



CIS 4190/5190: Lec 21 Mon Nov 18,  
2024

# Introduction to ML for Recommendations (And Graph Neural Networks)

# Recommender Systems are Everywhere

What media to consume



NETFLIX



# Recommender Systems are Everywhere

What news you see



# Recommender Systems are Everywhere

What products to buy

amazon.com<sup>®</sup>



ebay<sup>™</sup>



audible 



Walmart 

Image: <https://www.curiouskeeda.com/business/10-weird-products-available-on-amazon/>



# Recommender Systems are Everywhere

Who to date

match

coffee  
meets bagel

POF  
PlentyOfFish

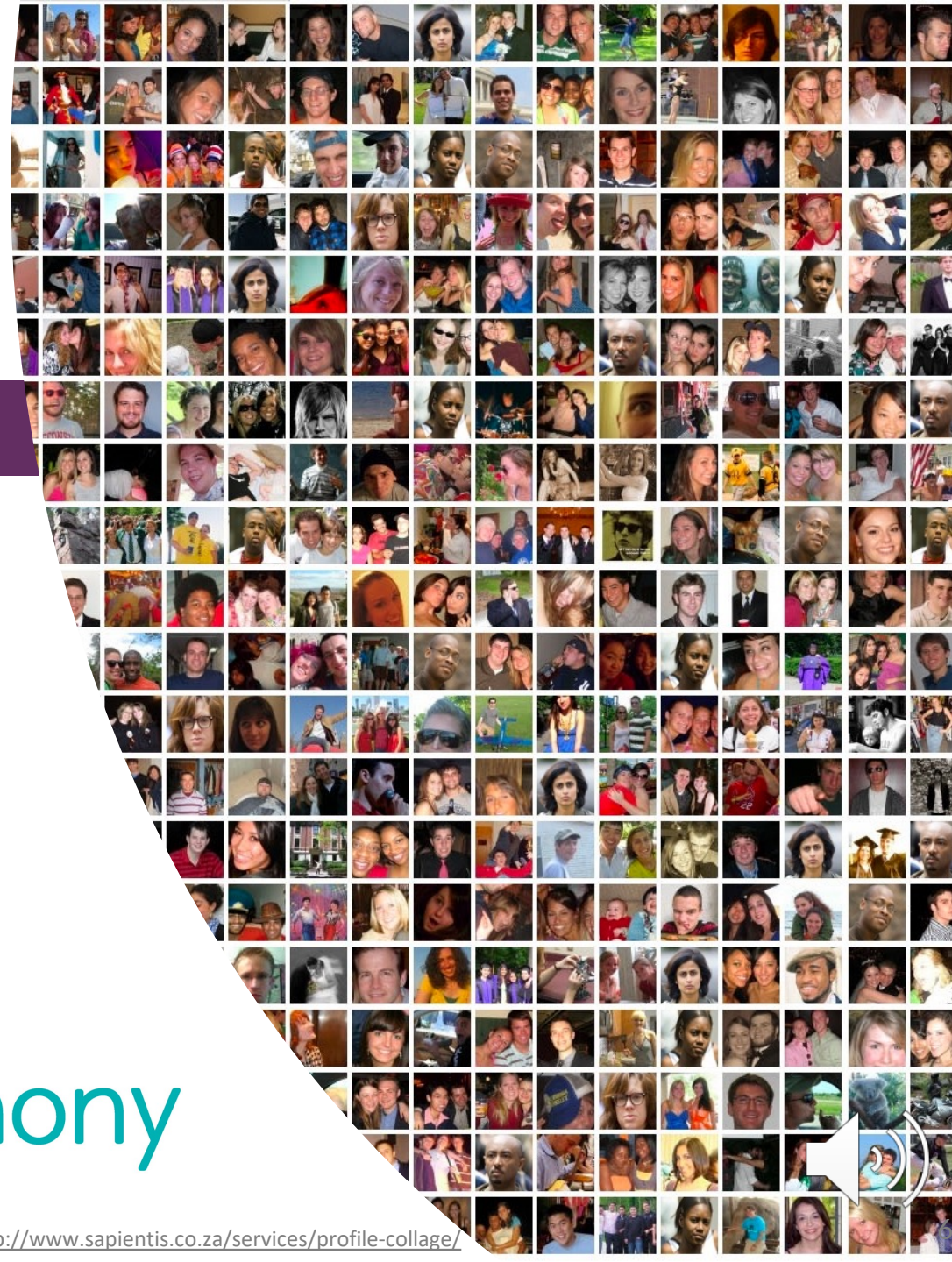
bumble

Hinge

tinder

okcupid

eHarmony



# Real Impact

Recommendations account for:

- 75% of movies watched on Netflix <sup>1</sup>
- 60% YouTube video clicks <sup>2</sup>
- 35% of Amazon sales <sup>1</sup>

Approximately 40% of committed relationships begin online <sup>3</sup>

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Sources:

1. McKinsey & Company (Oct 2013): <https://www.mckinsey.com/industries/retail/our-insights/how-retailers-can-keep-up-with-consumers> [Note: non-authoritative source; estimates only]
2. J. Davidson, et al. (2010). The YouTube video recommendation system. Proc. of the 4th ACM Conference on Recommender systems (RecSys). doi.org/10.1145/1864708.1864770
3. M. Rosenfeld, et al. (2019). Disintermediating your friends: How online dating in the United States displaces other ways of meeting. Proc. National Academy of Sciences 116(36).



# Stores Group Products Based on Consumer Buying Habits



Products that are commonly purchased together are displayed together



# Website Advertisements are Based on Our Online Activity



Users are tracked across websites to build consumer profiles

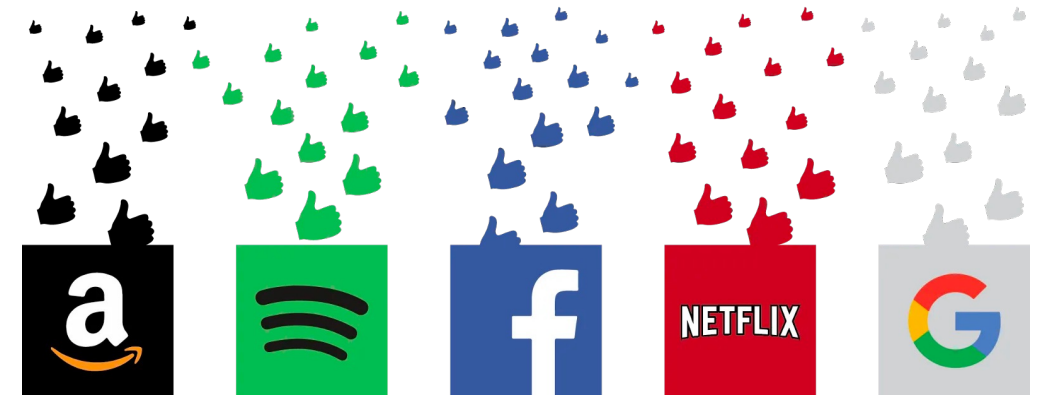




# Collaborative Filtering

# Popularity-Based Recommendations

- Just recommend whatever is currently popular
- Simple and often quite effective

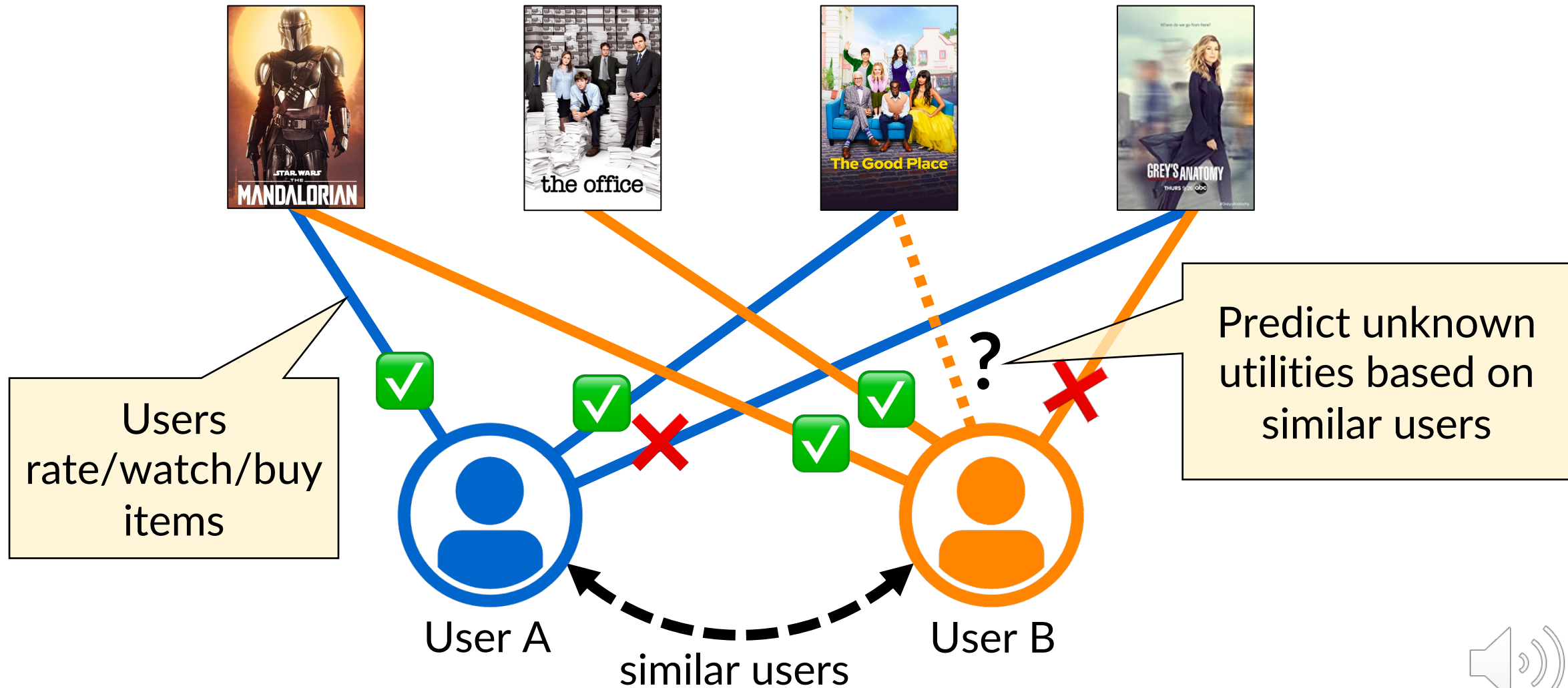


- This uses no information at all about the user!
  - Could improve by tailoring to the user: e.g. their geographical location, age, etc.



