#### Announcements

• Homework 3 due tonight at 8pm

## Lecture 20: Reinforcement Learning

CIS 4190/5190 Spring 2025

#### **Optimal Action-Value Function**

 Optimal Action-Value Function (or Q function): Expected reward if we start in s, take action a, and then act optimally thereafter:

$$Q^*(s,a) = \mathbb{E}\left(\sum_{t=0}^{\infty} \gamma^t \cdot r_t \mid s_0 = s, a_0 = a\right)$$

• Bellman equation:

$$Q^*(s,a) = \sum_{s' \in S} P(s' \mid s,a) \cdot \left( R(s,a,s') + \gamma \cdot \max_{a' \in A} Q^*(s',a') \right)$$

#### Q Iteration

• We have

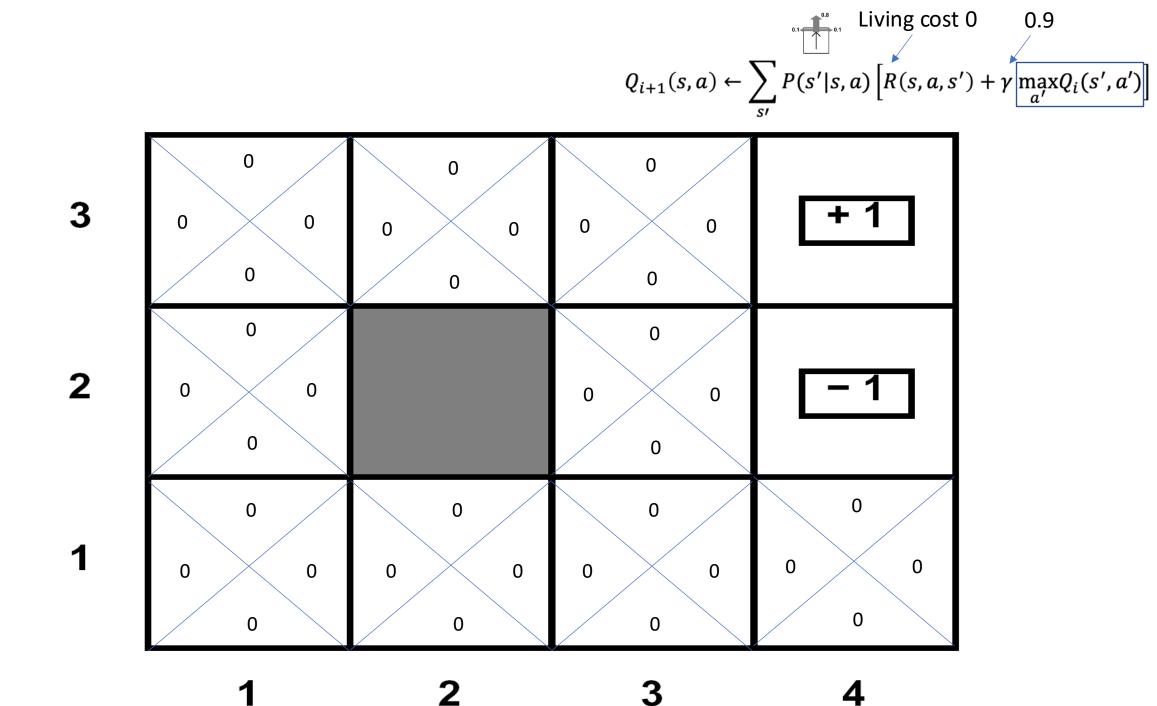
$$\pi^*(s) = \max_{a \in A} Q^*(s, a)$$

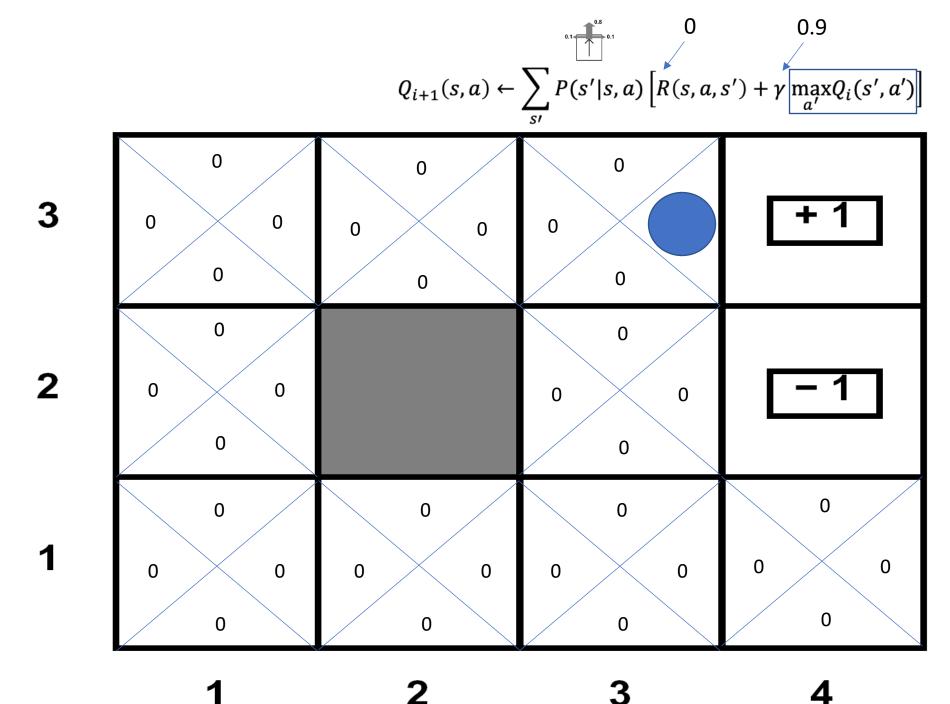
• Strategy: Compute  $Q^*$  and then use it to compute  $\pi^*$ 

#### Q Iteration

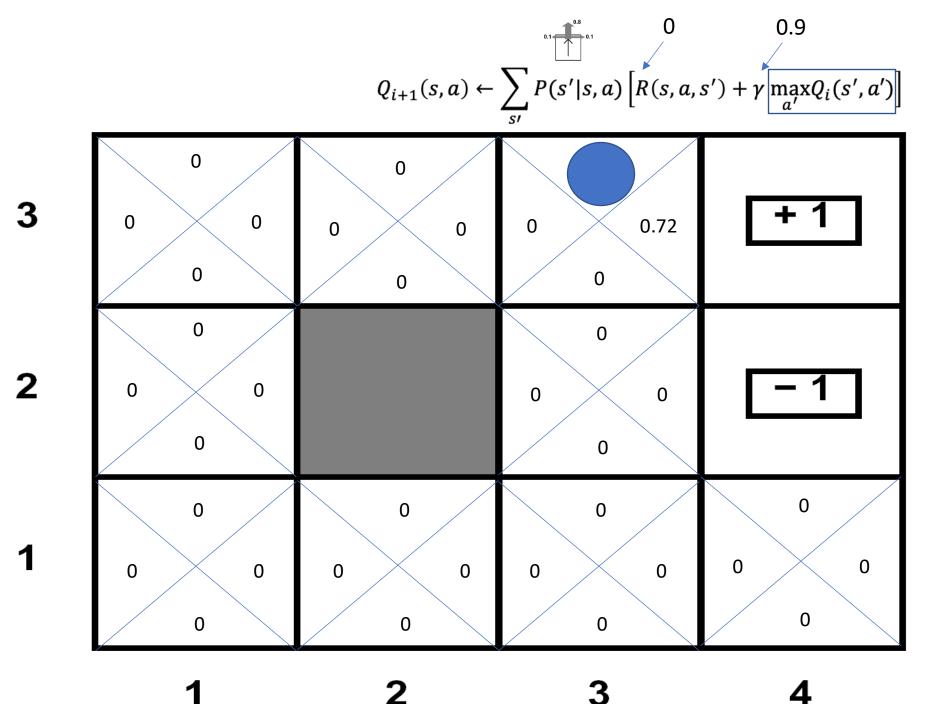
- Initialize  $Q_1(s, a) \leftarrow 0$  for all s, a
- For  $i \in \{1, 2, ...\}$  until convergence:

$$Q_{i+1}(s,a) \leftarrow \sum_{s' \in S} P(s' \mid s,a) \cdot \left( R(s,a,s') + \gamma \cdot \max_{a' \in A} Q_i(s',a') \right)$$

