ESE 568: Mixed Signal Design and Modeling

Lec 6: September 20th, 2017

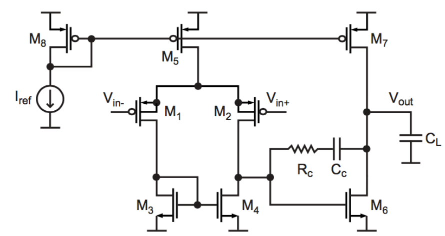
Noise Models and Advanced Opamp Design

Lecture Outline

- Opamp wrap-up
  - Noise Models
    - Transistor
    - Opamp noise
- Advanced Opamp Techniques

Basic 2-stage Opamp

Basic 2-Stage Opamp

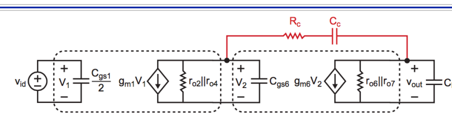


- This is a common "workhorse" opamp for medium performance applications
  - Relatively simple structure with reasonable performance

Basic 2-Stage Opamp

- Key issue: two-stages lead to two poles that are relatively close to each other
  - This leads to very poor phase margin unless very large CL is used
- Inclusion of a compensation capacitor across the second stage leads to pole splitting such that stable performance can be achieved with reasonable area
  - A compensation resistor is also desirable to help eliminate the impact of a RHP zero that occurs due to compensation

2-Stage Opamp Frequency Response



$$H(s) = \frac{v_{out}(s)}{v_{id}(s)} = \frac{K(1+s/w_z)}{(1+s/w_{p1})(1+s/w_{p2})}$$

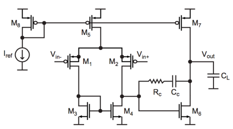
$$K = g_{m1}(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})$$

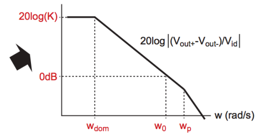
$$w_{p1} = \frac{1}{(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})C_c}$$

$$w_{p2} = \frac{g_{m6}C_c}{C_{gs6}C_L + C_c(C_{gs6}+C_L)}$$

$$w_z = -\left(\frac{g_{m6}}{C_c}\right)\frac{1}{1-g_{m6}R_c}$$

2-Stage Opamp Frequency Response





DC gain

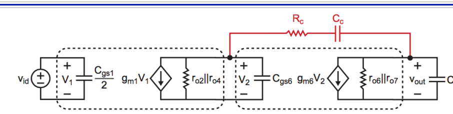
- **Determine K, w_{dom}, w_o, w_p**

Dominant pole

Parasitic pole

Unity-gain frequency

2-Stage Opamp Frequency Response



$$H(s) = \frac{v_{out}(s)}{v_{id}(s)} = \frac{K(1+s/w_z)}{(1+s/w_{p1})(1+s/w_{p2})}$$

DC gain

$$K = g_{m1}(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})$$

Unity-gain frequency

$$\omega_0 = \frac{g_{m1}}{C_c}$$

$$w_{p1} = \frac{1}{(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})C_c}$$

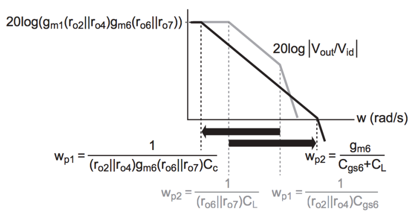
Dominant Pole

$$w_{p2} = \frac{g_{m6}C_c}{C_{gs6}C_L + C_c(C_{gs6}+C_L)}$$

Parasitic pole

$$w_z = -\left(\frac{g_{m6}}{C_c}\right)\frac{1}{1-g_{m6}R_c}$$

Impact of Pole Splitting with Compensation Cap



$$w_{p1} = \frac{1}{(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})C_c} \qquad w_{p2} = \frac{g_{m6}}{C_{gs6}+C_L}$$

$$w_{p2} = \frac{1}{(r_{o6}||r_{o7})C_L} \qquad w_{p1} = \frac{1}{(r_{o2}||r_{o4})C_{gs6}}$$

