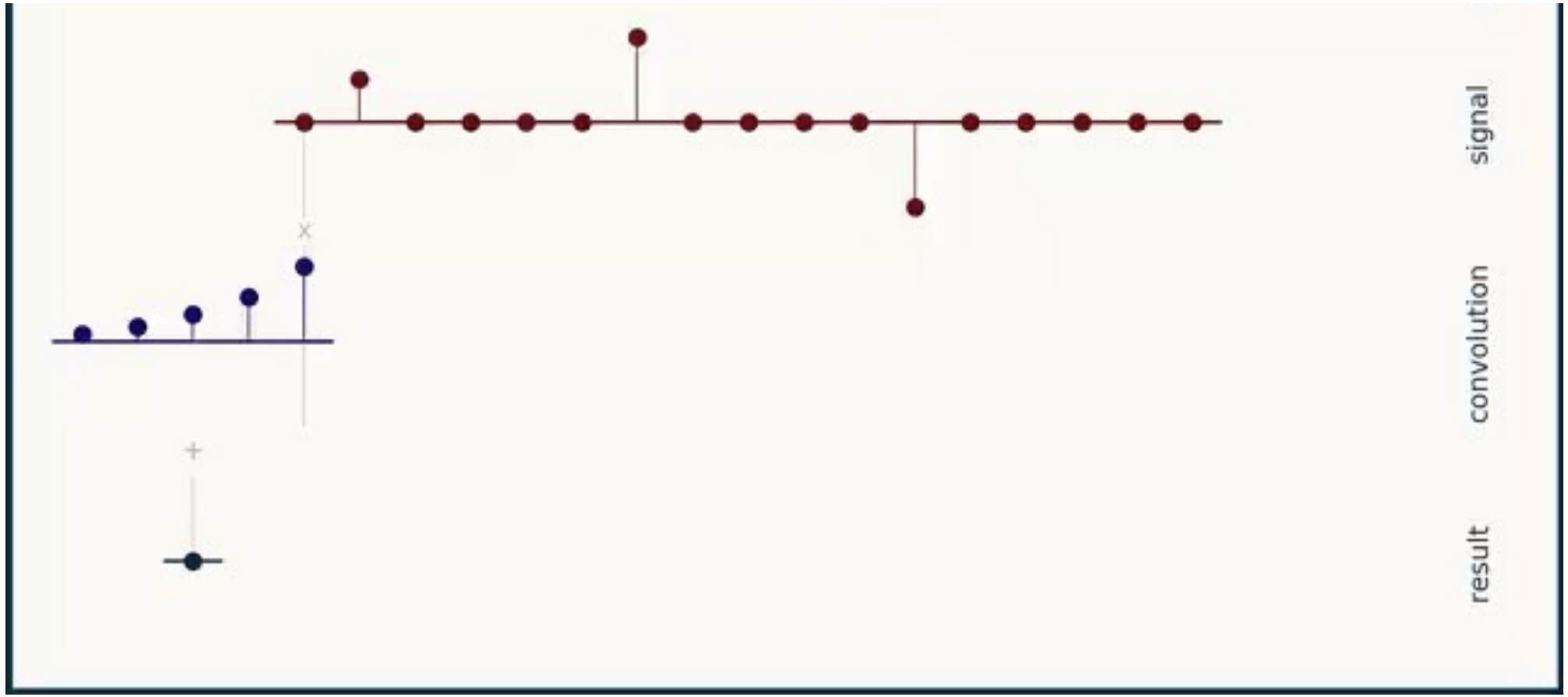
Strictly "cross-correlation", but we'll call it "convolution"

$$y[t] = \sum_{\tau=0}^{|\boldsymbol{k}|-1} k[\tau]x[t+\tau]$$

In neural networks, the weights $\boldsymbol{k}$ are learned. (Plus a bias)