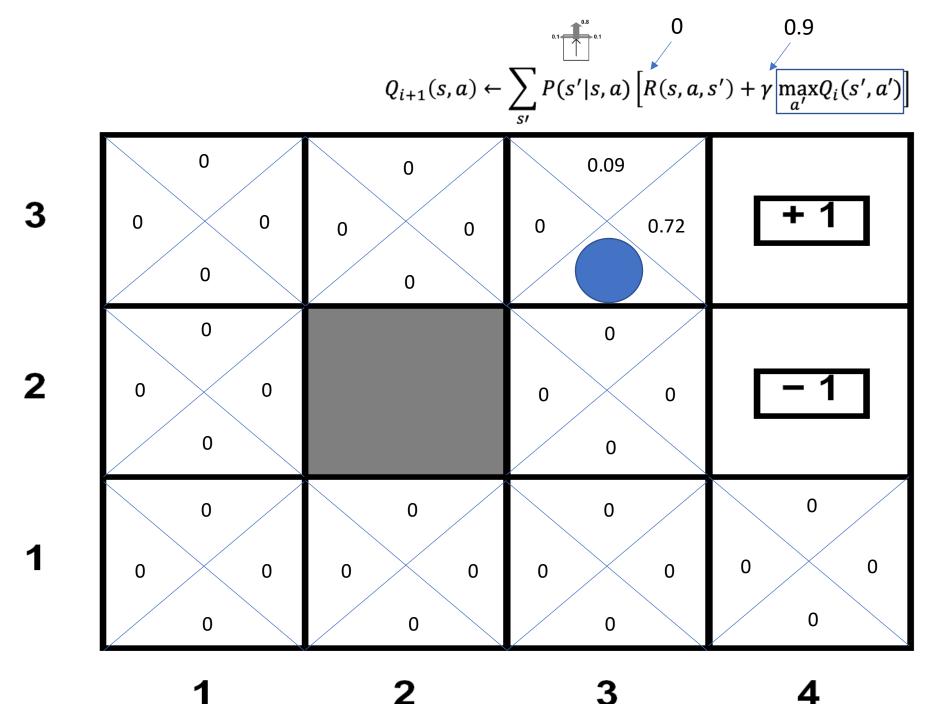




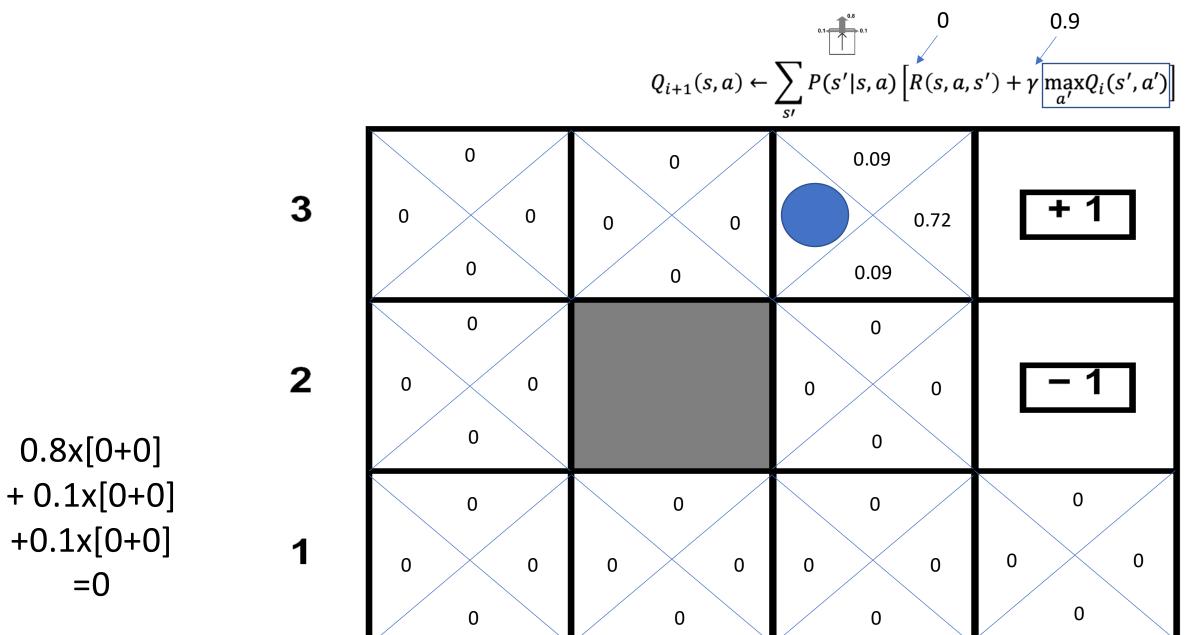
0.8x[0+0.9x1]+ 0.1x[0 + 0] +0.1x[0+0] =0.72