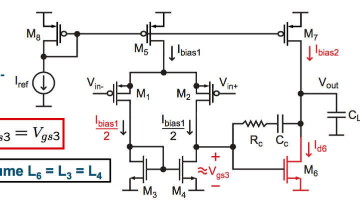
- Pole splitting allows the dominant pole frequency to be dramatically decreased and the main parasitic pole to be dramatically increased
  - We can achieve higher unity gain frequency with improved phase margin

2-Stage Opamp Offset: Systematic

$V_{in+} = V_{in-}$

$$V_{ds4} = V_{ds3} = V_{gs3}$$

assume $L_6 = L_3 = L_4$

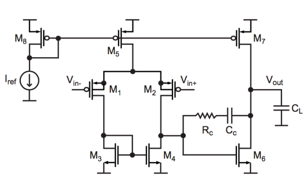


- **For mid-rail V_{out}, we need $I_{d6} = I_{bias2}$**

$$\Rightarrow \; I_{d6} = \frac{1}{2}\mu_n C_{ox}\frac{W_6}{L_6}(V_{gs3}-V_{TH})^2 = I_{bias2}$$

Also: $\frac{1}{2}\mu_n C_{ox}\frac{W_3}{L_3}(V_{gs3}-V_{TH})^2 = \frac{I_{bias1}}{2} \Rightarrow \boxed{\frac{W_6}{2W_3} = \frac{I_{bias2}}{I_{bias1}} = \frac{W_7}{W_5}}$

2-Stage Opamp Offset: Random



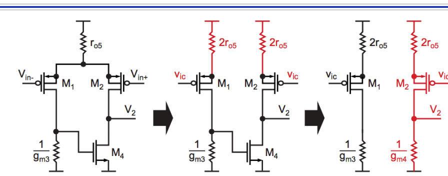
Minimize by making $g_{m3,4} < g_{m1,2}$

$$V_{OS} = \Delta V_{t(1-2)} + \Delta V_{t(3-4)}\left(\frac{g_{m3}}{g_{m1}}\right) + \frac{(V_{GS}-V_t)_{(1-2)}}{2}\left(\frac{-\Delta\frac{W}{L}_{(1-2)}}{\frac{W}{L}_{(1-2)}} - \frac{\Delta\frac{W}{L}_{(3-4)}}{\frac{W}{L}_{(3-4)}}\right)$$

Minimize by operating input transistors at small V_{gs}-V_t

Reference: Chap. 6 pages 471-472, Gray & Meyer, Analysis of and Design of Analog Integrated Circuits, 3rd Ed.

2-Stage Opamp CMRR



- **Differential gain**

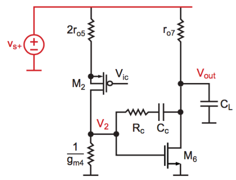
$$a_{vd1} = g_{m1}(r_{o2}||r_{o4})$$

- **Common-mode gain is calculated from the above as**

$$a_{vc1} = \frac{1/g_{m4}}{1/g_{m2}+2r_{o5}} \approx \frac{1}{2g_{m4}r_{o5}}$$

$$\Rightarrow \; CMRR = \frac{a_{vd1}}{a_{vc1}} = \boxed{2g_{m1}(r_{o2}||r_{o4})g_{m4}r_{o5}}$$

2-Stage Opamp PSRR+

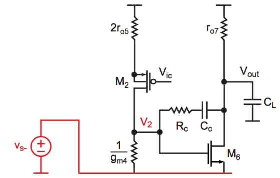


- **Calculation of impact of V_{s+} on V_{out}**

$$V_{out} = \frac{r_{o6}}{r_{o6}+r_{o7}}V_{s+} + g_{m6}(r_{o6}||r_{o7})\left(\frac{1}{2g_{m4}r_{o5}}\right)V_{s+}$$

$$\Rightarrow PSRR^+ = \frac{a_{vd}}{a_{v+}} \approx a_{vd} = g_{m1}(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})$$