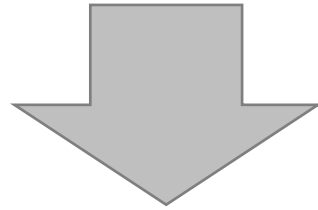
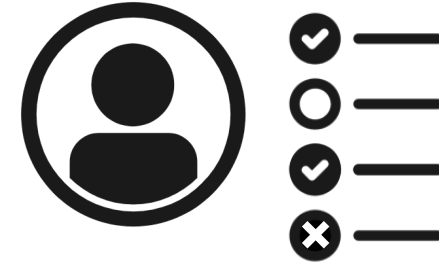
# The Recommendation Problem

Predict a user's rating for an item that they have not yet tried

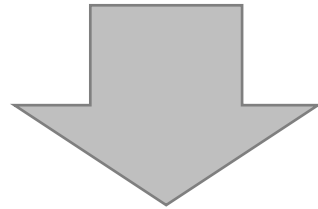


# Collaborative Filtering Steps

**Collect user-item utilities**



Identify similar users



Predict unknown item utilities  
based on other similar users



# Measuring User-Item Utilities

Utilities can be based on:

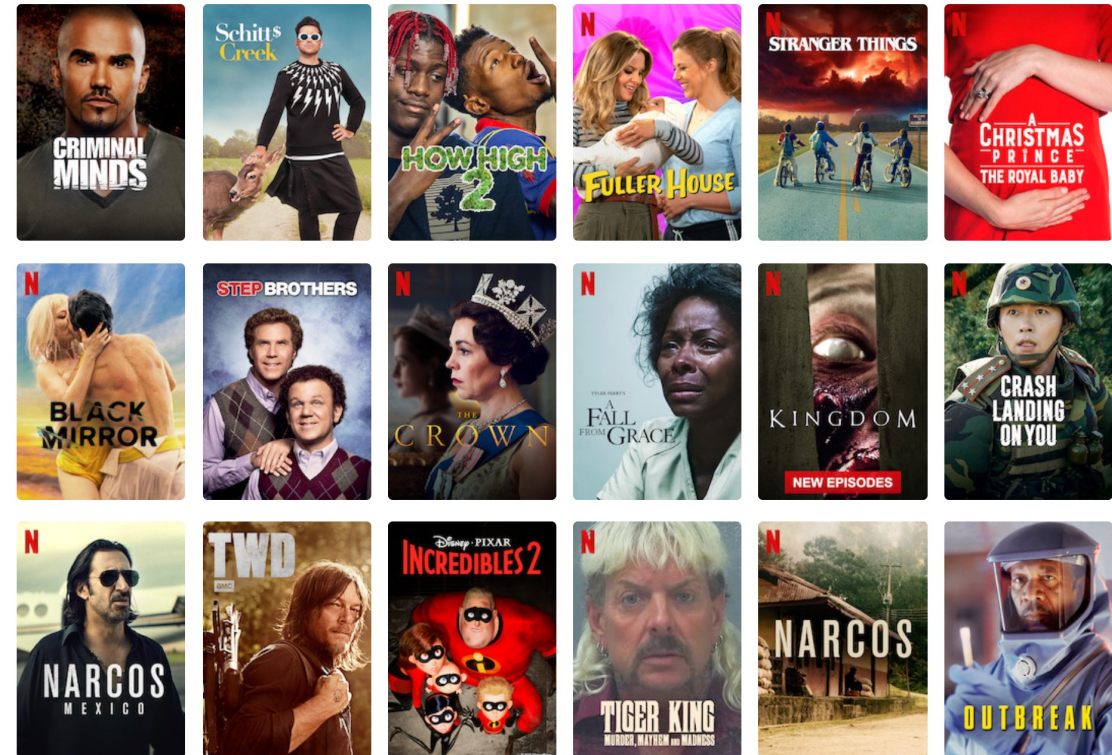
- Explicit rating
- Implicit rating
  - Inferred from user activity
  - e.g., User stops watching movie after 15 minutes
  - e.g., User repeatedly clicks on a particular dating profile

**NETFLIX**

✓ Elizabeth, choose 3 you like.

It will help us find TV shows & movies you'll love! Click the ones you like!

CONTINUE



For now, we are not considering user or item attributes/content  
e.g. genres and cast of movies/TV shows



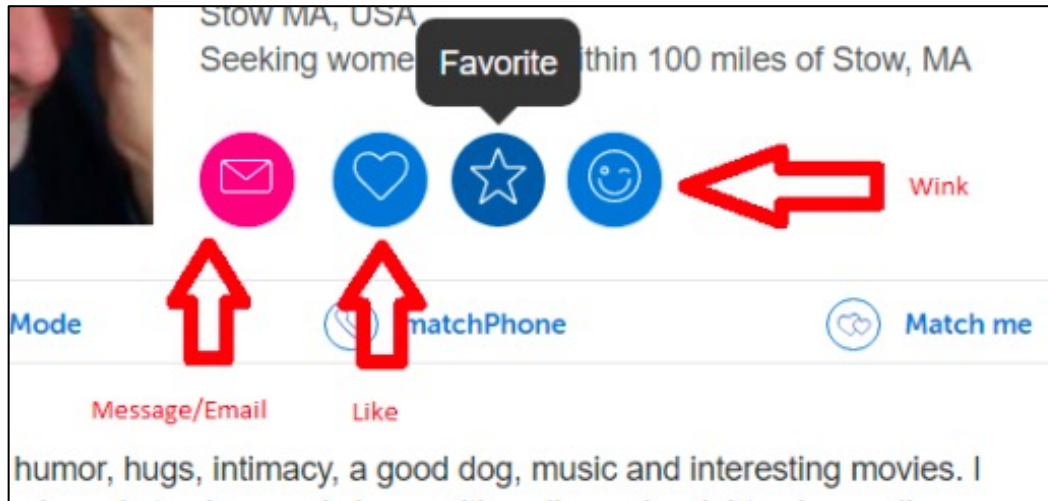
# Obtaining User Feedback



- Viewing profile, images, etc.

- Marking as a “favorite”

- Conversation
  - Swiping left/right
  - Messaging a person
  - “Liking” a profile
  - “Winking” at a person



# Challenges with Measuring Utility

Ratings can be misleading

- Sometimes users more likely to rate if experience is especially good or bad
- Users may have different scales
  - Can normalize user ratings, but their “scaling” might not even be linear.
- May need to consider credibility of individual raters (history of ratings)
- Bot farms may skew results through adversarial behavior

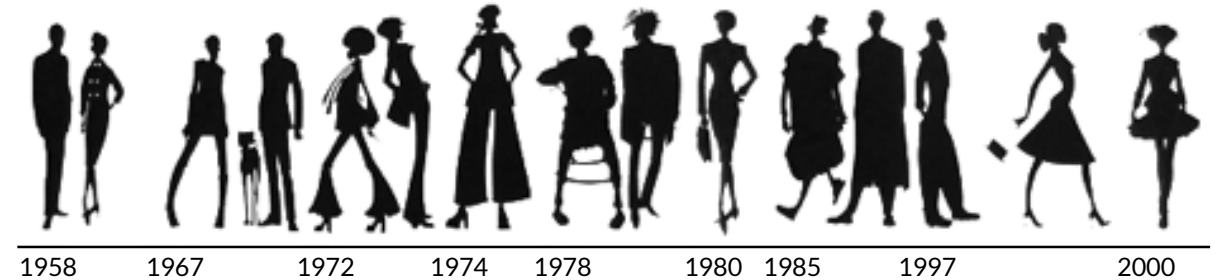
Reordered from class to improve flow



# Handling Time-Varying Preferences

Aspects of recommendations change over time:

- User preferences change
- Popularity of items change



Potential solution: weight more recent measurements over the past

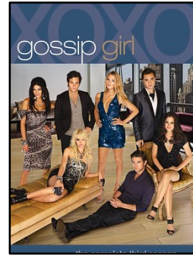
- Could use an exponentially weighted moving average
  - Decay old utilities. For example:
    - If user  $u$  has not newly rated item  $i$  at time  $t$ :  $x_{u,i}^{t+1} \leftarrow 0.95 x_{u,i}^t$
    - (Otherwise, set  $x_{u,i}$  to the new rating, of course.)








Reordered from class to improve flow





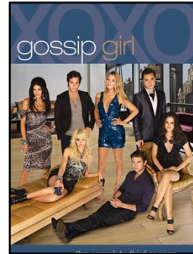
# User-Item Utility Matrix





|  | Gossip Girl | The Office | The Mandalorian | Criminal Minds | The Good Place | Grey's Anatomy | ... |
|--|-------------|------------|-----------------|----------------|----------------|----------------|-----|
|  Grace    | 4           | 5          | 4               | 1              | 5              | 3              | ... |
|  Eric     | 1           | 4          | 5               | 1              | 5              | 3              | ... |
|  Haren    | 5           | 5          | 5               | 1              | 3              | 4              | ... |
|  Sai      | 1           | 2          | 5               | 4              | 3              | 5              | ... |
|  Siyan   | 3           | 1          | 1               | 3              | 4              | 5              | ... |
|  Nikhil | 2           | 3          | 4               | 2              | 2              | 2              | ... |
|  Felix  | 1           | 1          | 1               | 5              | 2              | 2              | ... |



# User-Item Utility Matrix



|  | Gossip Girl | The Office | The Mandalorian | Criminal Minds | The Good Place | Grey's Anatomy | ... |
|--|-------------|------------|-----------------|----------------|----------------|----------------|-----|
|  Grace    | 4           | 5          | 4               | 1              | 5              | 3              | ... |
|  Eric     | 1           | 4          | 5               | 1              | 5              | 3              | ... |
|  Haren    | 5           | 5          | 5               | 1              | 5              | 3              | ... |
|  Sai      | 1           | 2          | 5               | 1              | 5              | 3              | ... |
|  Siyan   | 3           | 1          | 1               | 3              | 4              | 5              | ... |
|  Nikhil | 2           | 3          | 4               | 2              | 2              | 2              | ... |
|  Felix  | 1           | 1          | 1               | 5              | 2              | 2              | ... |

$x_u$

Let  $x_u$  be the item utilities for user  $u$

But of course, we don't have all the ratings. We will return to this soon!