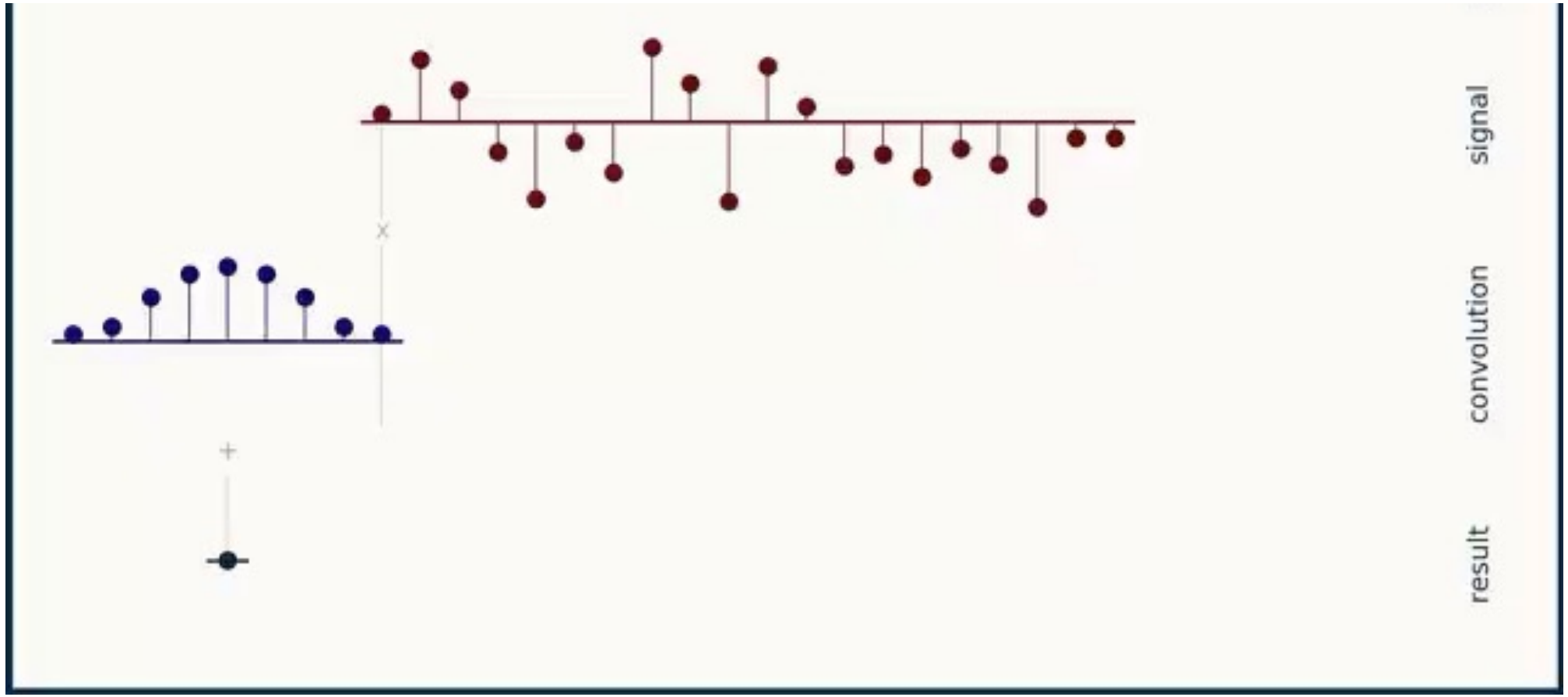
$$y[0] = k[0]x[0] + k[1]x[1] + k[2]x[2] = -25000 + 28000 - 30000$$
$$y[1] = k[0]x[1] + k[1]x[2] + k[2]x[3] = -28000 + 30000 - 21000$$
$$y[2] = k[0]x[2] + k[1]x[3] + k[2]x[4] = -30000 + 21000 - 18000$$