0.8x[0+0]+ 0.1x[0+0.9x1] +0.1x[0+0] =0.09

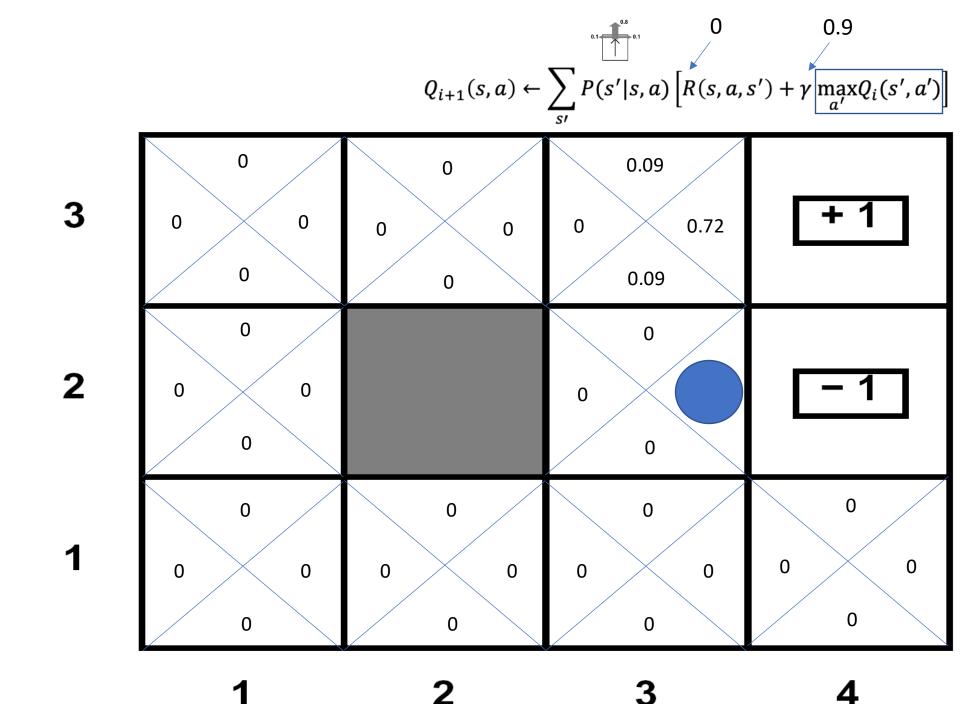


0.8x[0+0]+ 0.1x[0+0.9x1] +0.1x[0+0] =0.09

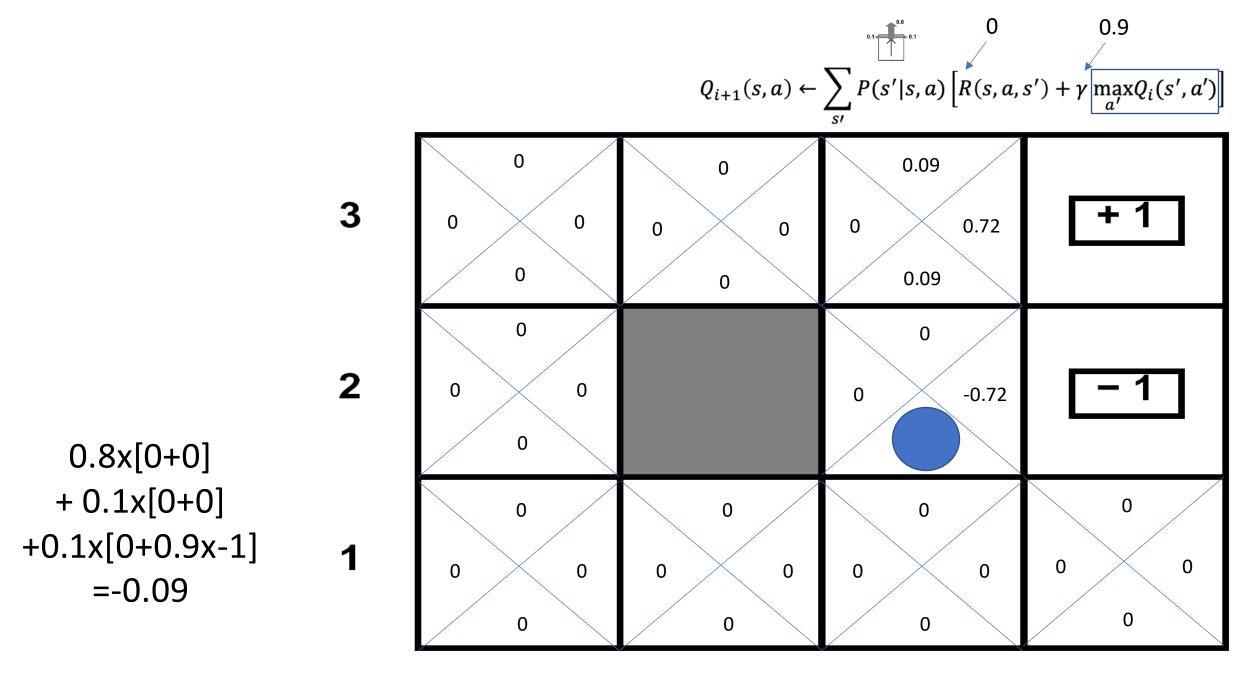


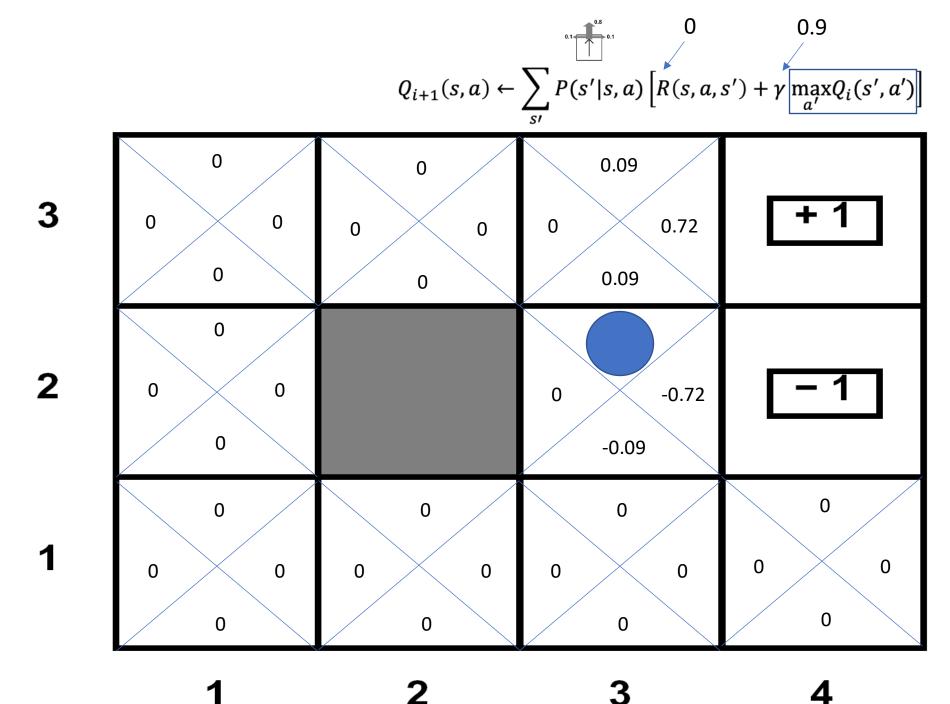
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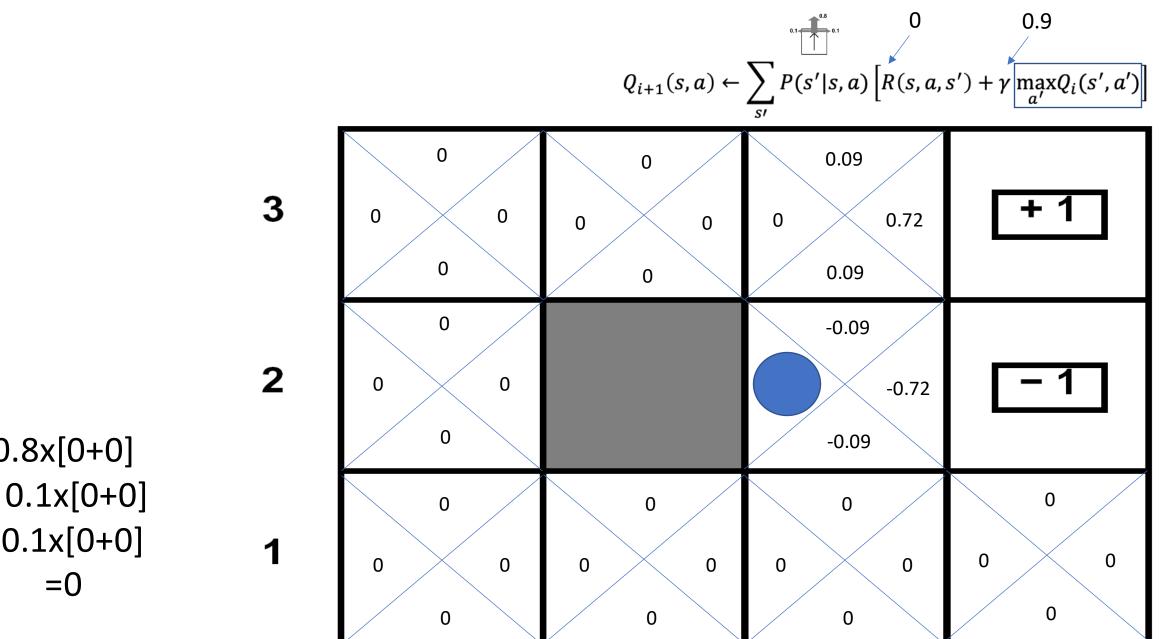


 $\begin{array}{l} 0.8x[0+0.9x-1] \\ + 0.1x[0+0] \\ + 0.1x[0+0] \\ = -0.72 \end{array}$ 



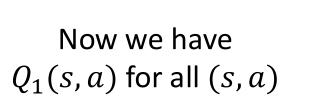


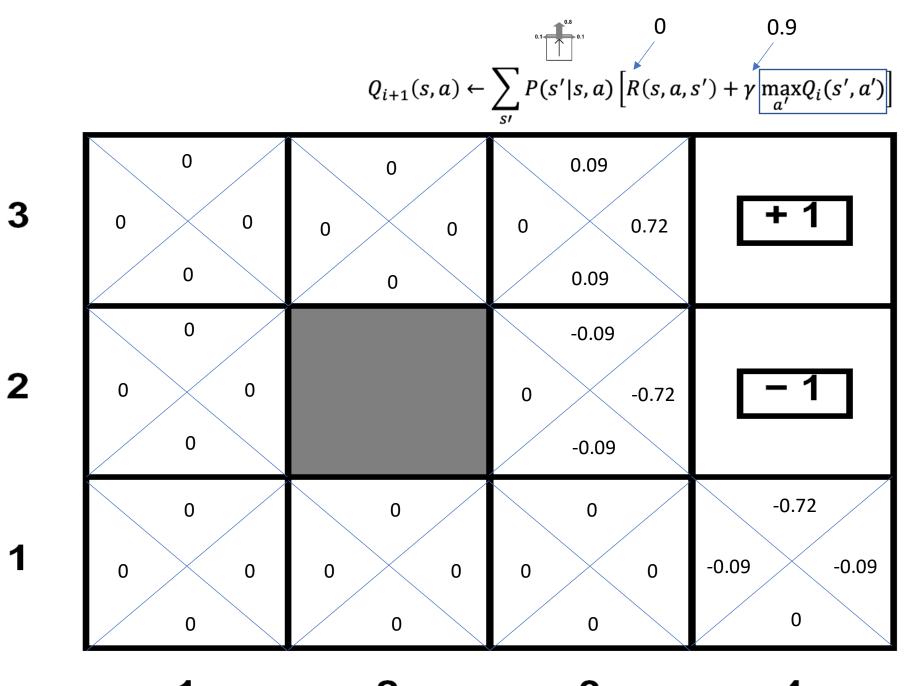
0.8x[0+0]+ 0.1x[0+0.9x-1] +0.1x[0+0] =-0.09



