Example OAT Typechecking Derivation

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This is an example derivation for the following OAT statements. Note that it is not a complete OAT program (it's technically not even a block because it isn't inside of braces).

Let S be the following OAT program:

```
var x1 = 0;
var x2 = x1 + x1;
x1 = x1 - x2;
return x1;
```

The following

$$\dfrac{\mathcal{D}_1 \quad \mathcal{D}_2 \quad \mathcal{D}_3 \quad \mathcal{D}_4}{H;G;\cdot;\texttt{int} \vdash_{ss} \texttt{var}\ x_1 = 0; \texttt{var}\ x_2 = x_1 + x_1; x_1 = x_1 - x_2; \texttt{return}\ x_1; \Rightarrow x_1:\texttt{int}, x_2:\texttt{int}, \cdot; \top}\ \textsc{typ\_stmts}$$

$\mathcal{D}_1 =$

$$\dfrac{\dfrac{\dfrac{}{H;G;\cdot \vdash\ 0\ :\ \texttt{int}}\ \textsc{typ\_int} \qquad x_1 \notin \cdot}{H;G;\cdot \vdash \texttt{var}\ x_1 = 0 \Rightarrow \cdot, x_1:\texttt{int}}\ \textsc{typ\_decl}}{H;G;\cdot;\texttt{int} \vdash\ \texttt{var}\ x_1 = 0;\ \Rightarrow\ \cdot, x_1:\texttt{int}; \bot}\ \textsc{typ\_stmtdecl}$$

$\mathcal{D}_2' =$

$$\dfrac{\dfrac{x_1:\texttt{int} \in \cdot, x_1:\texttt{int}}{H;G;\cdot, x_1:\texttt{int} \vdash_{lhs}\ x_1\ :\ \texttt{int};\ \top}\ \textsc{typ\_local}}{H;G;\cdot, x_1:\texttt{int} \vdash\ x_1\ :\ \texttt{int}}\ \textsc{typ\_lhs}$$

$\mathcal{D}_2 =$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash\ +:\texttt{(int,int) -> int}}\ \textsc{typ\_intOps} \qquad \mathcal{D}_2' \qquad \mathcal{D}_2'}{H;G;\cdot, x_1:\texttt{int} \vdash\ x_1 + x_1\ :\ \texttt{int}}\ \textsc{typ\_bop}}{H;G;\cdot \vdash \texttt{var}\ x_2 = x_1 + x_1 \Rightarrow \cdot, x_1:\texttt{int}, x_2:\texttt{int}}\ \textsc{typ\_decl} \qquad x_2 \notin \cdot, x_1:\texttt{int}}{H;G;\cdot, x_1:\texttt{int};\texttt{int} \vdash\ \texttt{var}\ x_2 = x_1 + x_1;\ \Rightarrow\ \cdot, x_1:\texttt{int}, x_2:\texttt{int}; \bot}\ \textsc{typ\_stmtdecl}$$

$\mathcal{D}_{31} =$

$$\dfrac{\dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash_{lhs} x_1 \ : \ \texttt{int}; \top}\ \textsc{typ\_local}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_1 \ : \ \texttt{int}}\ \textsc{typ\_lhs}$$

$\mathcal{D}_{32} =$

$$\dfrac{\dfrac{x_2 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash_{lhs} x_2 \ : \ \texttt{int}; \top}\ \textsc{typ\_local}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_2 \ : \ \texttt{int}}\ \textsc{typ\_lhs}$$

$\mathcal{D}'_3 =$

$$\dfrac{\dfrac{}{\vdash \texttt{-} : \texttt{(int,int) -> int}}\ \textsc{typ\_intOps} \qquad \mathcal{D}_{31} \qquad \mathcal{D}_{32}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_1 \texttt{ - } x_2 \ : \ \texttt{int}}\ \textsc{typ\_bop}$$

$\mathcal{D}_3 =$

$$\dfrac{\dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash_{lhs} x_1 \ : \ \texttt{int}; \top}\ \textsc{typ\_local} \qquad \mathcal{D}_5 \qquad \dfrac{}{H \vdash \texttt{int} \ \leq \ \texttt{int}}\ \textsc{sub\_sub\_int}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}; \texttt{int} \vdash x_1 = x_1 \texttt{ - } x_2 \texttt{;} \ \Rightarrow \ \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}; \bot}\ \textsc{typ\_assn}$$

$\mathcal{D}_4 =$

$$\dfrac{\dfrac{\dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash_{lhs} x_1 \ : \ \texttt{int}; \top}\ \textsc{typ\_local}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_1 \ : \ \texttt{int}}\ \textsc{typ\_lhs} \qquad \dfrac{}{H \vdash \texttt{int} \ \leq \ \texttt{int}}\ \textsc{sub\_sub\_int}}{H; G; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}; \texttt{int} \vdash \texttt{return } x_1 \texttt{;} \ \Rightarrow \ \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}; \top}\ \textsc{typ\_retT}$$