# Collaborative Filtering

- **Given:**

- User-Item Utility Matrix  $X_{u,i} = \begin{cases} \text{rating}_{u,i} & \text{if user}_u \text{ rated product}_i \\ \text{N/A} & \text{otherwise} \end{cases}$
- Assume fixed set of  $n$  users and  $m$  products
- **Not given any information about the products!**

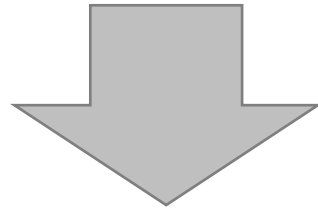
- **Problem:** Predict what  $X_{u,i}$  would be if it is observed
  - Not quite supervised or unsupervised learning!

# Collaborative Filtering Steps

Collect user-item utilities



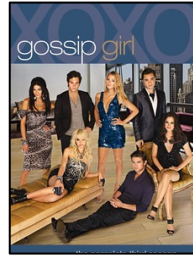
**Identify similar users**



Predict unknown item utilities  
based on other similar users



# Correlations Between Users

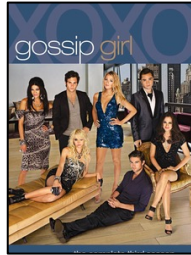


Similar users

|        | Gossip Girl | The Office | The Mandalorian | Criminal Minds | The Good Place | Grey's Anatomy | ... |
|--------|-------------|------------|-----------------|----------------|----------------|----------------|-----|
| Grace  | 4           | 5          | 4               | 1              | 5              | 3              | ... |
| Eric   | 1           | 4          | 5               | 1              | 5              | 3              | ... |
| Haren  | 5           | 5          | 5               | 1              | 3              | 4              | ... |
| Sai    | 1           | 2          | 5               | 4              | 3              | 5              | ... |
| Siyan  | 3           | 1          | 1               | 3              | 4              | 5              | ... |
| Nikhil | 2           | 3          | 4               | 2              | 2              | 2              | ... |
| Felix  | 1           | 1          | 1               | 5              | 2              | 2              | ... |



# Correlations Between Users



Dissimilar users



|        | Gossip Girl | The Office | The Mandalorian | Criminal Minds | The Good Place | Grey's Anatomy | ... |
|--------|-------------|------------|-----------------|----------------|----------------|----------------|-----|
| Grace  | 4           | 5          | 4               | 1              | 5              | 3              | ... |
| Eric   | 1           | 4          | 5               | 1              | 5              | 3              | ... |
| Haren  | 5           | 5          | 5               | 1              | 3              | 4              | ... |
| Sai    | 1           | 2          | 5               | 4              | 3              | 5              | ... |
| Siyan  | 3           | 1          | 1               | 3              | 4              | 5              | ... |
| Nikhil | 2           | 3          | 4               | 2              | 2              | 2              | ... |
| Felix  | 1           | 1          | 1               | 5              | 2              | 2              | ... |



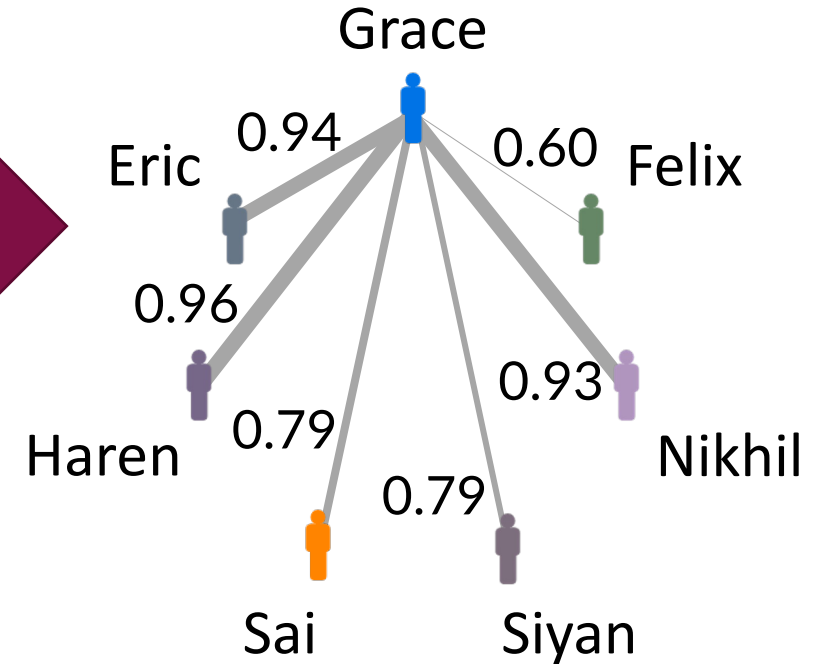
# Collaborative Filtering

## User-Item Utility Matrix

|        | Gossip Girl | The Office | The Mandalorian | Criminal Minds | The Good Place | Grey's Anatomy | ... |
|--------|-------------|------------|-----------------|----------------|----------------|----------------|-----|
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| Eric   | 1           | 4          | 5               | 1              | 5              | 3              | ... |
| Haren  | 5           | 5          | 5               | 1              | 3              | 4              | ... |
| Sai    | 1           | 2          | 5               | 4              | 3              | 5              | ... |
| Siyon  | 3           | 1          | 1               | 3              | 4              | 5              | ... |
| Nikhil | 2           | 3          | 4               | 2              | 2              | 2              | ... |
| Felix  | 1           | 1          | 1               | 5              | 2              | 2              | ... |



## User Similarities



We could then predict unknown item utilities for Grace based on other similar users

Open issues:

- Choice of distance metric
- Dealing with sparse data
- How to combine known user utilities to do the prediction



# User Distances



# Distance Metrics: Measuring Similarity Between Users

There are many ways to measure user similarity:

- Euclidean similarity
- Cosine similarity
- Pearson correlation

Pros:

- Straightforward to use as a similarity metric

- Euclidean similarity:

$$\text{similarity}(user_u, user_v) = \frac{1}{1 + \|\mathbf{x}_u - \mathbf{x}_v\|_2} \in (0, 1]$$

- Cosine similarity:

$$\text{similarity}(user_u, user_v) = \frac{\mathbf{x}_u \cdot \mathbf{x}_v}{\|\mathbf{x}_u\| \|\mathbf{x}_v\|} \in [0, 1]$$

Cons:

- Assumes utilities are calibrated across users
  - i.e., some users might give overall higher ratings than others



# Distance Metrics: Measuring Similarity Between Users

There are many ways to measure user similarity:

- Euclidean similarity
- Cosine similarity
- Pearson correlation

Measures the linear correlation between two users' utilities; value  $\in [-1,1]$

- Recall, this is formally defined as:

$$\rho = \frac{\text{covariance}(\mathbf{x}_u, \mathbf{x}_v)}{\text{stdev}(\mathbf{x}_u) \times \text{stdev}(\mathbf{x}_v)} = \frac{E[(x_{ui} - \bar{x}_u)(x_{vi} - \bar{x}_v)]}{\text{stdev}(\mathbf{x}_u) \times \text{stdev}(\mathbf{x}_v)}$$



# Distance Metrics: Measuring Similarity Between Users

There are many ways to measure user similarity:

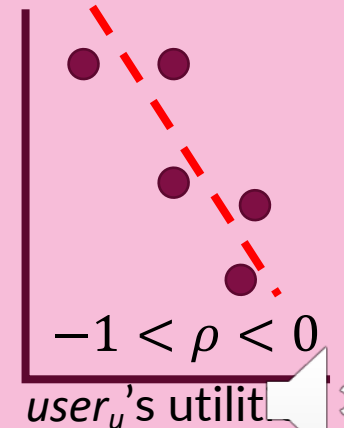
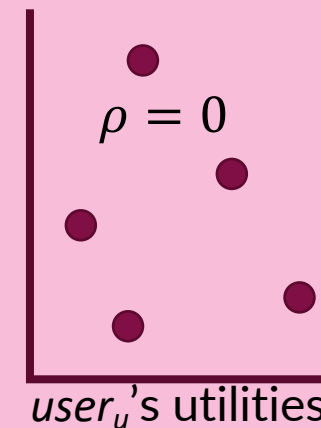
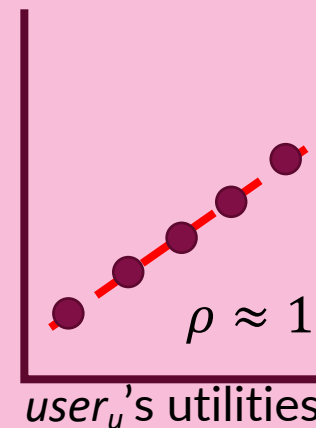
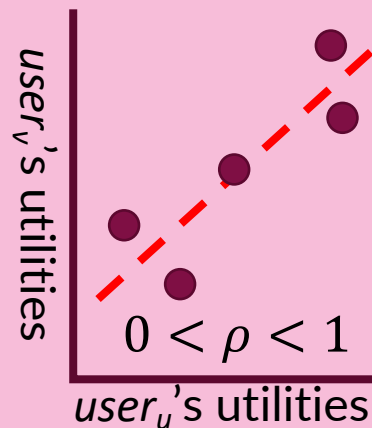
- Euclidean similarity
- Cosine similarity
- Pearson correlation

Measures the linear correlation between two users' utilities; value  $\in [-1,1]$

- Measuring correlations between users' utilities allows it to handle different scale calibrations
- Related to the slope (+/-) and quality of linear regression fit to the paired points

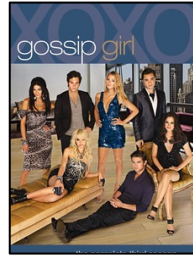
Pearson correlation coefficient  $\rho$  is:

- 1 if there is a perfect linear relationship with pos. slope
- 0 if no linear relationship exists
- -1 if perfect linear relationship with neg. slope