2-Stage Opamp PSRR-



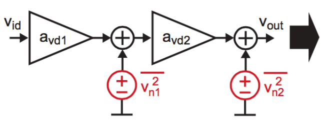
- **Calculation of impact of V_{s-} on V_{out}**

$$V_{out} \approx \frac{r_{o7}}{r_{o6}+r_{o7}}V_{s-}$$ @ high frequencies M6 looks diode connected

$$\Rightarrow a_- = \frac{V_{out}}{V_{s-}} \approx 1$$

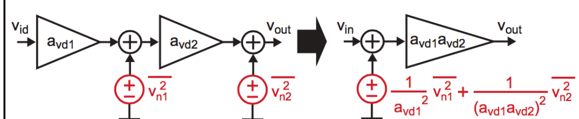
$$\Rightarrow PSRR^- = \frac{a_{vd}}{a_{v-}} \approx a_{vd} = g_{m1}(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})$$

2-Stage Opamp Noise Analysis



- Each opamp stage will contribute noise
  - Typically the spectral density of the noise will be of the same order at each stage
- Input referral of the noise reveals that the second stage noise will have much less impact than the first stage noise
  - Input-referred noise calculations of an opamp need only focus on the first stage

2-Stage Opamp Noise Analysis



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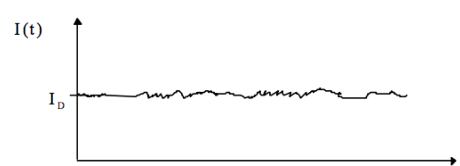
Noise Sources

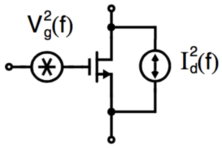
- Shot Noise
  - Discrete random event of charge jumping potential barrier (i.e PN junction)
- Thermal Noise
  - Random thermal motion of electrons
- Flicker Noise: 1/F Noise
  - Usually due to impurities causing traps in material which capture and emit carriers randomly

Shot noise



- Random fluctuations of the DC current
  - Negligible compared to thermal and flicker noise

MOSFET Noise Model



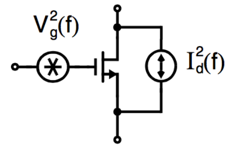
- Flicker noise as voltage on the gate

$$V_g^2(f) = \frac{K}{WLC_{ox}f}$$

- Thermal noise as current in the channel

$$I_d^2(f) = 4kT\gamma g_m$$

MOSFET Noise Model



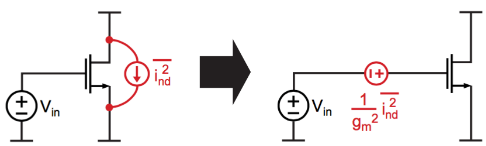
- Flicker noise as voltage on the gate

$$V_g^2(f) = \frac{K}{WLC_{ox}f}$$

- Thermal noise as current in the channel

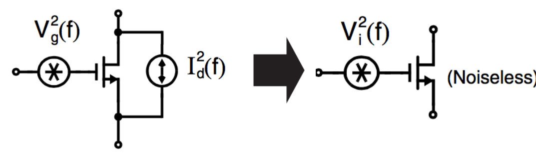
$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m$$

Input-referred MOSFET Noise



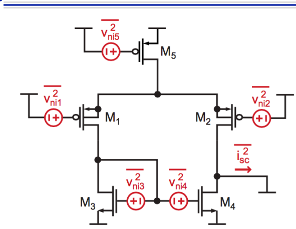
$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m \qquad V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m}$$

Input-referred MOSFET Noise



$$V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{K}{WLC_{ox}f}$$

Analysis of Opamp Noise (First Stage)



Assume:
$g_{m1} = g_{m2}$
$g_{m3} = g_{m4}$

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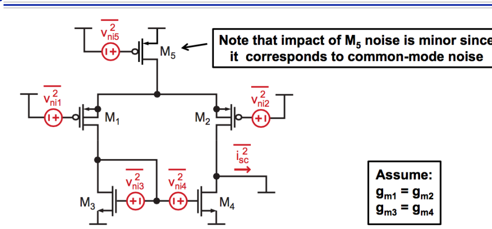