# Convolutional Filtering in 1D



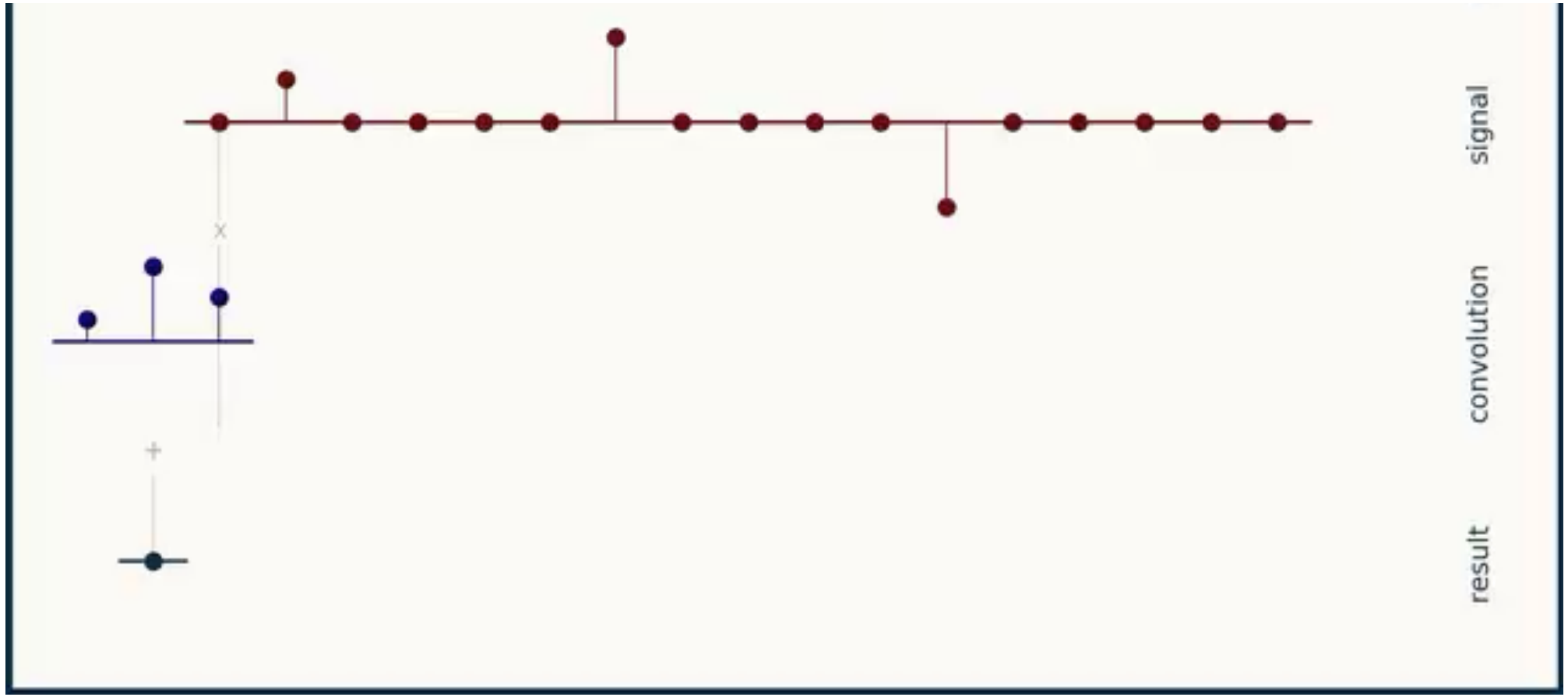

https://gitlab.com/brohrer/

# Convolutional Filtering in 1D




No good positive match, but good *negative* match?

# Convolutional Filtering in 1D




https://gitlab.com/brohrer/

# Convolutional filtering in 2D

- 1-D convolution is defined by the following operation:

$$y[t] = \sum_{\tau=0}^{|k|-1} k[\tau]x[t+\tau]$$

- With a 2-D signal $\boldsymbol{x}$ and 2-D $h \times w$ kernel $\boldsymbol{k}$, 2-D convolution is defined by the following operation:

$$y[s,t] = \sum_{\tau=0}^{h-1} \sum_{\gamma=0}^{w-1} k[\tau,\gamma]x[s+\tau,t+\gamma]$$

Again, in convolutional neural networks, the weights $\boldsymbol{k}$ will be learned.

# Convolutional filtering in 2D

$$y[s,t] = \sum_{\tau=0}^{h-1} \sum_{\gamma=0}^{w-1} k[\tau,\gamma] x[s+\tau, t+\gamma]$$

- To compute:
  - Slide kernel over image
  - Take the element-wise multiplication over the window and sum



Image: https://towardsdatascience.com/intuitively-understanding-convolutions-for-deep-learning-1f6f42faee1