The Utility Matrix is Sparse

# The Utility Matrix is Sparse



Blanks indicate the user has not rated the item

In practice, the matrix would be much sparser

|        | Gossip Girl | The Office | The Mandalorian | Criminal Minds | The Good Place | Grey's Anatomy |     |
|--------|-------------|------------|-----------------|----------------|----------------|----------------|-----|
| Grace  |             | 5          |                 | 1              | 5              |                | ... |
| Eric   |             | 4          | 5               |                | 5              | 3              | ... |
| Haren  | 5           |            | 5               |                | 3              | 4              | ... |
| Sai    |             | 2          |                 |                |                |                | ... |
| Siyan  | 3           | 1          |                 | 3              |                | 5              | ... |
| Nikhil |             |            |                 | 2              | 2              |                | ... |
| Felix  | 1           |            | 1               |                | 2              |                | ... |

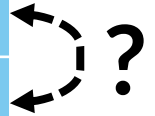
The goal of collaborative filtering is to predict values for blanks in the utility matrix

# Measuring User Similarity with Sparse Utility Data



Can only measure similarity between users using their overlapping items

|  |        | The Office | The Mandalorian | Criminal Minds | The Good Place | Grey's Anatomy | ... |
|--|--------|------------|-----------------|----------------|----------------|----------------|-----|
|  | Grace  | 5          |                 | 1              | 5              |                | ... |
|  | Eric   | 4          | 5               |                | 5              | 3              | ... |
|  | Haren  | 5          | 5               |                | 3              | 4              | ... |
|  | Sai    |            | 2               |                |                |                | ... |
|  | Siyan  | 3          | 1               | 3              |                | 5              | ... |
|  | Nikhil |            |                 | 2              | 2              |                | ... |
|  | Felix  | 1          | 1               |                | 2              |                | ... |



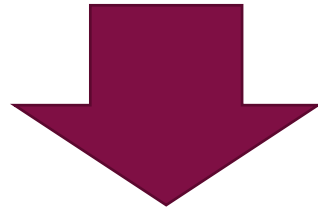
# Nearest-Neighbor Collaborative Filtering

# Collaborative Filtering Steps

Collect user-item utilities



Identify similar users



Predict unknown item utilities  
based on other similar users

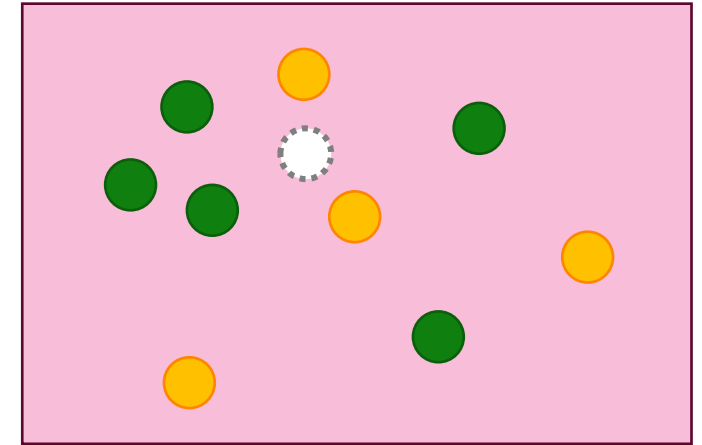




# Nearest-Neighbor Collaborative Filtering

- A type of user-to-user collaborative filtering
- Very simple, yet effective

**Idea:** predict utility of item  $i$  based on the most-similar users who recorded a utility for that item



- Let  $\mathcal{N}$  be the neighborhood set: the most similar users to user  $u$  who have rated  $i$
- Let  $w_{uv}$  be a weight  $\in [0,1]$  based on the similarity of users  $u$  and  $v$

- Predict user  $u$ 's utility for item  $i$  as 
$$\hat{x}_{ui} = \underbrace{\bar{x}_u}_{\text{Offset to this user's mean}} + \underbrace{\sigma_u}_{\text{Scale to this user's range}} \left( \sum_{v \in \mathcal{N}} \underbrace{\frac{(x_{vi} - \bar{x}_v)}{\sigma_v}}_{\text{mean-center and normalize other's utilities}} \times \underbrace{\frac{w_{uv}}{\sum_{v' \in \mathcal{N}} w_{uv'}}}_{\text{normalize weights to sum to 1}} \right)$$



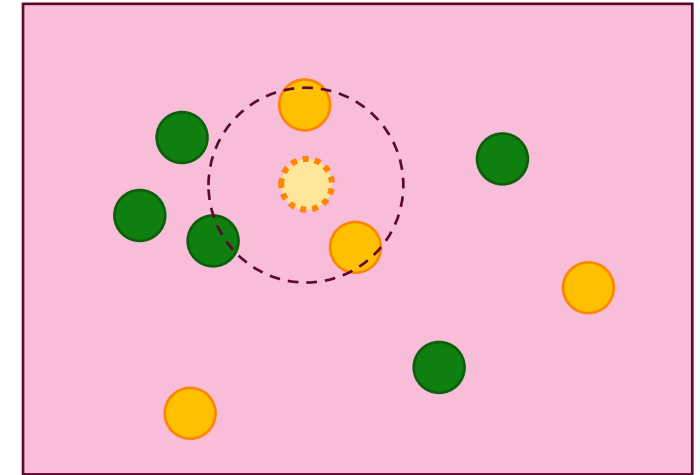
# Nearest-Neighbor Collaborative Filtering

## Ways to select the neighborhood set $\mathcal{N}$ :

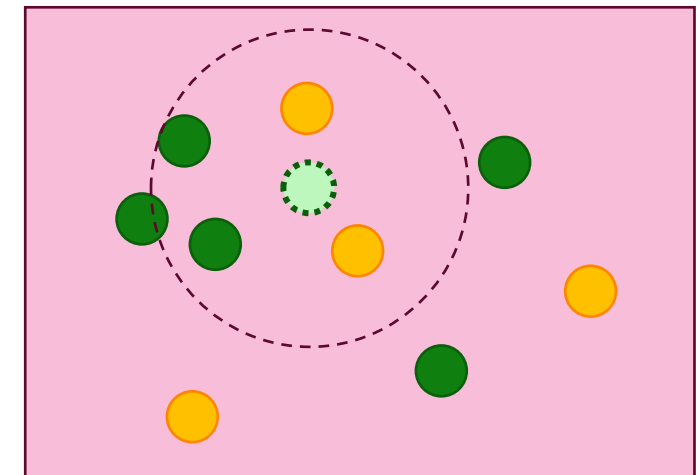
- Based on a threshold of similarity
- Choose top- $k$  neighbors by similarity
- Cluster users (e.g. using  $k$ -means clustering), and choose the entire cluster

## Combining utilities:

- Mean-centering
- Standardize by user's stdev



$k = 3 \rightarrow$  orange



$k = 5 \rightarrow$  green



# Matrix Factorization-Based Collaborative Filtering

# Matrix Factorization-Based Collaborative Filtering



## Idea:

- Represent each item as a vector  $\mathbf{q}_i \in \mathbb{R}^d$
- Represent each user as a vector  $\mathbf{p}_u \in \mathbb{R}^d$
- Approximate user  $u$ 's utility for item  $i$  as

$$\hat{x}_{ui} = \mathbf{q}_i^T \mathbf{p}_u$$

These vectors **factorize** the utility matrix



# Matrix Factorization-Based Collaborative Filtering

## Determining the factors:

- Just factorize the user-item utility matrix  $U$  directly via singular value decomposition (SVD)?
  - This will only work if we knew the full matrix, which we don't
- A better way is to fit the model with regularization

$$\min_{q^*, p^*} \sum_{x_{ui} \in U} (x_{ui} - q_i^T p_u)^2 + \sum_i \|q_i\|_2^2 + \sum_u \|p_u\|_2^2$$

- Solve via stochastic gradient descent or alternating least squares
- For details, see:
  - Koren, et al. (2009) Matrix factorization techniques for recommender systems. *Computer* 42 (8), ACM. <https://datajobs.com/data-science-repo/Recommender-Systems-%5BNetflix%5D.pdf>



# Injecting Prior Knowledge About Items (and Users)

# Pros & Cons of The Methods So Far

## Advantages:

- No domain knowledge needed
  - Item details are irrelevant, only user behavior matters
- Heterogeneous preferences
  - Captures that users may have diverse preferences

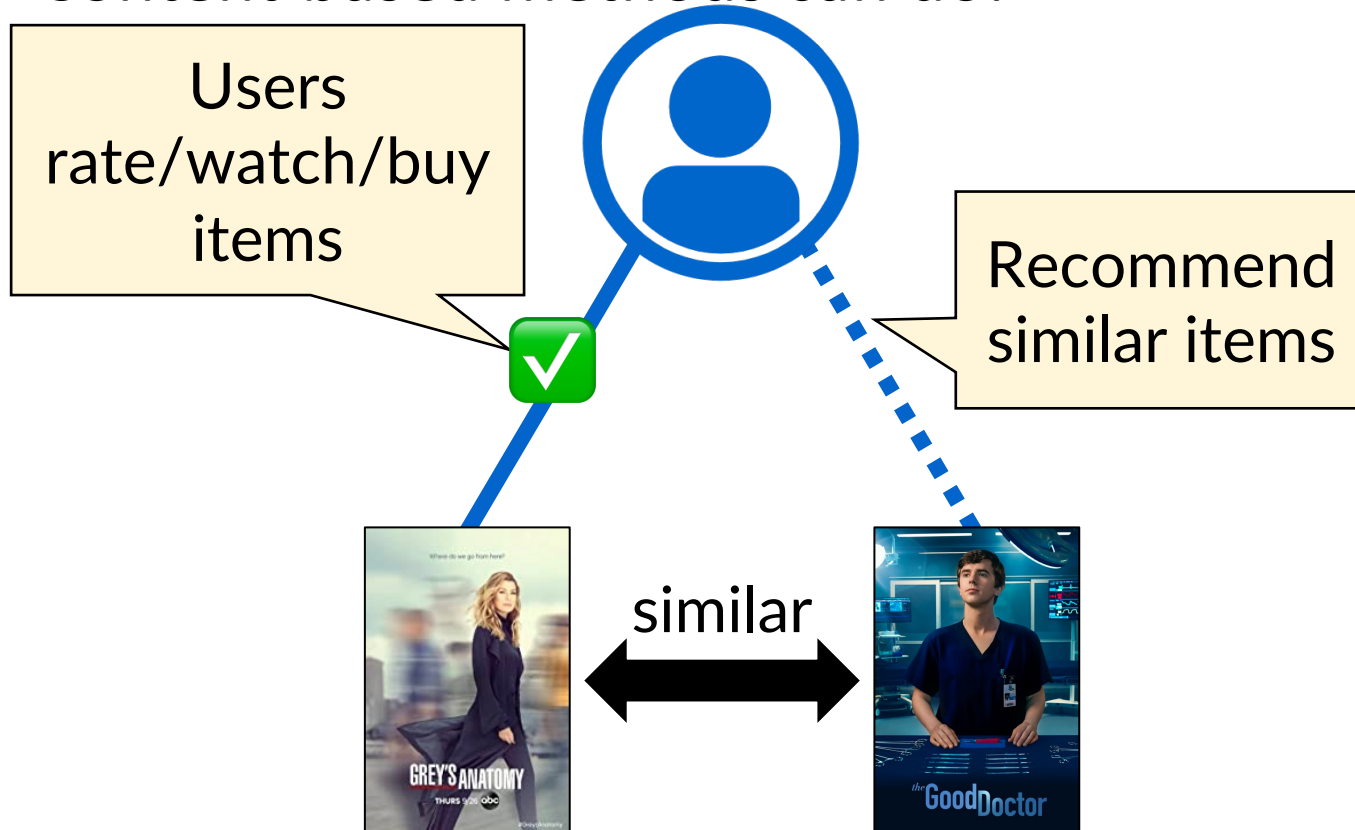
## Disadvantages:

- Suffers when data is sparse
  - Cannot generalize across items
    - Does not consider item content, and so cannot generalize to similar items
    - e.g. New items have no user feedback, and so the system cannot make recommendations for them
  - Cannot generalize across users



# “Content-Based Methods”

- Vanilla collaborative filtering doesn't consider user or item attributes/content
- Content-based methods can do:

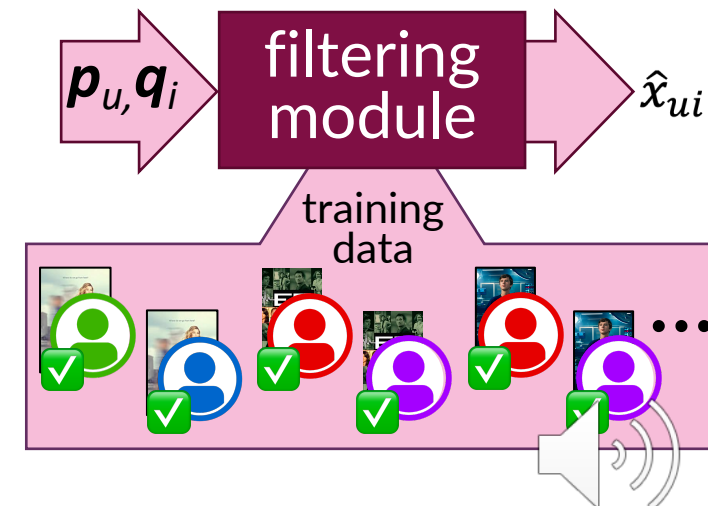
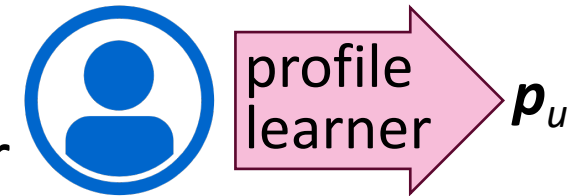
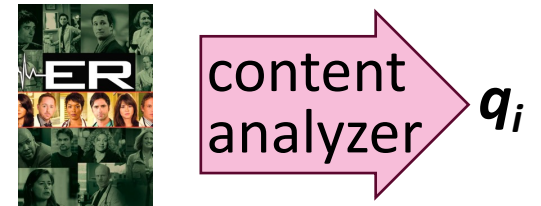




# Incorporating User / Item Knowledge

## Steps:

1. Content analysis: Characterize item as feature vector
  - e.g., TF-IDF features of description, image features, etc.
2. Profile learning: Characterize user as feature vector
  - e.g., demographic details, or true/predicted ratings for representative items
3. Filtering module: Learns a classification/regression model for predicting user's utility for an item
  - Train model on items each user has rated

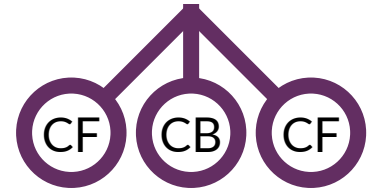


**Q: What happens with a new item or new user?**

# Hybrid Approaches

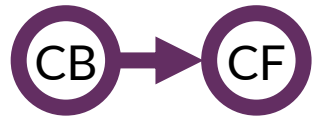
# Hybrid Recommenders

**Idea:** Combine multiple recommenders to improve performance



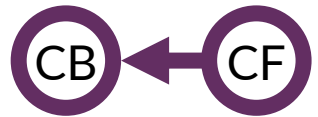
Combining separate recommenders

- Can use any ensemble technique: linear weighting, stacking, etc.
- Recall – the Netflix prize winner was a blend of over 800+ recommenders



Adding content-based aspects to collaborative models

- e.g., content-based user profiles to help build collaborator neighborhoods



Adding collaborative-based aspects to content-based models



Models combining content and collaboration



# Hybrid Recommenders

Most systems that we use nowadays are hybrid recommenders:

**NETFLIX**

- Shows other similar users are watching
- Shows similar to others the user has rated/viewed

**amazon.com**

- Items other similar users have purchased
- Items that are similar to user's past purchases

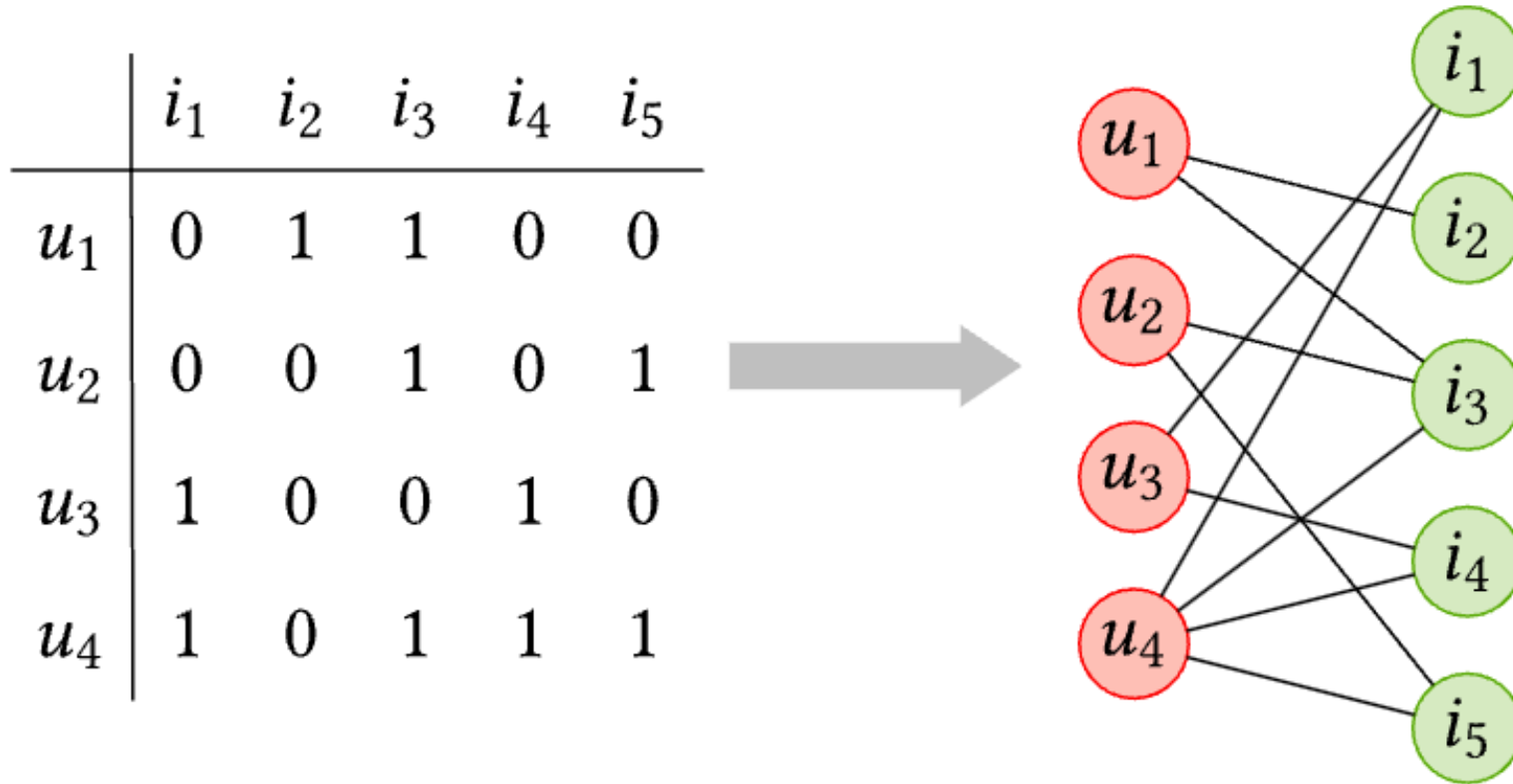
**okcupid**

- Profiles that other similar users have liked/viewed
- Profiles selected based on user's personal preferences