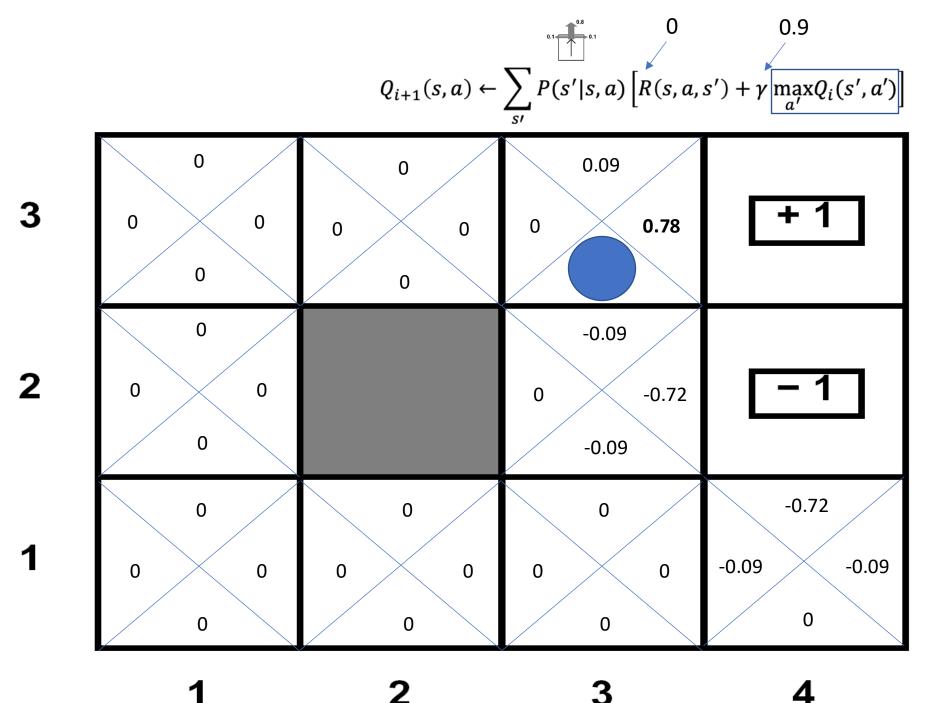
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0.8x[0+0] + 0.1x[0+0]+0.1x[0+0]

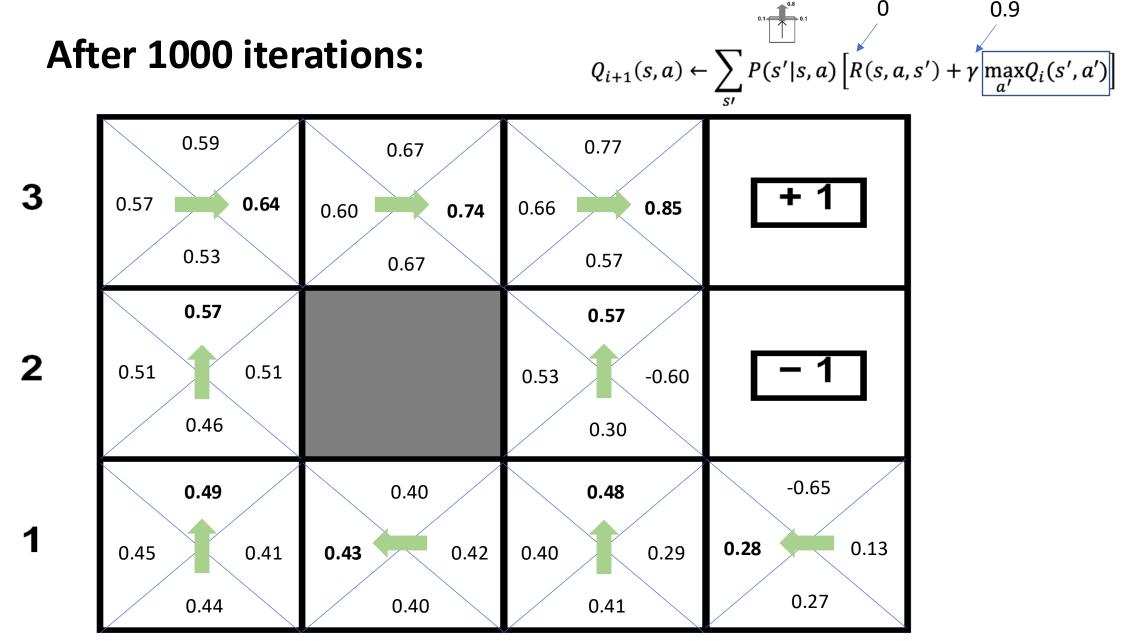




0.1 0.1 0 0.9  $Q_{i+1}(s,a) \leftarrow \sum P(s'|s,a) \left[ R(s,a,s') + \gamma \max_{a'} Q_i(s',a') \right]$ 0.09 0.09 -0.09 -0.72 -0.09 -0.72 -0.09 -0.09 

0.8x[0+0.9x1]+ 0.1x[0+0.9x0.72]+0.1x[0+0]=0.7848 

0.8x[0+0] + 0.1x[0+0.9x1] +0.1x[0+0] =0.09



After 1000 iterations:

0.9

#### Q Iteration

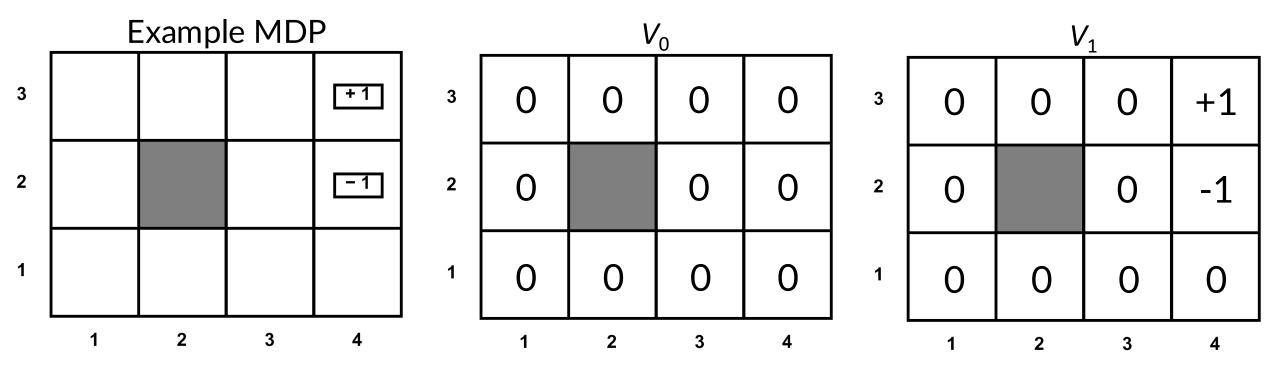
- Information propagates outward from terminal states
- Eventually all state-action pairs converge to correct Q-value estimates

#### Aside: Value Iteration

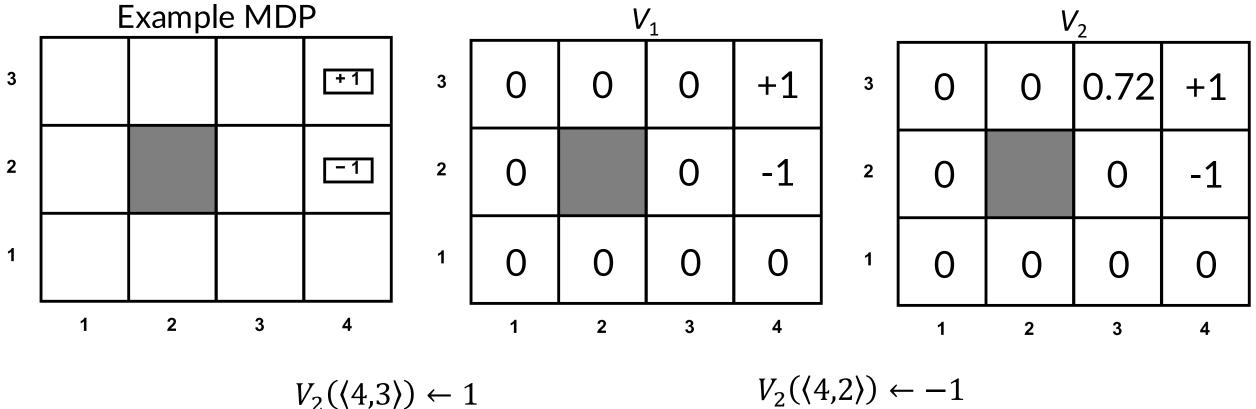
- Analogous to Q-Policy iteration but for computing the value function
- Initialize  $V_1(s) \leftarrow 0$  for all s
- For  $i \in \{1, 2, ...\}$  until convergence:

$$V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s' \mid s, a) \cdot (R(s, a, s') + \gamma \cdot V_i(s'))$$