# Example: Edge Detection via Convolution

## Example Edge Detection Kernels

| | | |
|---|---|---|
| -1 | -1 | -1 |
| 2 | 2 | 2 |
| -1 | -1 | -1 |

Horizontal lines

| | | |
|---|---|---|
| -1 | 2 | -1 |
| -1 | 2 | -1 |
| -1 | 2 | -1 |

Vertical lines

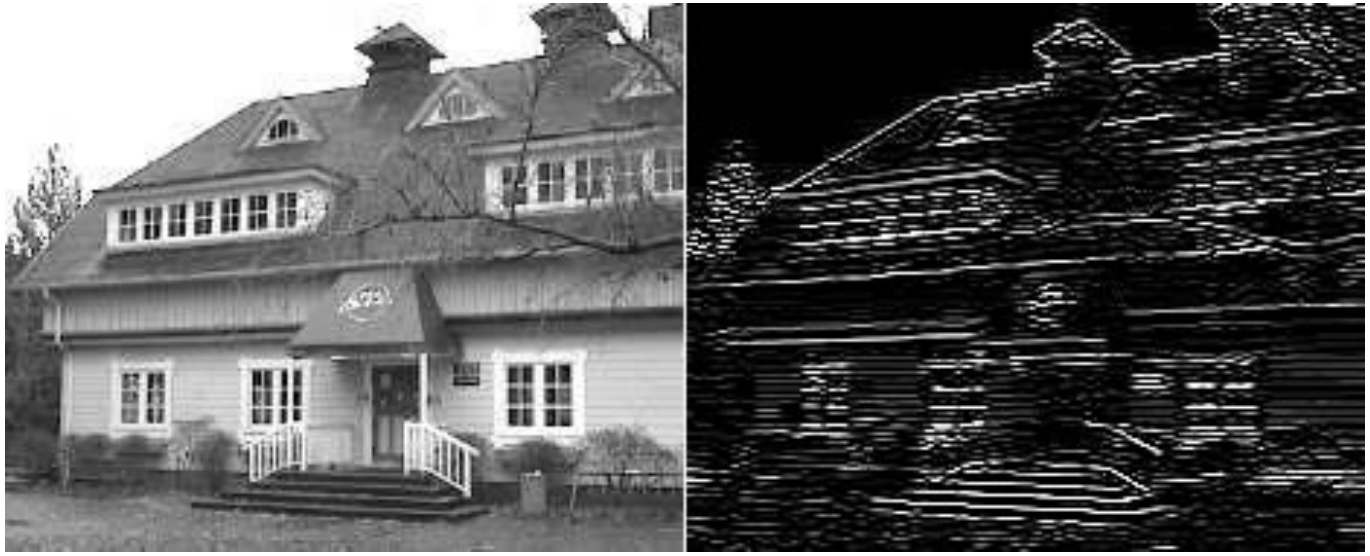
| | | |
|---|---|---|
| -1 | -1 | 2 |
| -1 | 2 | -1 |
| 2 | -1 | -1 |

45 degree lines

| | | |
|---|---|---|
| 2 | -1 | -1 |
| -1 | 2 | -1 |
| -1 | -1 | 2 |

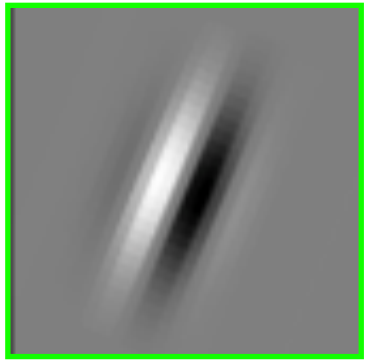
135 degree lines

## Result of Convolution with Horizontal Kernel



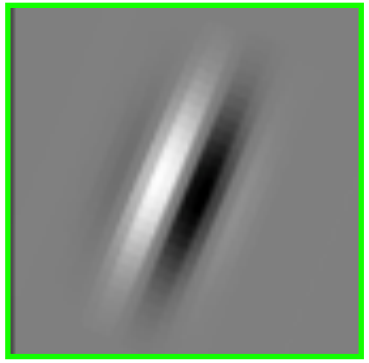
https://aishack.in/tutorials/image-convolution-examples/

# Back To Our Example



graphic credit: S. Lazebnik

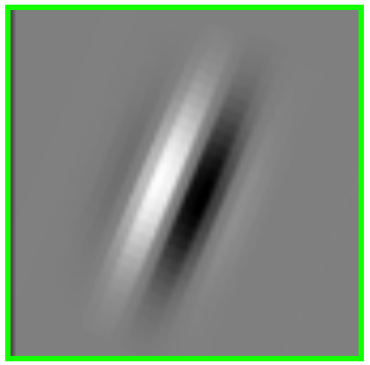
# Back To Our Example



$$\text{output}[0,0] = \sum_{\tau=0}^{k-1}\sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[0+\tau, 0+\gamma]$$

graphic credit: S. Lazebnik

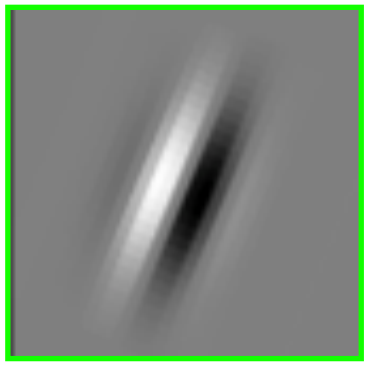
# Back To Our Example



$$\text{output}[0,1] = \sum_{\tau=0}^{k-1}\sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[0+\tau, 1+\gamma]$$

graphic credit: S. Lazebnik

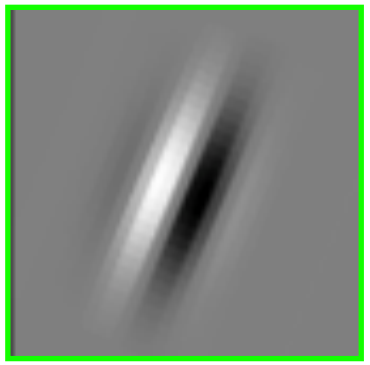
# Back To Our Example



$$\text{output}[0,2] = \sum_{\tau=0}^{k-1}\sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[0+\tau, 2+\gamma]$$