# Graph Neural Networks for Recommender Systems

# User-Item Bipartite Graph in Recommender Systems



# GNNs in Collaborative Filtering

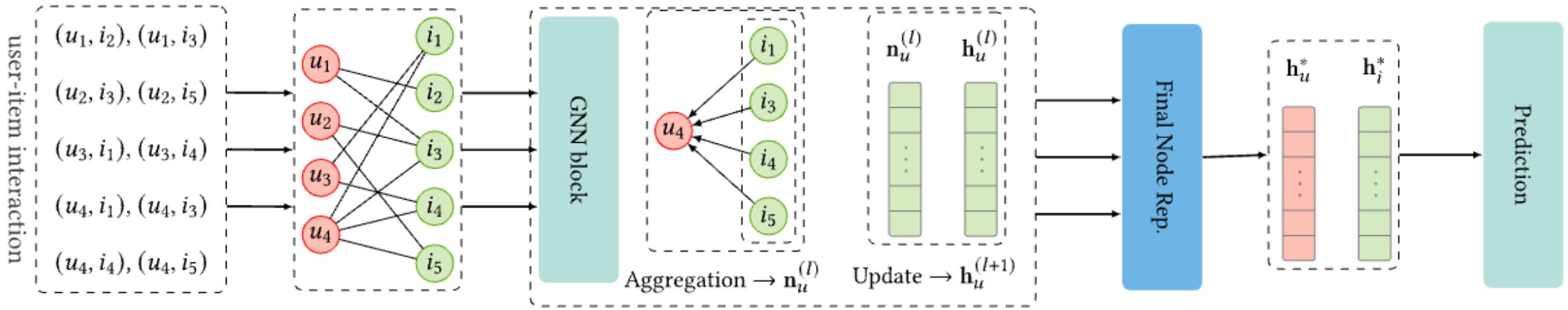


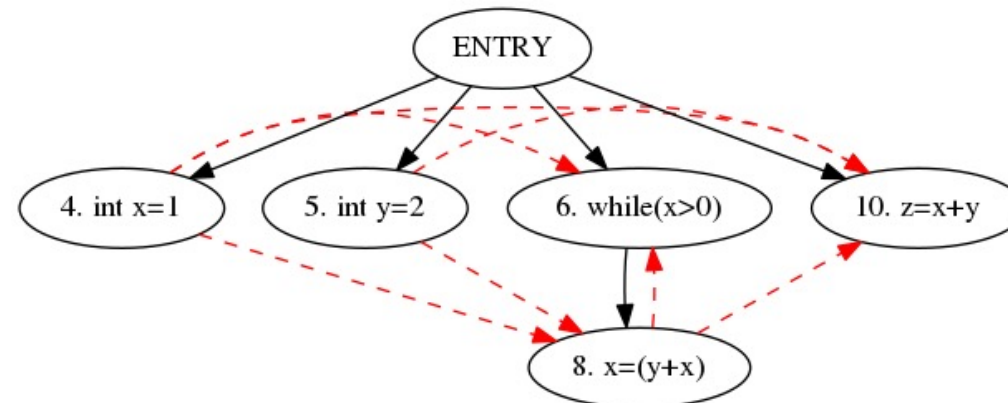
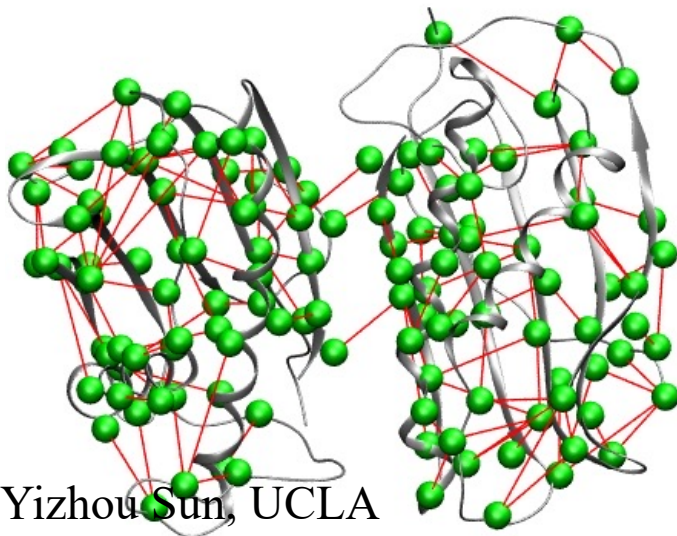
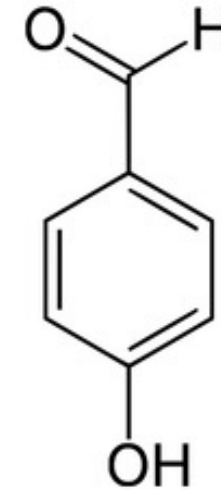
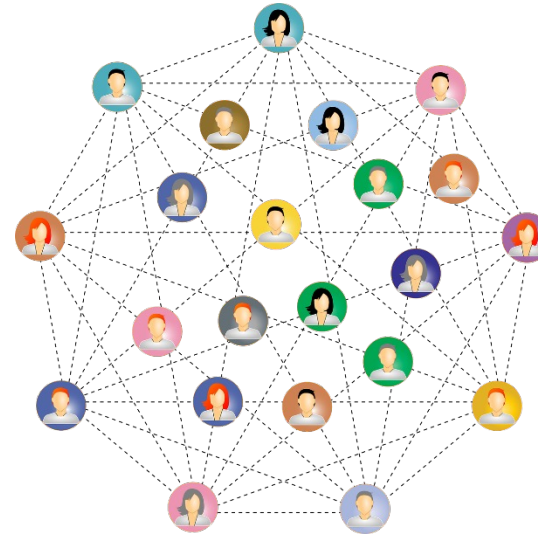
Fig. 2. The overall framework of GNN in user-item collaborative filtering.

A Detour into GNNs (not necessarily for Recommendation Systems)



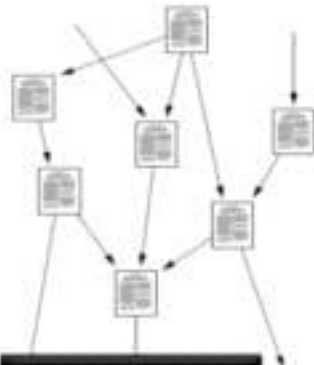
# Graph Analysis

- Graphs are ubiquitous
  - Social networks
  - Proteins
  - Chemical compounds
  - Program dependence graph
  - ...

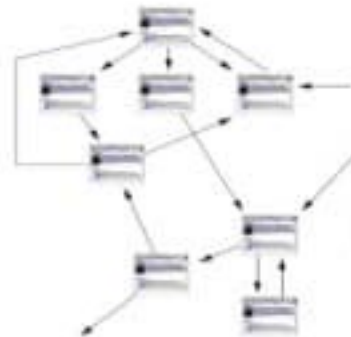




Molecules



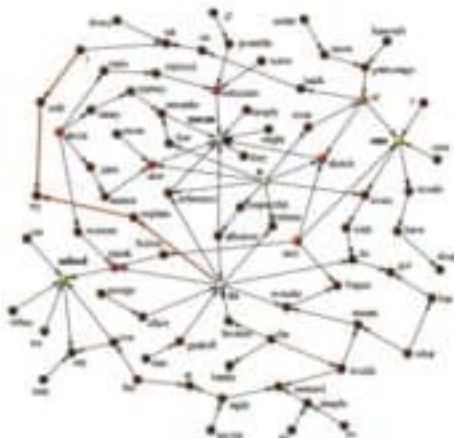
Knowledge



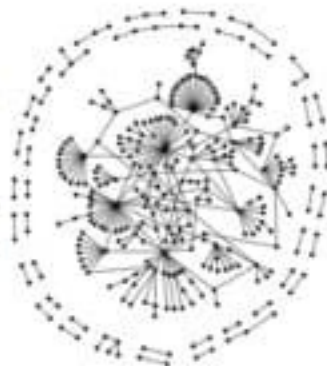
Information



Brain/neurons



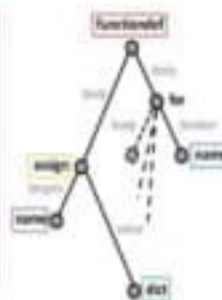
Genes



Communication

```
function foo() {  
  // ...  
  return true;  
}
```

Software



Social

# Notations

- An attributed graph  $G = (V, E)$ 
  - $V$ : vertex set
  - $E$ : edge set
  - $A$ : adjacency matrix
  - $X \in R^{d_0 \times |V|}$ : feature matrix for all the nodes
  - $N(v)$ : neighbors of node  $v$
  - $h_v^l$ : Representation vector of node  $v$  at Layer  $l$ 
    - *Note*  $h_v^0 = x_v$
  - $H^l \in R^{d_l \times |V|}$ : representation matrix

# One layer of a GNN

- For a node  $v$  at layer  $t$

$$h_v^{(l)} = f \left( \underbrace{h_v^{(l-1)}}_{\text{representation vector from previous layer for node } v}, \underbrace{\{h_u^{(l-1)} \mid u \in \mathcal{N}(v)\}}_{\text{representation vectors from previous layer for node } v\text{'s neighbors}} \right)$$

representation vector  
from previous layer for  
node  $v$

representation vectors  
from previous layer for  
node  $v$ 's neighbors

- Two key things that must happen in  $f(\cdot)$ :
  - **Aggregation: combine all neighbor features into one new feature for the neighborhood  $n_v^{(t)}$**
  - **Update: Combine current central node feature at previous layer + aggregated feature of the neighborhood above**

# Common Choices for Aggregation

$$\text{Aggregation: } n_v^{(l-1)} = \text{Aggregator}_{l-1} \left( \{h_u^{(l-1)}, \forall u \in \mathcal{N}_v\} \right)$$

- Mean:  $n_v^{(l-1)} = \sum_{u \in \mathcal{N}(v)} \frac{h_u^{(l-1)}}{|\mathcal{N}(v)|}$

- Modified Mean (“graph convolutions”):  $n_v^{(l-1)} = \sum_{u \in \mathcal{N}(v)} \frac{h_u^{(l-1)}}{\sqrt{|\mathcal{N}(v)||\mathcal{N}(u)|}}$

- Attention mechanisms:  $n_v^{(l-1)} = \sum_{u \in \mathcal{N}(v)} \alpha_{vu} h_u^{(l-1)}$

$\alpha_{vu}$  are weights computed by attention

# Common Choices for Update

$$\text{Update } h_v^{(l)} = \text{Updater}_l \left( h_v^{(l-1)}, n_v^{(l-1)} \right)$$

Often a simple linear layer + non-linear activation function

$$\text{e.g. } h_v^{(l)} = \delta \left( W^{(l-1)} \cdot \text{concat} \left( h_v^{(l-1)}, n_v^{(l-1)} \right) + b^{(l-1)} \right)$$