 $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s'|s,a)[R(s,a,s') + \gamma V_i(s')]$ 

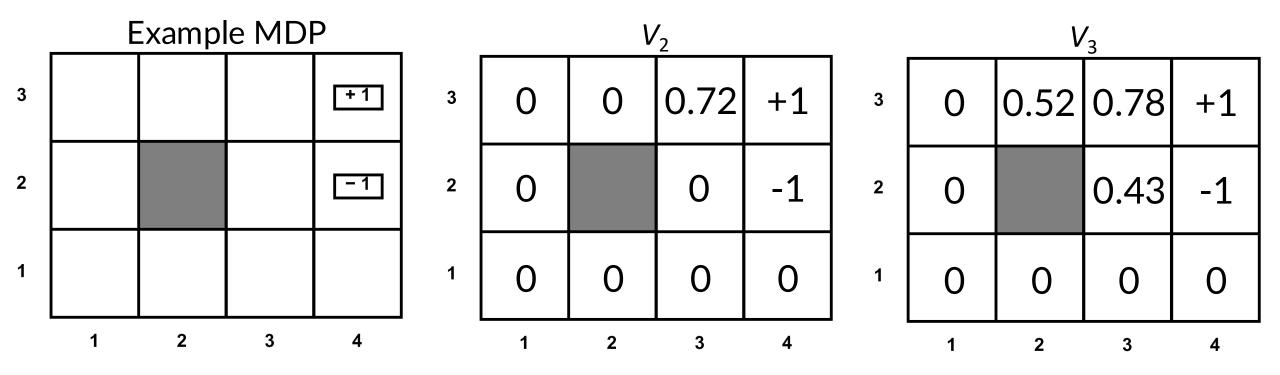


0.9 0  $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s'|s,a) [R(s,a,s') + \gamma V_i(s')]$ 



3

 $V_{i+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} P(s'|s,a)[R(s,a,s') + \gamma V_i(s')]$ 



## **Reinforcement Learning**

- Q iteration can be used to compute the optimal Q function when *P* and *R* are **known**
- How can we adapt it to the setting where these are unknown?
  - Observation: Every time you take action a from state s, you obtain one sample s' ~ P( ·| s, a) and observe R(s, a, s')
  - Use single sample instead of full P

• Can we learn  $\pi^*$  without explicitly learning *P* and *R*?

$$Q_{i+1}(s,a) \leftarrow \sum_{s' \in S} P(s' \mid s,a) \cdot \left( R(s,a,s') + \gamma \cdot \max_{a' \in A} Q_i(s',a') \right)$$

• Can we learn  $\pi^*$  without explicitly learning *P* and *R*?

$$Q_{i+1}(s,a) \leftarrow \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[ R(s,a,s') + \gamma \cdot \max_{a' \in A} Q_i(s',a') \right]$$

• Q Learning update:

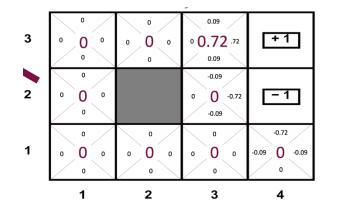
$$Q_{i+1}(s,a) \leftarrow R(s,a,s') + \gamma \cdot \max_{a' \in A} Q_i(s',a')$$

- **Q Iteration:** Update for all (*s*, *a*, *s'*) at each step
- **Q Learning:** Update just for current (s, a), and approximate with the state s' we actually reached (i.e., a single sample  $s' \sim P(\cdot | s, a)$ )

- **Problem:** Forget everything we learned before (i.e.,  $Q_i(s, a)$ )
- **Solution:** Incremental update:

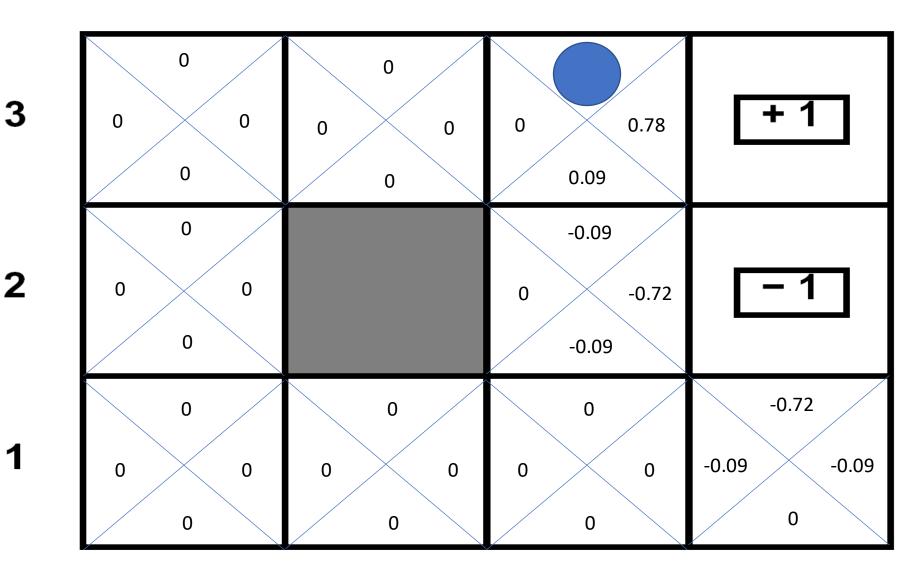
$$Q_{i+1}(s,a) \leftarrow (1-\alpha) \cdot Q_i(s,a) + \alpha \cdot \left(R(s,a,s') + \gamma \cdot \max_{a' \in A} Q_i(s',a')\right)$$

 $0.1 \qquad 0.9$  $Q(s,a) \leftarrow Q(s,a) + \alpha \left( R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a) \right)$ 



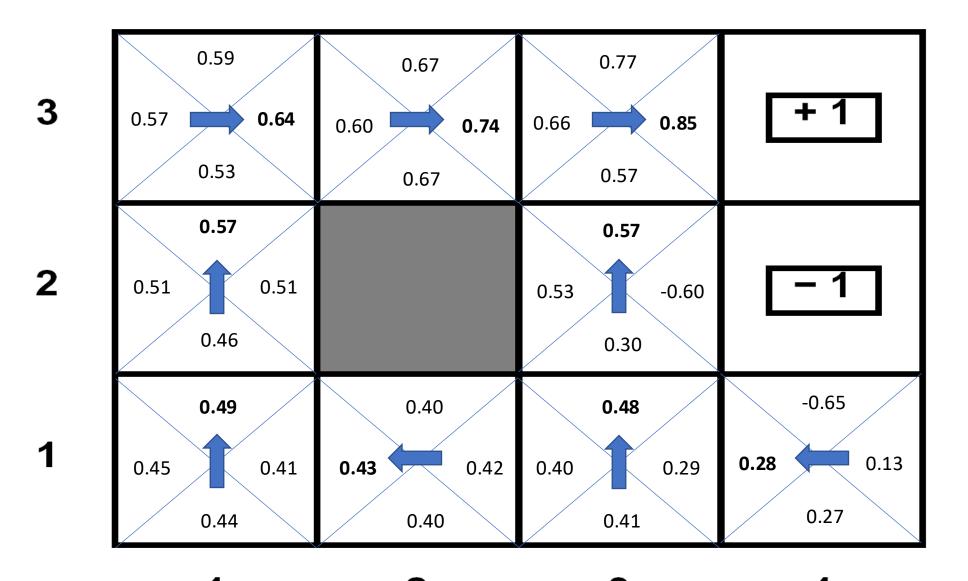
Sample  $R + \gamma \max Q =$ 0+0.9x0.78 = 0.702 **2** 