graphic credit: S. Lazebnik

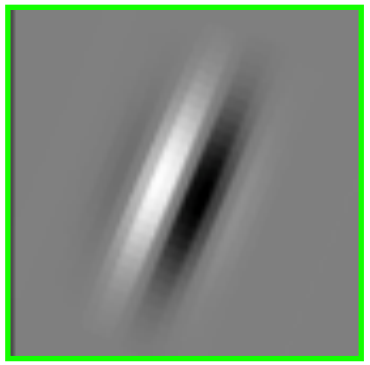
# Back To Our Example

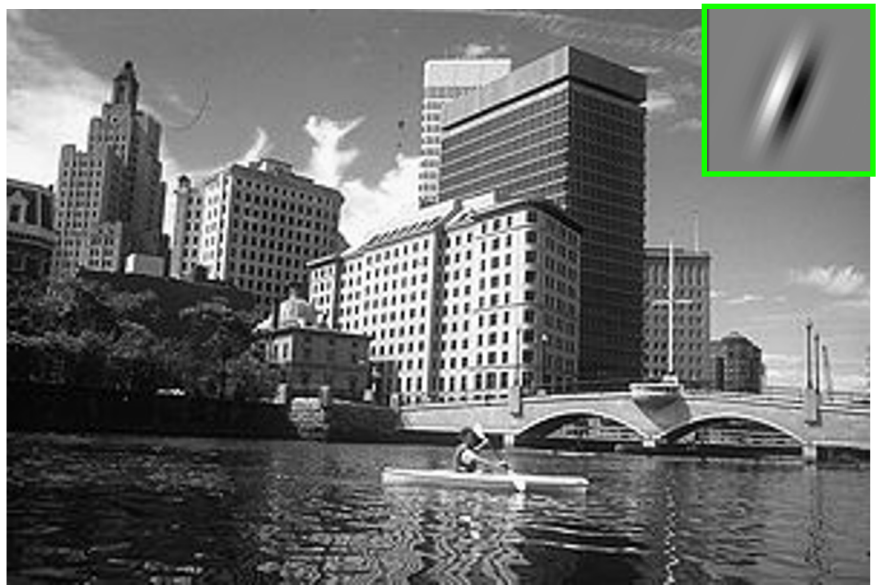


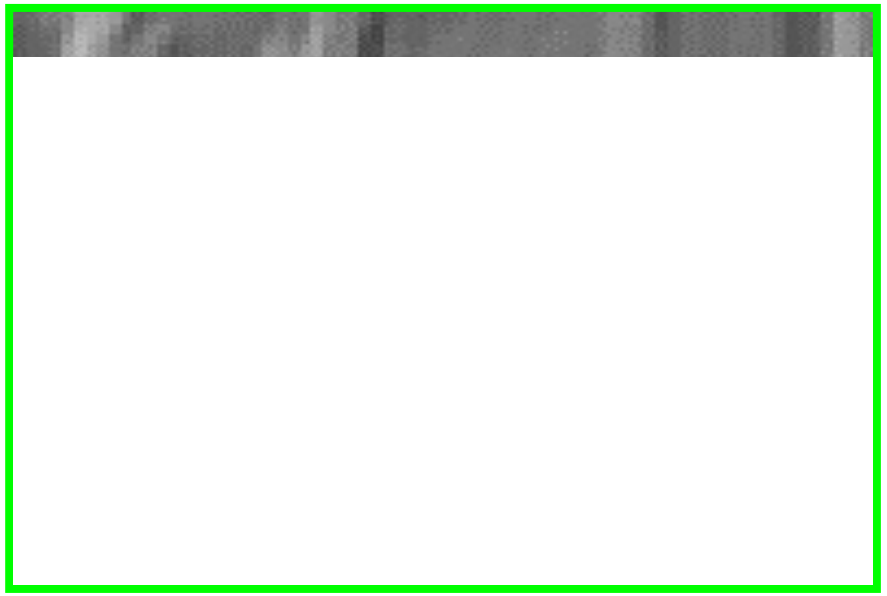
$$\text{output}[i,j] = \sum_{\tau=0}^{k-1}\sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[i+\tau, j+\gamma]$$