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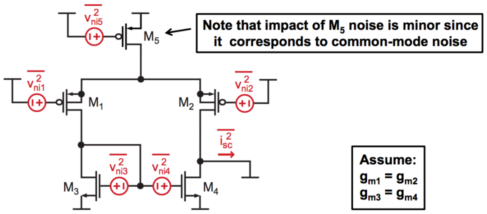
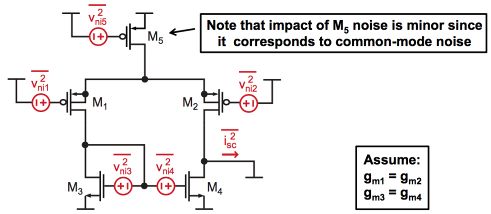


$$\overline{i_{sc}^2} = g_{m1}^2\left(\overline{v_{ni1}^2} + \overline{v_{ni2}^2}\right) + g_{m3}^2\left(\overline{v_{ni3}^2} + \overline{v_{ni4}^2}\right)$$

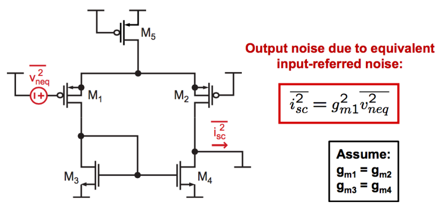
Analysis of Opamp Noise (First Stage)



$$\overline{i_{sc}^2} = g_{m1}^2\left(\overline{v_{ni1}^2} + \overline{v_{ni2}^2}\right) + g_{m3}^2\left(\overline{v_{ni3}^2} + \overline{v_{ni4}^2}\right)$$

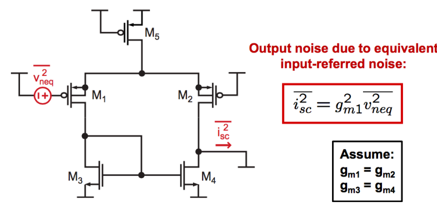
$$\Rightarrow \overline{i_{sc}^2} = 2g_{m1}^2\overline{v_{ni1}^2} + 2g_{m3}^2\overline{v_{ni3}^2}$$

Input-referred Opamp Noise (First Stage)



- **Output noise due to individual devices**

Input-referred Opamp Noise (First Stage)



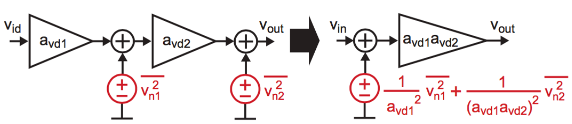
- **Output noise due to individual devices**

$$\overline{i_{sc}^2} = 2g_{m1}^2\overline{v_{ni1}^2} + 2g_{m3}^2\overline{v_{ni3}^2} = g_{m1}^2\overline{v_{neq}^2}$$

$$\overline{v_{neq}^2} = 2\overline{v_{ni1}^2} + 2\left(\frac{g_{m3}}{g_{m1}}\right)^2\overline{v_{ni3}^2}$$

Want $g_{m1} > g_{m3}$ for low noise

2-Stage Opamp Noise Analysis



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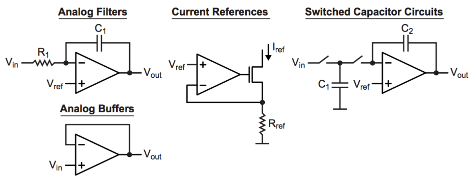
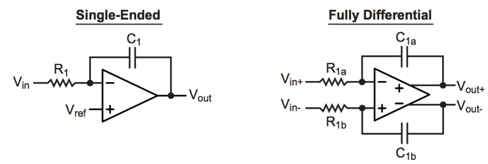
Advanced Opamp Topologies

Opamps Utilized in Wide Range of Applications



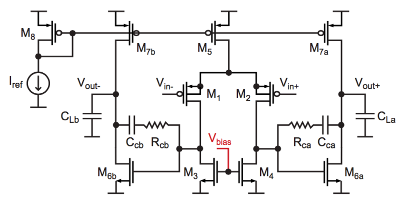
- Each application comes with different opamp requirements
  - Integrated opamps are typically custom designed for their specific application

Single-Ended Vs. Fully Differential Topologies



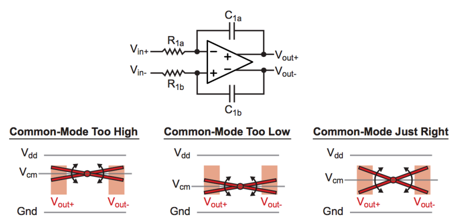
- Analog circuits are sensitive to noise from the power supply and other coupling mechanisms
- Fully differential topologies can offer rejection of common-mode noise (such as from supplies)
  - Information is encoded as the difference between two signals
  - More complex implementation than single-ended designs