New Q = 0.09+0.1X(0.702-0.09) = 0.1512



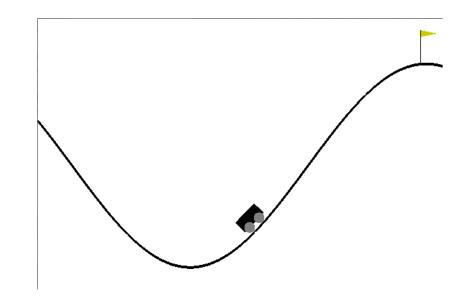
After 100,000 actions:

 $Q(s,a) \leftarrow Q(s,a) + \alpha \left( R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a) \right)$ 



# Policy for Gathering Data

- Strategy 1: Randomly explore all (*s*, *a*) pairs
  - Not obvious how to do so!
  - E.g., if we act randomly, it may take a very long time to explore states that are difficult to reach
- Strategy 2: Use current best policy
  - Can get stuck in local minima
  - E.g., we may never discover a shortcut if it sticks to a known route to the goal
- Return to this question later



# Summary

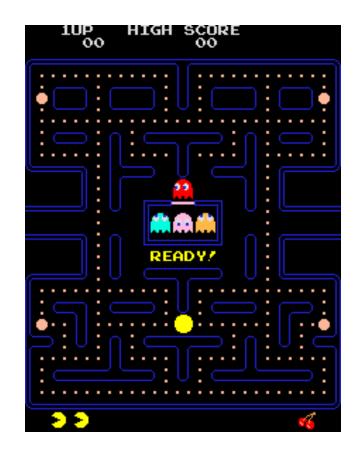
- **Q iteration:** Compute optimal Q function when the transitions and rewards are known
- **Q learning:** Compute optimal Q function when the transitions and rewards are unknown

#### • Extensions

- Various strategies for exploring the state space during learning
- Handling large or continuous state spaces

# **Curse of Dimensionality**

- How large is the state space?
  - Gridworld: One for each of the n cells
  - Pacman: State is (player, ghost<sub>1</sub>, ..., ghost<sub>k</sub>), so there are n<sup>k</sup> states!
- **Problem:** Learning in one state does not tell us anything about the other states!
- Many states  $\rightarrow$  learn very slowly

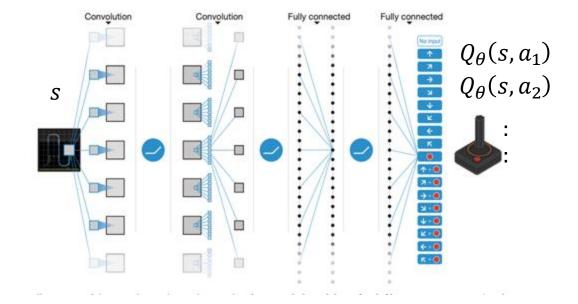


#### **State-Action Features**

- Can we learn **across** state-action pairs?
- Yes, use features!
  - $\phi(s,a) \in \mathbb{R}^d$
  - Then, learn to predict  $Q^*(s, a) \approx Q_{\theta}(s, a) = f_{\theta}(\phi(s, a))$
  - Enables generalization to similar states

#### Neural Network Q Function

- Examples: Distance to closest ghost, distance to closest dot, etc.
  - Can also use neural networks to **learn** features (e.g., represent Pacman game state as an image and feed to CNN)!



### Deep Q Learning

• Learning: Gradient descent with the squared Bellman error loss:

$$\left(\underbrace{\left(R(s,a,s')+\gamma\cdot\max_{a'}Q_{\theta}(s',a')\right)-Q_{\theta}(s,a)\right)^{2}}_{\text{``Label'' }y}\right)$$

# Deep Q Learning

#### • Iteratively perform the following:

• Take an action  $a_i$  and observe  $(s_i, a_i, s_{i+1}, r_i)$ 

• 
$$y_i \leftarrow r_i + \gamma \cdot \max_{\substack{a' \in A}} Q_{\theta}(s_{i+1}, a')$$
  
•  $\theta \leftarrow \theta - \alpha \cdot \frac{d}{d\theta} (Q_{\theta}(s_i, a_i) - y_i)^2$ 

- Note: Pretend like  $y_i$  is constant when taking the gradient
- For finite state setting, recover incremental update if the "parameters" are the Q values for each state-action pair

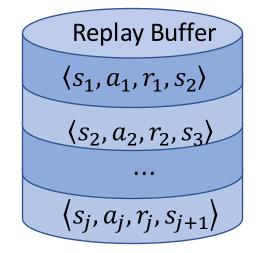
# **Experience Replay Buffer**

#### Problem

- Sequences of states are highly correlated
- Tend to overfit to current states and forget older states

#### Solution

- Keep a replay buffer of observations (as a priority queue)
- Gradient updates on samples from replay buffer instead of current state



**Priority Queue** 

#### Advantages

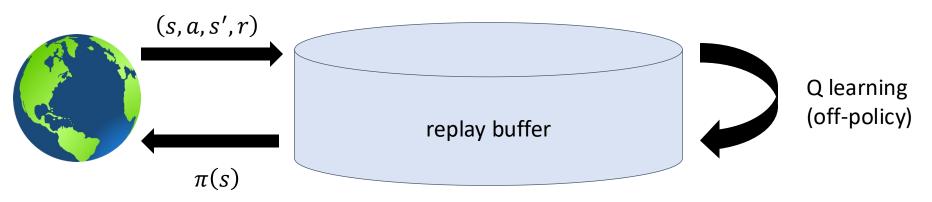
- Breaks correlations between consecutive samples
- Can take multiple gradient steps on each observation

# Deep Q Learning with Replay Buffer

#### • Iteratively perform the following:

- Take an action  $a_i$  and add observation  $(s_i, a_i, s_{i+1}, r_i)$  to replay buffer D
- For  $k \in \{1, ..., K\}$ :
  - Sample  $(s_{i,k}, a_{i,k}, s_{i+1,k}, r_{i,k})$  from D
  - $y_{i,k} \leftarrow r_{i,k} + \gamma \cdot \max_{a' \in A} Q_{\theta}(s_{i+1,k}, a')$

• 
$$\theta \leftarrow \theta - \alpha \cdot \frac{d}{d\theta} (Q_{\theta}(s_{i,k}, a_{i,k}) - y_{i,k})^2$$



### Target Q Network

#### Problem

• Q network occurs in the label  $y_i!$ 

• 
$$\theta \leftarrow \theta - \alpha \cdot \frac{d}{d\theta} \left( Q_{\theta}(s_i, a_i) - r_i - \gamma \cdot \max_{a' \in A} Q_{\theta}(s_{i+1}, a') \right)^2$$

• Thus, labels change as Q network changes (distribution shift)

#### Solution

- Use a separate **target Q network** for the occurrence in  $y_i$
- Only update target network occasionally

• 
$$\theta \leftarrow \theta - \alpha \cdot \frac{d}{d\theta} \left( \underbrace{Q_{\theta}(s_{i}, a_{i})}_{Q_{\theta}(s_{i}, a_{i})} - r_{i}\gamma \cdot \max_{a' \in A} \underbrace{Q_{\theta'}(s_{i+1}, a')}_{Q_{\theta'}(s_{i+1}, a')} \right)^{2}$$
  
Original Q Network Target Q Network

# Deep Q Learning with Target Q Network

#### • Iteratively perform the following:

- Take an action  $a_i$  and add observation  $(s_i, a_i, s_{i+1}, r_i)$  to replay buffer D
- For  $k \in \{1, ..., K\}$ :
  - Sample  $(s_{i,k}, a_{i,k}, s_{i+1,k}, r_{i,k})$  from D
  - $y_{i,k} \leftarrow r_{i,k} + \gamma \cdot \max_{a' \in A} Q_{\theta'}(s_{i+1,k}, a')$

• 
$$\theta \leftarrow \theta - \alpha \cdot \frac{d}{d\theta} (Q_{\theta}(s_{i,k}, a_{i,k}) - y_{i,k})^2$$

• Every N steps,  $\theta' \leftarrow \theta$ 

### Deep Q Learning for Atari Games



#### Image Sources:

https://towardsdatascience.com/tutorial-double-deep-q-learning-with-dueling-network-architectures-4c1b3fb7f756 https://deepmind.com/blog/going-beyond-average-reinforcement-learning/ https://jaromiru.com/2016/11/07/lets-make-a-dqn-double-learning-and-prioritized-experience-replay/