graphic credit: S. Lazebnik

# Back To Our Example



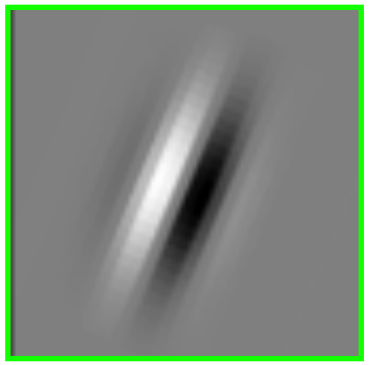


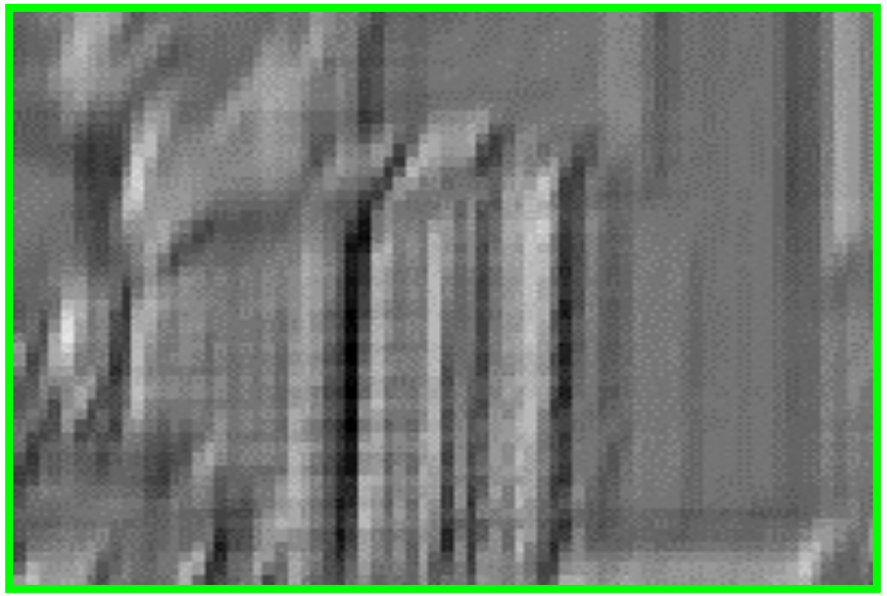


$$\text{output}[i,j] = \sum_{\tau=0}^{k-1}\sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[i+\tau, j+\gamma]$$

graphic credit: S. Lazebnik

# Back To Our Example





$$\text{output}[i,j] = \sum_{\tau=0}^{k-1}\sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[i+\tau, j+\gamma]$$

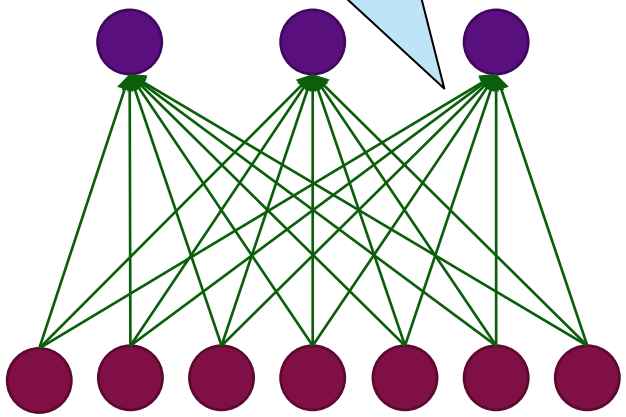
graphic credit: S. Lazebnik

# Convolutions Are Frugal

From fully connected layers to convolutions

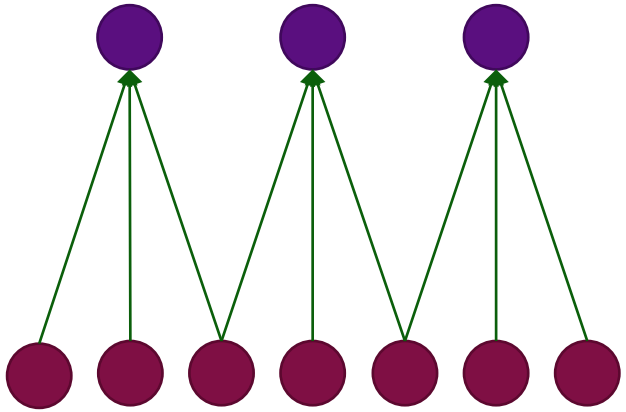
# Convolutional Layer: Local Connectivity

Hence “fully connected” / “fc” layers.