# Compare with CNN

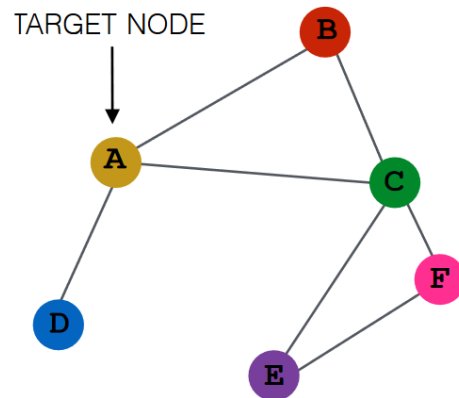
- Recall CNN
  - Regular graph
- GNN
  - Extend to irregular graph structure

|                 |                 |                 |   |   |
|-----------------|-----------------|-----------------|---|---|
| 1 <sub>x1</sub> | 1 <sub>x0</sub> | 1 <sub>x1</sub> | 0 | 0 |
| 0 <sub>x0</sub> | 1 <sub>x1</sub> | 1 <sub>x0</sub> | 1 | 0 |
| 0 <sub>x1</sub> | 0 <sub>x0</sub> | 1 <sub>x1</sub> | 1 | 1 |
| 0               | 0               | 1               | 1 | 0 |
| 0               | 1               | 1               | 0 | 0 |

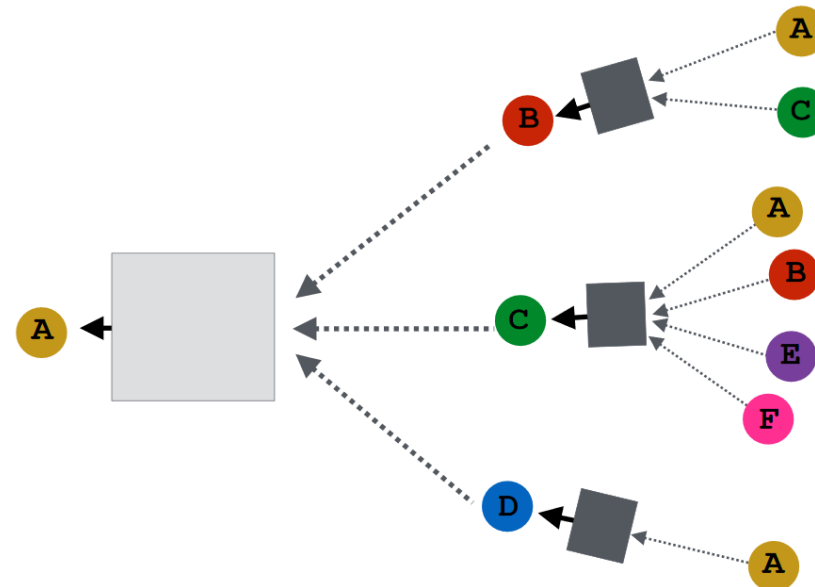
Image

|   |  |  |
|---|--|--|
| 4 |  |  |
|   |  |  |
|   |  |  |

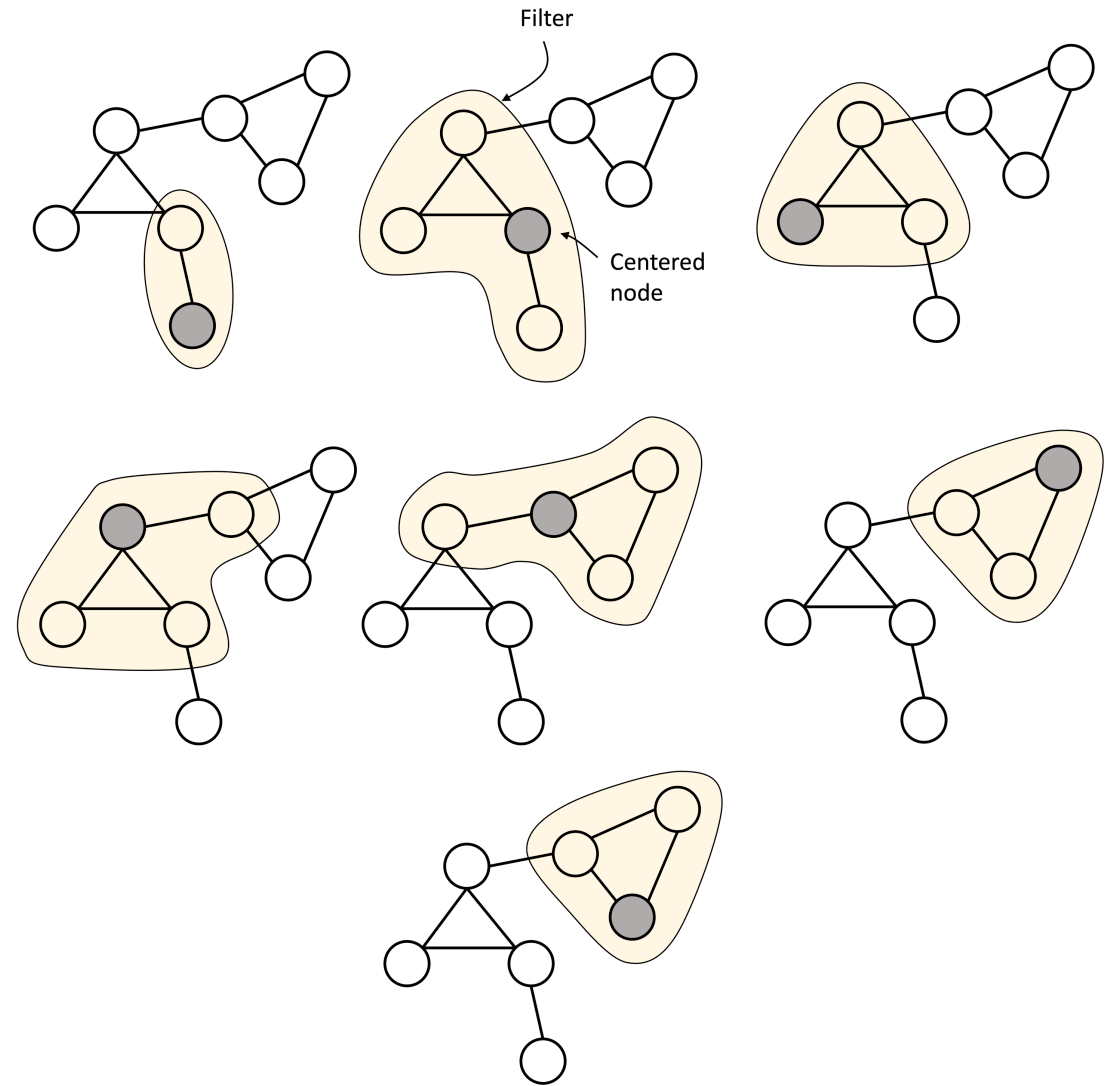
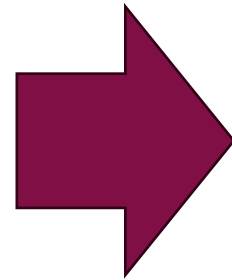
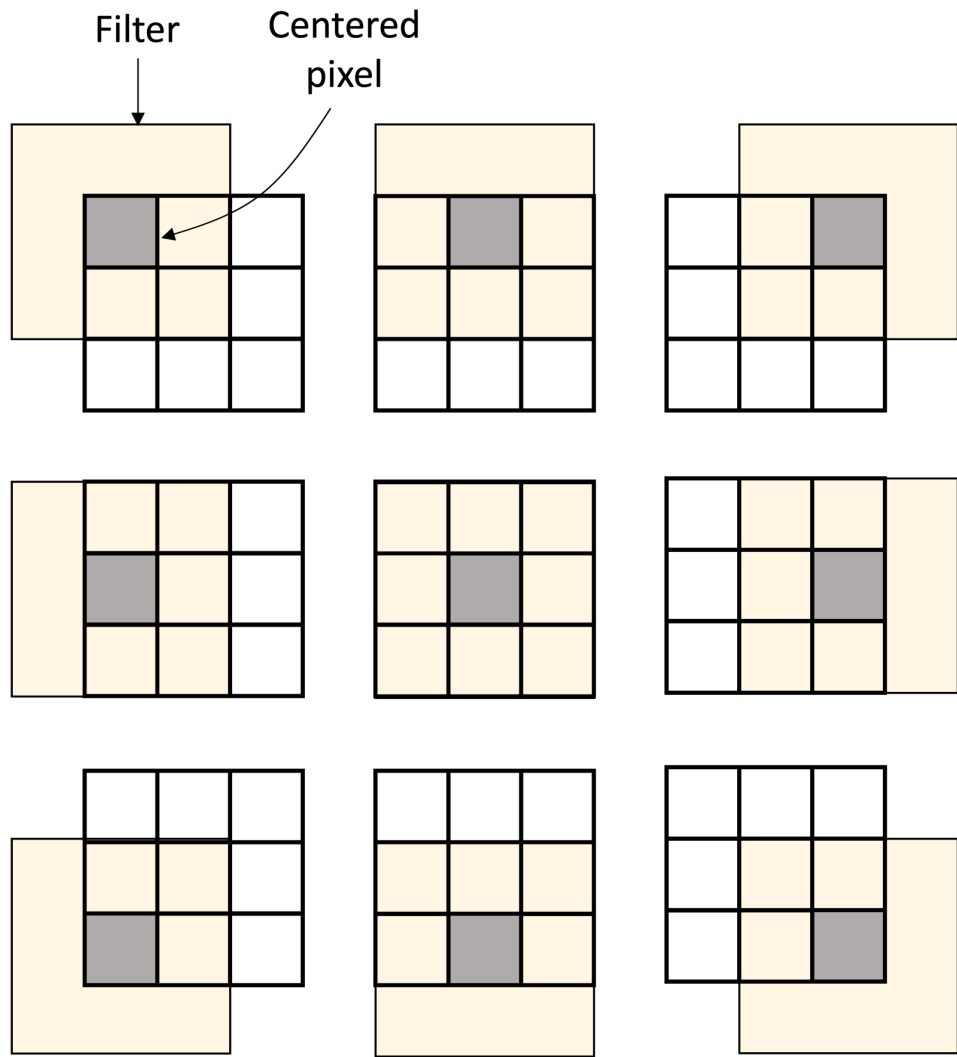
Convolved Feature