Fully Differential Basic Two Stage Opamp



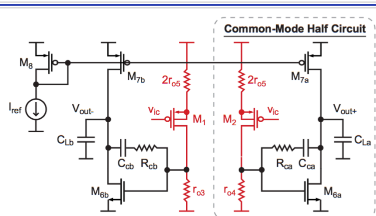
- We can separate this into differential and common mode circuits, similar to a single-ended differential amplifier
  - Differential behavior same as the single-ended opamp
    - Note that we have twice the effective range in input/output swing due to the differential signaling
  - Common mode setting needs to be dealt with (V_{bias})

Common Mode Influence



- Maximum swing for fully differential signals requires
  - Accurate setting of the common mode value
  - Suppression of common mode noise

Common-Mode Gain From Input

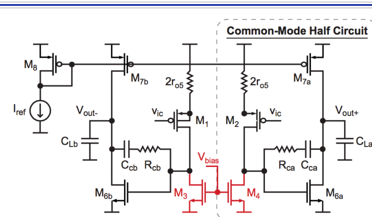


- Analysis is same as for single-ended design
  - Can be simplified to common-mode "half-circuit"

$$a_{vc} = \frac{r_{o4}}{1/g_{m2} + 2r_{o5}} g_{m6a} \left(r_{o6a} || r_{o7a}\right)$$

  - Common-mode output is sensitive to common mode input

Common-Mode Gain From Input Bias

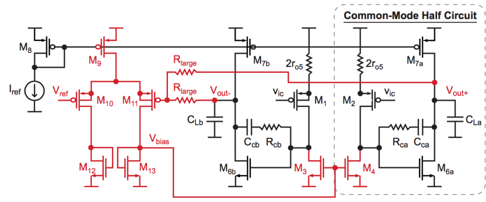


- Common mode "half circuit" can still be used

$$a_{vbias} \approx \left(g_{m4} r_{o4}\right) g_{m6a} \left(r_{o6a} || r_{o7a}\right)$$

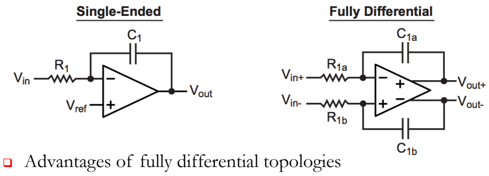
  - Common-mode output is extremely sensitive to Vbias!

Common Mode Feedback Biasing (CMFB)



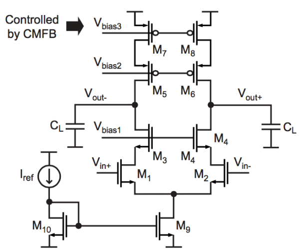
- Use an auxiliary circuit to accurately set the common mode output value to a controlled value V_{ref}
  - Need to be careful not to load the outputs with the common mode sensing circuit (R_{large} in this case)
  - Need to design CMFB to be stable

Single-Ended Versus Fully Differential



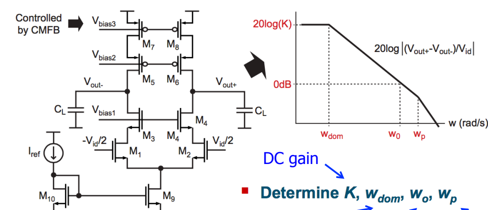
- Advantages of fully differential topologies
  - Improved CMRR and PSRR across a wide frequency range
  - Twice the effective signal swing
- Disadvantages of fully differential topologies
  - Power and complexity
- Most opamp topologies can be modified to support either single-ended or fully differential signaling

Telescopic Opamp (Fully Differential)



- Popular for high frequency applications
  - Single stage design
  - Limitation: input and output swing quite limited