Outgoing layer

Incoming layer



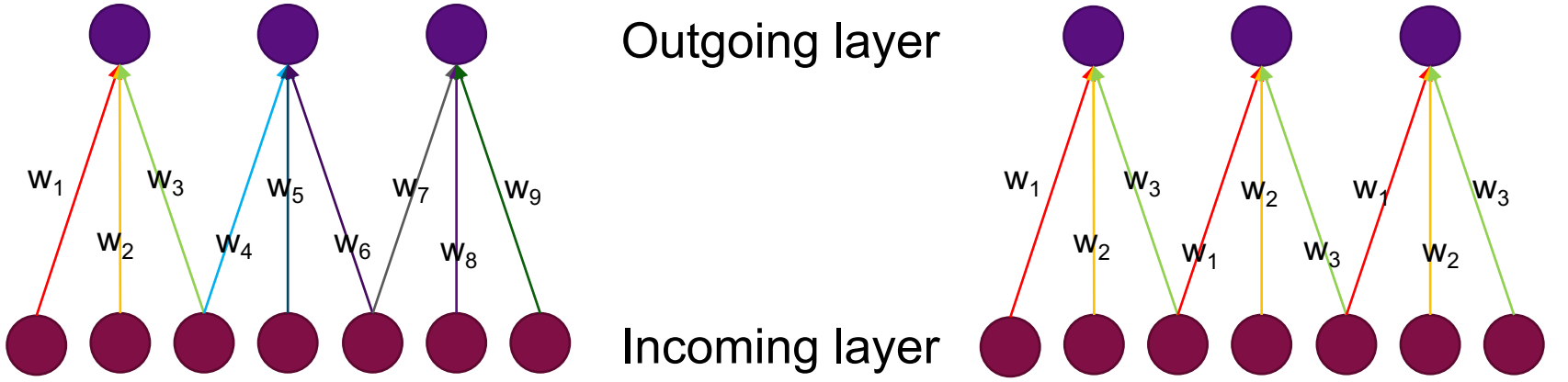
**Global** connectivity

**Local** connectivity

- # input units (neurons): 7
- # hidden units: 3
- Number of parameters (ignoring bias)
  - Global connectivity: 3 x 7 = 21
  - Local connectivity: 3 x 3 = 9

Slide credit: Jia-Bin Huang

# Convolutional Layer: Weight Sharing




**Without** weight sharing

**With** weight sharing

- # input units (neurons): 7
- # hidden units: 3
- Number of parameters (ignoring bias)
  - Without weight sharing: 3 x 3 = 9
  - With weight sharing : 3 x 1 = 3

Slide credit: Jia-Bin Huang

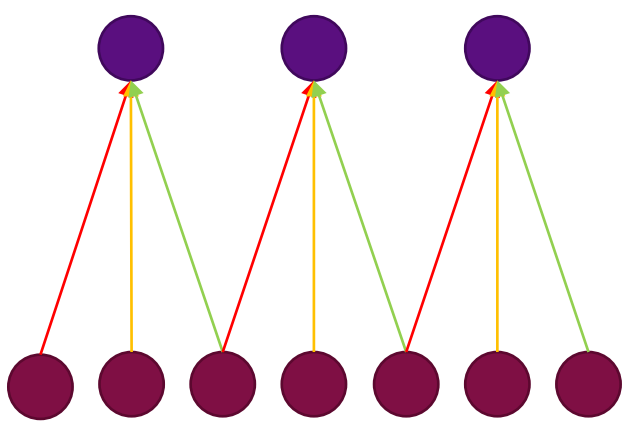
# From Convolutions to Convolutional Layers

# Extending convolutions

- We have just discussed the connection between normal “fully connected” layers and convolutions.

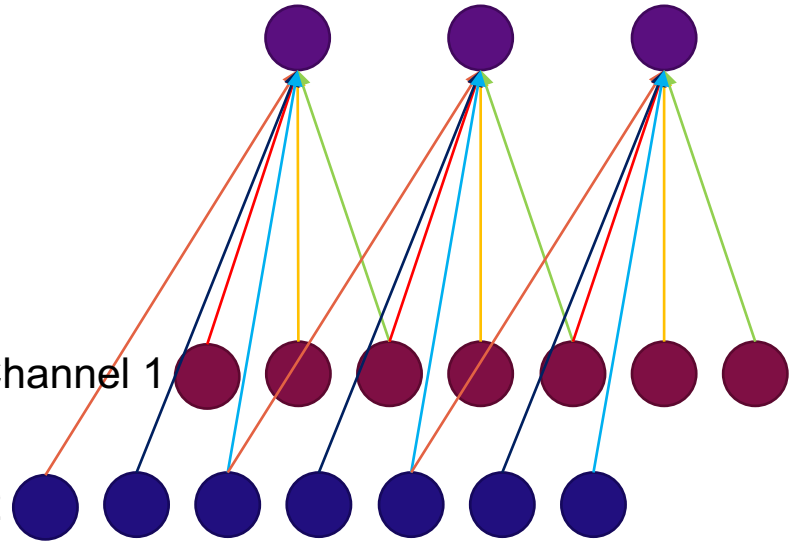
- But convolutional layers in neural networks extend this a bit more (next 2 slides):
  - They can handle multiple input channels (e.g. RGB channels in color image)
  - They can also handle multiple *output* channels
  - They can modify the inputs to maintain desired activation sizes