### Aside: Policy Gradient Algorithm

- Directly train policy  $\pi_{\theta}(a \mid s)$  mapping states to action distributions
- Policy gradient theorem gives the gradient update:

$$\theta \leftarrow \theta + \eta \cdot \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta} \left( a_{i,t} \middle| s_{i,t} \right) \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'} \right)$$

• Can be combined with Q learning to form "actor-critic algorithms"

# Policy for Gathering Data

• First, detour on **multi-armed bandits** 

#### **Multi-Armed Bandits**

- State: None! (To be precise, a single state  $S = \{s_0\}$ )
- Action: Item to recommend (often called arms)
- Transitions: Just stay in the same state
- Rewards: Random payoff for each arm
  - Denote  $R(a) = R(s_0, a)$ , where a is the chosen action

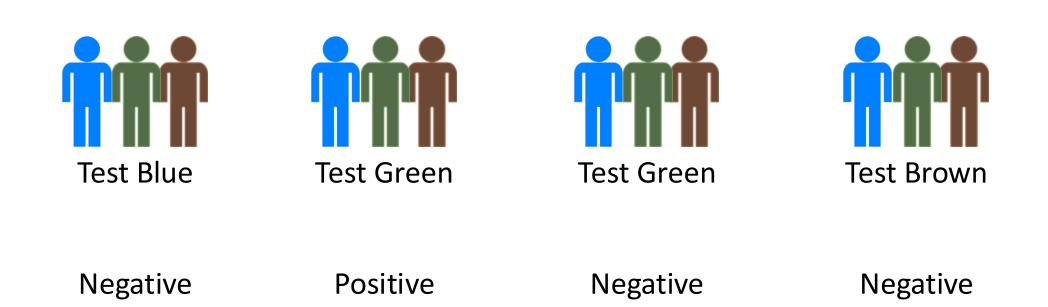
# Application: Ad Targeting

#### Setting

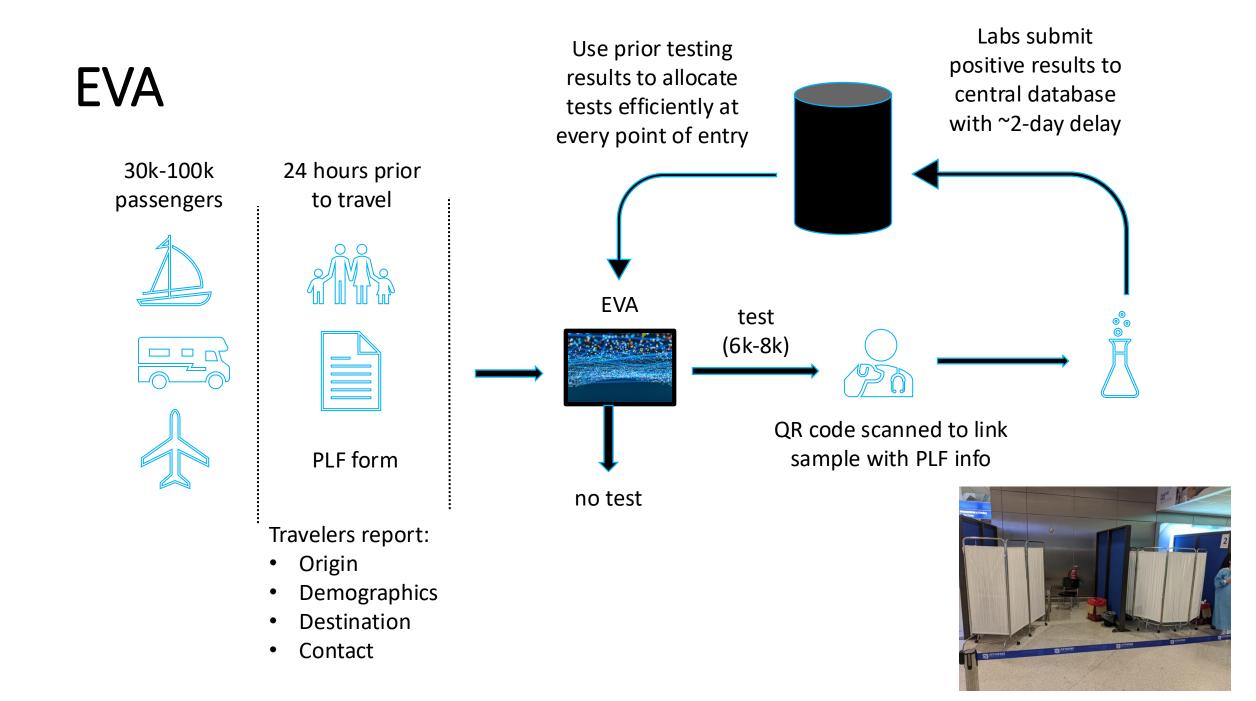
- Google wants to show the most popular ad for a search term (e.g., "lawyer")
- There are a fixed number of ads to choose from



### Application: Targeted COVID-19 Testing



H. Bastani, K. Drakopoulos, V. Gupta, et al. Efficient and Targeted COVID-19 Border Testing via Reinforcement Learning.



# Why Bandits?

#### Bandit feedback

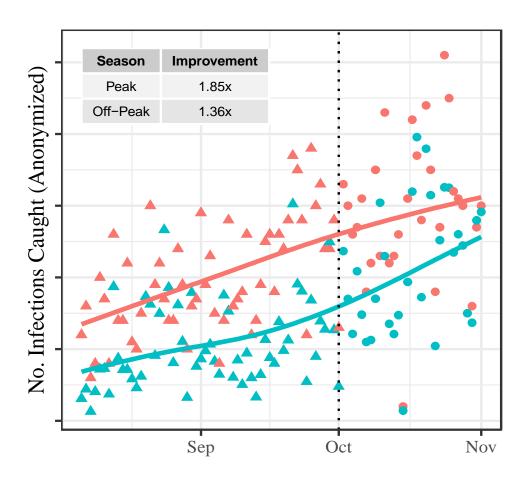
- Only observe positive/negative if the traveler is tested
- Technically "semi-bandit feedback"

#### Nonstationarity

- Infection rate for different passenger types changes over time
- Need to continue to explore and collect data over time

# Cases Caught

- 1.85 × improvement compared to random testing
- $1.25-1.45 \times \text{improvement vs.}$ targeting based on public data

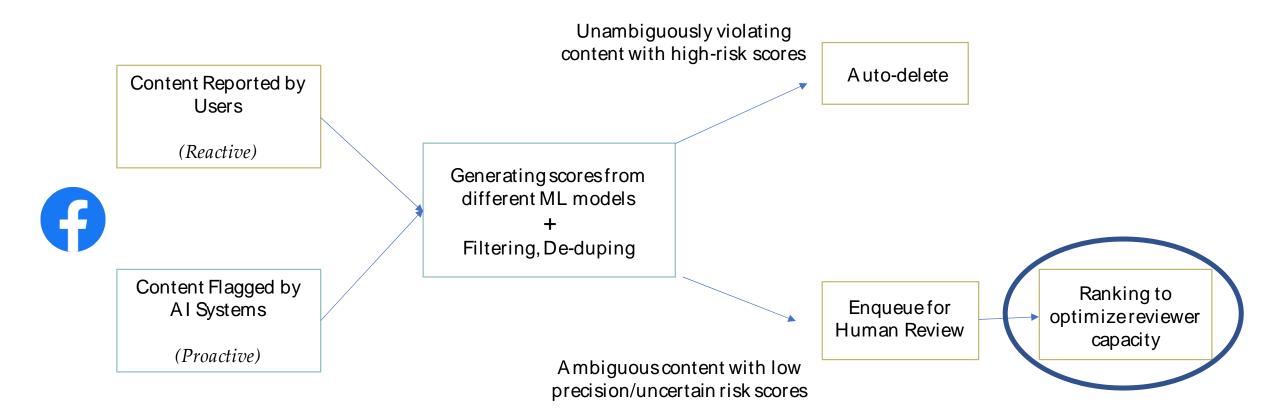


#### Problem

- Millions of pieces of content are posted on Meta platforms each day
- Too much to manually review all content
- How to moderate to make sure no harmful?

#### Solution

- ML to prioritize potentially harmful content for manual review
- Featurize content and predict likelihood that it is harmful



- What about new "types" of content?
  - E.g., new kind of racial slur
  - Cold start problem!
- Use multi-armed bandits!