Telescopic Opamp Frequency Response



DC gain

- **Determine K, w_{dom}, w_o, w_p**

Dominant pole

Parasitic pole

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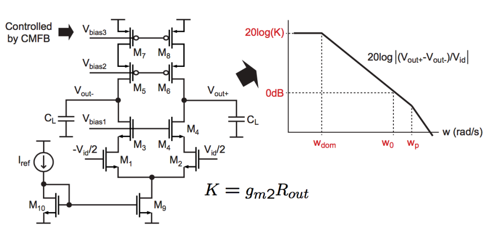
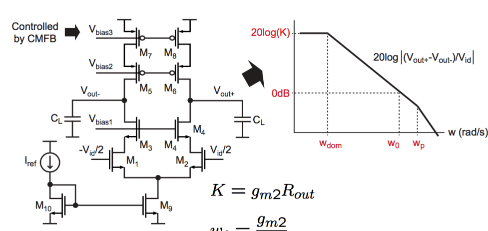
Telescopic Opamp Frequency Response



$$K = g_{m2}R_{out}$$

where $R_{out} = ((g_{m4}r_{o4})r_{o2})||((g_{m6}r_{o6})r_{o8})$

Telescopic Opamp Frequency Response

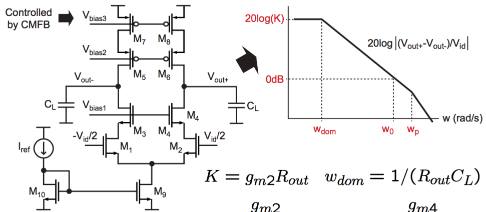


$$K = g_{m2}R_{out}$$

$$w_o = \frac{g_{m2}}{C_L}$$

where $R_{out} = ((g_{m4}r_{o4})r_{o2})||((g_{m6}r_{o6})r_{o8})$

Telescopic Opamp Frequency Response

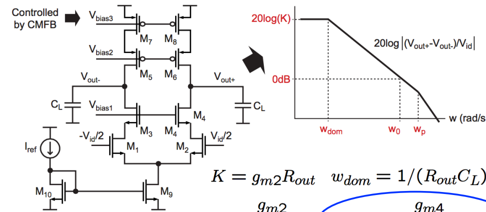


$$K = g_{m2}R_{out} \quad w_{dom} = 1/(R_{out}C_L)$$

$$w_o = \frac{g_{m2}}{C_L} \quad w_p \approx \frac{g_{m4}}{C_{gs4} + C_{s4,d2}}$$

where $R_{out} = ((g_{m4}r_{o4})r_{o2})||((g_{m6}r_{o6})r_{o8})$

Telescopic Opamp Frequency Response



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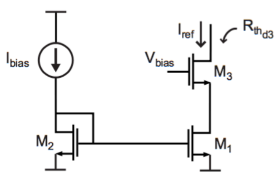
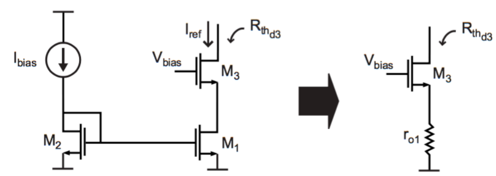
where $R_{out} = ((g_{m4}r_{o4})r_{o2})||((g_{m6}r_{o6})r_{o8})$

$$w_{p2} = \frac{g_{m6}}{C_{gs6}+C_L}$$

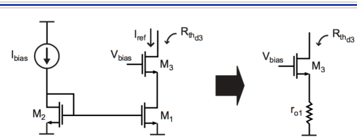
Cascoded Current Source



Cascoded Current Source

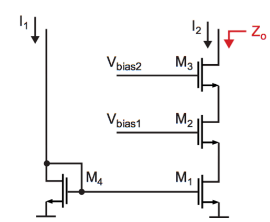


Cascode Current Mirror



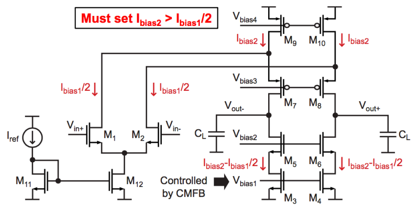
- Offers increased output resistance
  - Reduces small signal dependence of output current on the output voltage of the current source
  - We derived: $R_{th_{d3}} \approx (g_{m3}r_{o3})r_{o1}$
- Output resistance boosted by intrinsic gain of M_3, $g_{m3}r_{o3}$

Cascode Current Mirror



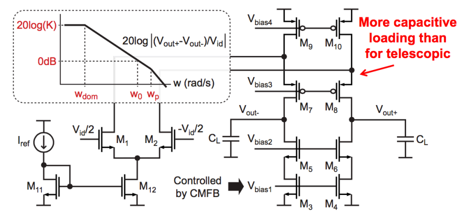
- What is Z_o?