# Convolutional Layer with >1 input “channels” / “maps”



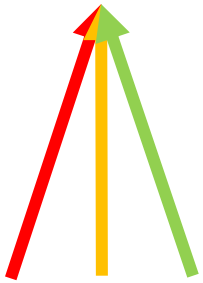
Outgoing layer

Incoming layer

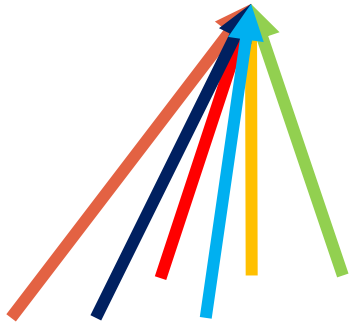



**Single** input channel

**Multiple** input channels



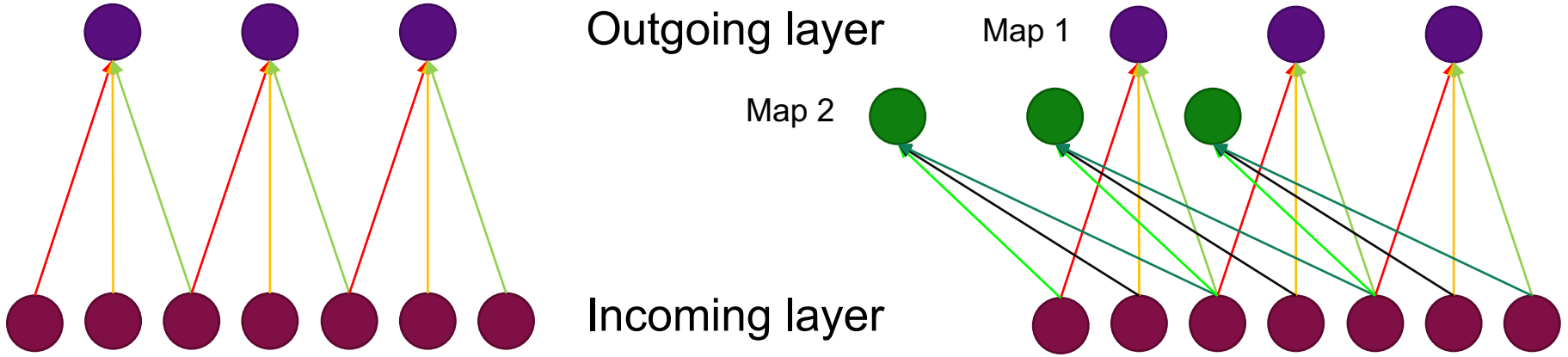
Filter weights



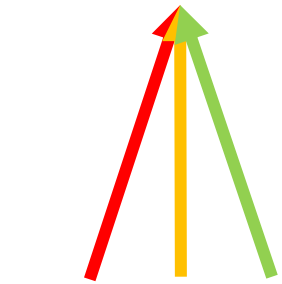
Filter weights

Slide credit: Jia-Bin Huang

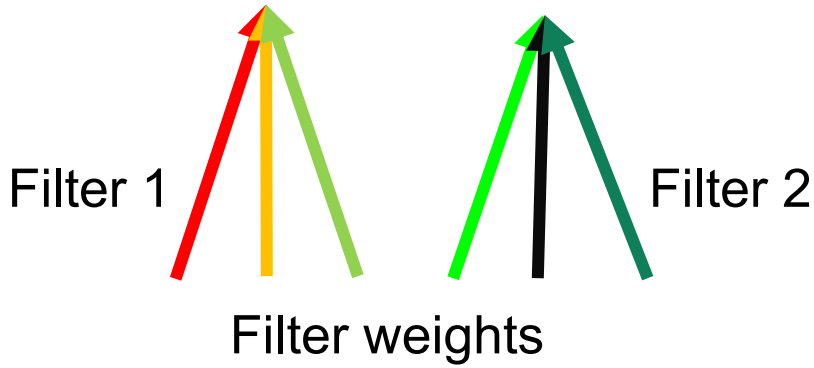
# Convolutional Layer with >1 output "channels" / "maps"




**Single** output map




**Multiple** output maps




Slide credit: Jia-Bin Huang