- Multi-armed bandit
  - Each "step" corresponds to one piece of content
- Action: Whether to manually review content
- **Reward:** 1 if content is harmful, 0 otherwise
  - Intuition: Goal is to maximize amount of harmful content caught
  - Include an  $\alpha$  penalty for flagging content to avoid flagging everything

### **Multi-Armed Bandits**

#### Many applications

- Cold-start for news/ad/movie recommendations
- A/B testing
- Flagging potentially harmful content on a social media platform
- Prioritizing medical tests
- Learning dynamically
- Many practical RL problems are multi-armed bandits

### **Exploration-Exploitation Tradeoff**

- For  $t \in \{1, 2, ..., T\}$ 
  - Compute reward estimates  $r_{t,a} = \frac{\sum_{i=1}^{t-1} r_i \cdot 1(a_i = a)}{\sum_{i=1}^{t-1} 1(a_i = a)}$
  - Choose action  $a_t$  based on reward estimates
  - Add  $(a_t, r_t)$  to replay buffer
- Question: How to choose actions?
  - Exploration: Try actions to better estimate their rewards
  - Exploitation: Use action with the best estimated reward to maximize payoff

### Multi-Armed Bandit Algorithms

- Naïve strategy: *ε*-Greedy
  - Choose action  $a_t \sim \text{Uniform}(A)$  with probability  $\epsilon$
  - Choose action  $a_t = \underset{a \in A}{\arg \max r_{t,a}}$  with probability  $1 \epsilon$
- Can we do better?

### **Multi-Armed Bandit Algorithms**

- Upper confidence bound (UCB)
  - Choose action  $a_t = \arg \max_{a \in A} \left\{ r_{t,a} + \frac{\text{const}}{\sqrt{N_t(a)}} \right\}$
  - $N_t(a) = \sum_{i=1}^{t-1} 1(a_i = a)$  is the number of times action a has been played

#### Thompson sampling

- Choose action  $a_t = \underset{a \in A}{\operatorname{arg max}} \{r_{t,a} + \epsilon_{t,a}\}$ , where  $\epsilon_{t,a} \sim N\left(0, \frac{\operatorname{const}}{N_t(a)}\right)$
- Both come with theoretical guarantees

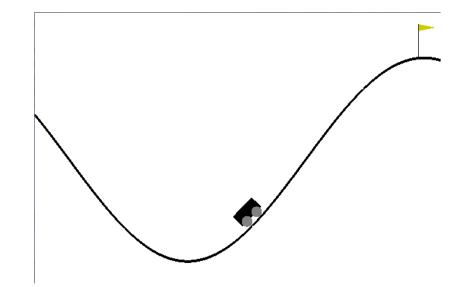
# **Exploration in Reinforcement Learning**

#### • $\epsilon$ -greedy:

- Play current best with probability  $1-\epsilon$  and randomly with probability  $\epsilon$
- Can reduce  $\epsilon$  over time
- Works okay, but exploration is undirected

## **Exploration in Reinforcement Learning**

- *ε*-greedy suffers additional issues due to state space
- Policy learning is an effective practical solution
  - No theoretical guarantees due to local minima



### **Exploration in Finite MDPs**

- Upper confidence bound (UCB)
  - Choose action  $a_t = \arg \max_{a \in A} \left\{ Q_t(s, a) + \frac{\text{const}}{\sqrt{N_t(s, a)}} \right\}$  (inflate less visited states)
  - Visitation count  $N_t(s, a) = \sum_{i=1}^{t-1} 1(s_i = s, a_i = a)$  is the number of times action a has been played in state s

#### Thompson sampling

- Choose action  $a_t = \underset{a \in A}{\operatorname{arg max}} \{Q_t(s, a) + \epsilon_{t,s,a}\}$ , where  $\epsilon_{t,s,a} \sim N\left(0, \frac{\operatorname{const}}{N_t(s,a)}\right)$
- Both come with theoretical guarantees

### **Exploration in Continuous MDPs**

- Can we adapt these ideas to continuous MDPs?
  - Thompson sampling is more suitable

#### Bootstrap DQN

- Train ensemble of k different Q-function estimates  $Q_{\theta_1}, \dots, Q_{\theta_k}$  in parallel
- Original idea was to use online bootstrap, but training from different random initial  $\theta$ 's worked as well
- In each episode, act optimally according to  $Q_{\theta_i}$  for  $i \sim \text{Uniform}(\{1, \dots, k\})$

### **Exploration in Continuous MDPs**

- Can we adapt these ideas to continuous MDPs?
  - Thompson sampling is more suitable
- Soft Q-learning
  - Sample actions according to  $a \sim \operatorname{Softmax}\left(\left[\beta \cdot \hat{Q}_{\theta}(s, a)\right]_{a \in A}\right)$

- Intuition: Rather than focus on optimism with respect to reward, focus on exploring where we are uncertain
- How to determine uncertainty?
- Candidate strategy
  - Train a **dynamics model** to predict s' = f(s, a)
  - Take actions where f(s, a) has high variance (e.g., use bootstrap)
- Problems?
  - What if s' includes spurious components, like a TV screen playing a movie?

- Learn a feature map  $\phi(s) \in \mathbb{R}^d$
- Model 1: Train a model to predict state transitions:

$$\widehat{\phi}(s') = f_{\theta}(\phi(s), a)$$

- Feature map lets the model "ignore" spurious components of s such as a TV
- **Problem:** We could just learn  $\phi(s) = \vec{0}$ ?

- Learn a feature map  $\phi(s) \in \mathbb{R}^d$
- Model 1: Train a model to predict state transitions:

$$\widehat{\phi}(s') = f_{\theta}(\phi(s), a)$$

• Model 2: Train a model to predict action to achieve a transition:

$$\hat{a} = g_{\theta}(\phi(s), \phi(s'))$$

- "Inverse dynamics model" that avoids collapsing  $\phi$ 

• Curiosity reward is

$$R(s, a, s') = \|\hat{\phi}(s') - \phi(s')\|_{2}^{2}$$

• In other words, reward agent for exercising transitions that *f* cannot yet predict accurately

# **Offline Reinforcement Learning**

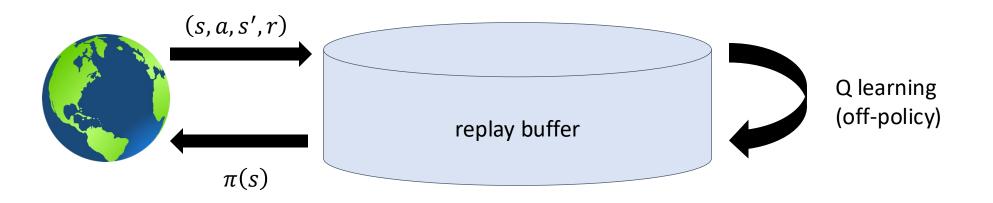
- Offline reinforcement learning: How can we learn without actively gathering new data?
  - E.g., learn how to perform a task from videos of humans performing the task
  - Also known as **off-policy** or **batch** reinforcement learning
- **Recall:** Drawback of Q learning was we need an exploration strategy
- However, this also enables us to use Q learning with offline data!

# **Offline Reinforcement Learning**

#### • Iteratively perform the following:

- Take an action  $a_i$  and add observation  $(s_i, a_i, s_{i+1}, r_i)$  to replay buffer D
- For  $k \in \{1, ..., K\}$ :
  - Sample  $(s_{i,k}, a_{i,k}, s_{i+1,k}, r_{i,k})$  from D
  - $y_{i,k} \leftarrow r_{i,k} + \gamma \cdot \max_{a' \in A} Q_{\theta}(s_{i+1,k}, a')$

• 
$$\phi \leftarrow \phi - \alpha \cdot \frac{d}{d\theta} (Q_{\theta}(s_{i,k}, a_{i,k}) - y_{i,k})^2$$



### **Offline Reinforcement Learning**

- Iteratively perform the following:
  - Take an action  $a_i$  and add observation  $(s_i, a_i, s_{i+1}, r_i)$  to replay buffer D
  - For  $k \in \{1, ..., K\}$ :
    - Sample  $(s_{i,k}, a_{i,k}, s_{i+1,k}, r_{i,k})$  from D
    - $y_{i,k} \leftarrow r_{i,k} + \gamma \cdot \max_{a' \in A} Q_{\theta}(s_{i+1,k}, a')$

• 
$$\phi \leftarrow \phi - \alpha \cdot \frac{d}{d\theta} (Q_{\theta}(s_{i,k}, a_{i,k}) - y_{i,k})^2$$