INPUT GRAPH

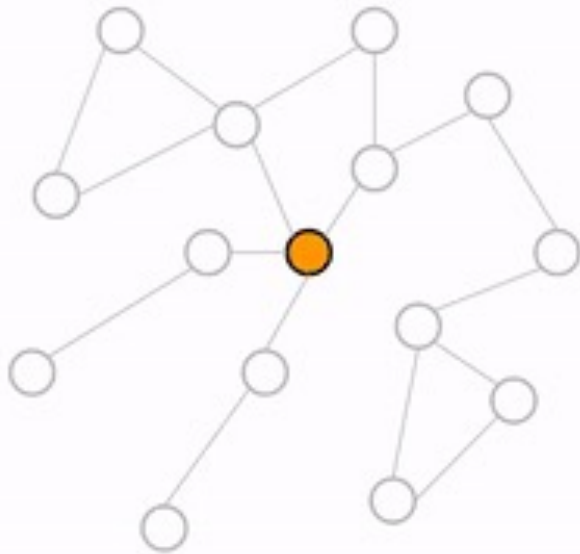


# “Graph Convolutions”





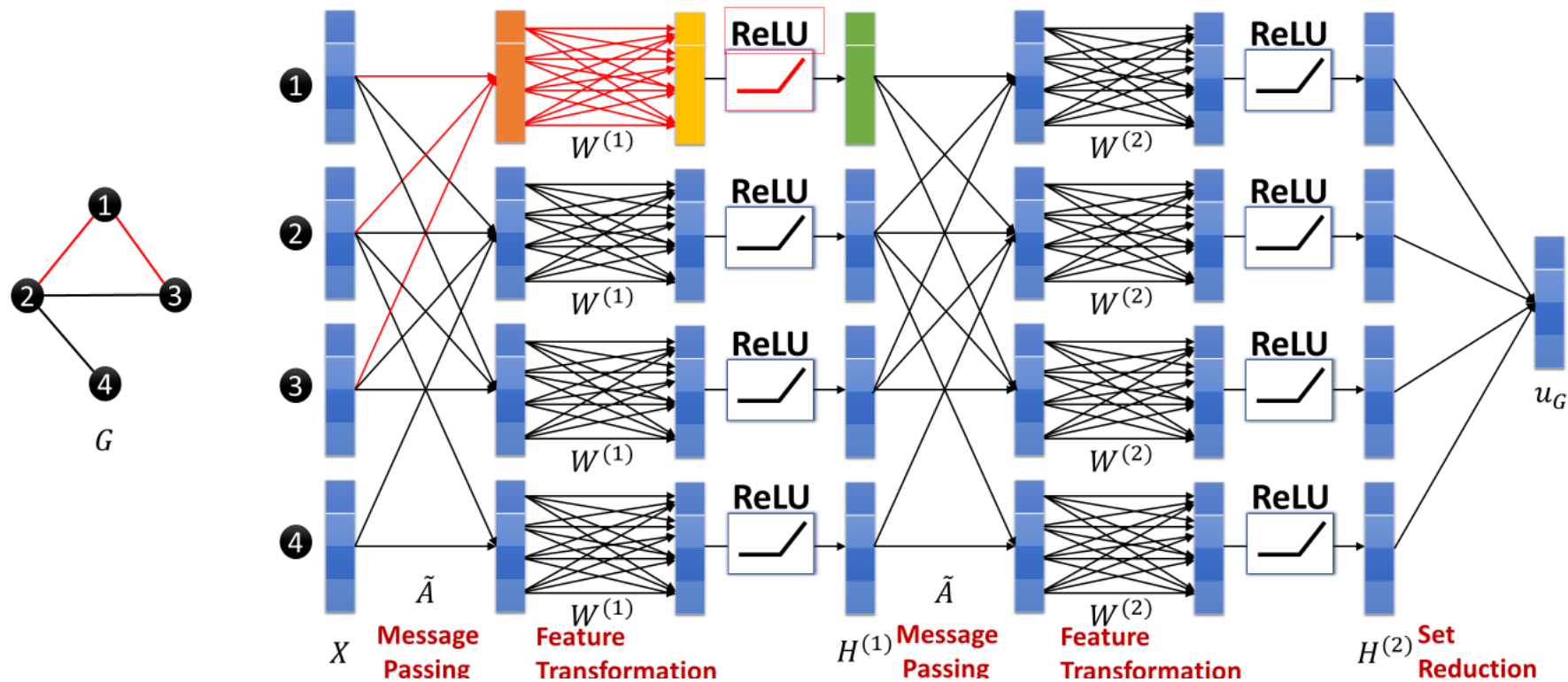
## Input Layer



» **Node 1** → **One-hot vector [0,0,1,0,0]**

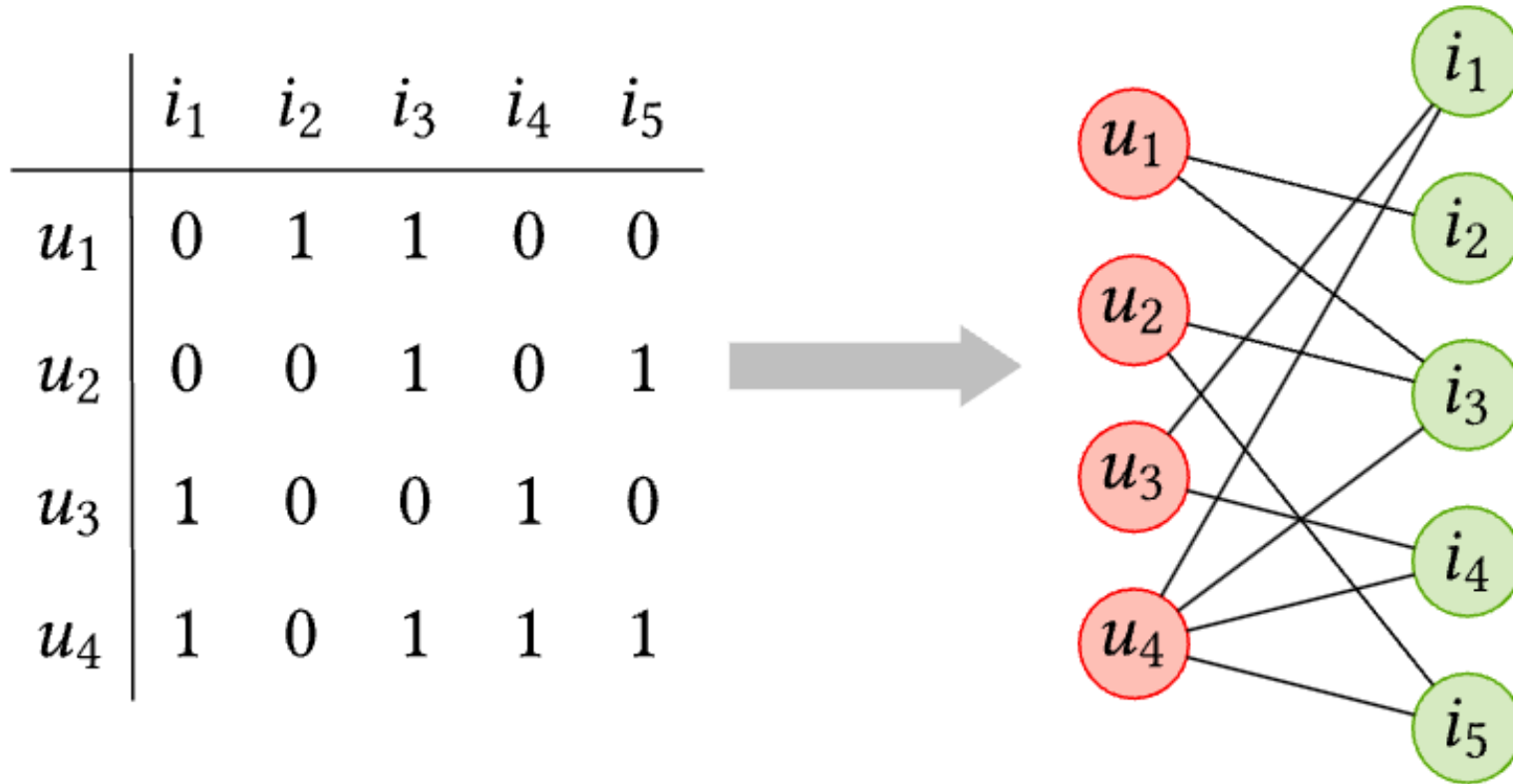
# A toy example of 2-layer GCN on a 4-node graph

- Computation graph



GNNs in Recommender Systems?

# User-Item Bipartite Graph in Recommender Systems



# GNNs in Collaborative Filtering

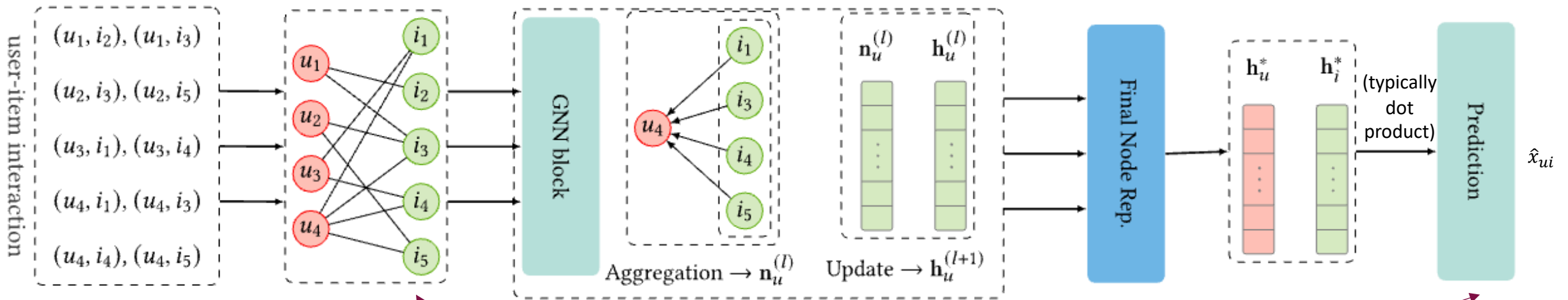


Fig. 2. The overall framework of GNN in user-item collaborative filtering.