Folded Cascode Opamp



- Modified version of telescopic opamp
  - Significantly improved input/output swing
  - High BW (better than two stage, worse than telescopic)
  - Only single stage of gain (lower than telescopic)

Folded Cascode Frequency Response



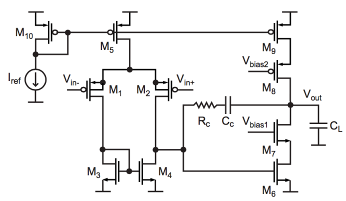
$K = g_{m2} R_{out} \quad w_{dom} = 1/(R_{out} C_L)$

$w_o = \frac{g_{m2}}{C_L} \quad w_p \approx \frac{g_{m8}}{C_{gs8} + C_{d2,d10,s8}}$

R_{o10} is lower than for telescopic due to higher drain current in M_{10}

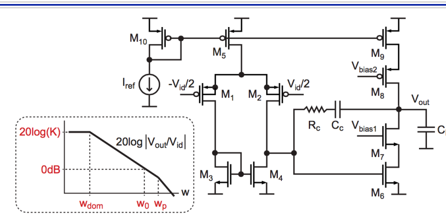
where $R_{out} = ((g_{m6} r_{o6}) r_{o4}) || ((g_{m8} r_{o8}) r_{o10})$

Two Stage with Cascoded Output Stage



- Higher DC gain than with two stage or folded cascode
  - Two gain stages with boosted gain on the output stage
- Same output swing as folded cascode
  - Lower than for basic two stage

Two Stage with Cascoded Output Stage –Frequency Response



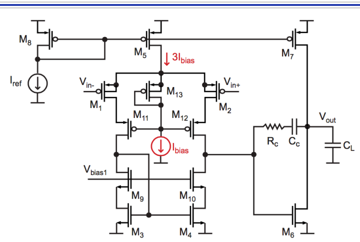
$K = g_{m2}(r_{o2}||r_{o4}) g_{m6} R_{out} \quad w_{dom} = 1/((r_{o2}||r_{o4}) C_M)$

$w_o = \frac{g_{m2}}{C_c} \quad w_p \approx \frac{g_{m6}}{C_L}$

where $R_{out} = ((g_{m7} r_{o7}) r_{o6}) || ((g_{m8} r_{o8}) r_{o9})$

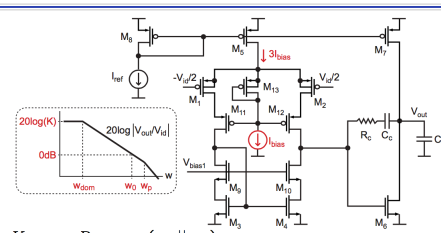
$C_M \approx (g_{m6} R_{out}) C_c$

Two Stage with Cascoded Input Stage



- Compared to two stage with cascoded output
  - Similar DC gain
  - Improved output swing
  - Reduced input swing

Two Stage with Cascoded Input Stage –Frequency Response



$K = g_{m2} R_{out1} g_{m6} (r_{o6}||r_{o7})$

$w_{dom} = 1/(R_{out1} C_M) \quad w_o = \frac{g_{m2}}{C_c} \quad w_p \approx \frac{g_{m6}}{C_L}$

where $R_{out1} = ((g_{m12} r_{o12}) r_{o2}) || ((g_{m10} r_{o10}) r_{o4})$

$C_M \approx (g_{m6}(r_{o6}||r_{o7})) C_c$

Big Ideas

- Opamp topologies can be configured to process fully differential signals
  - Provides improved immunity to noise from common-mode perturbations such as power supply noise
  - Increases effective signal swing by a factor of two
  - Carries additional complexity for CMFB and increased power consumption
- Integrated opamps are often custom designed for a given application
  - Each application places different demands on DC gain, bandwidth, signal swing, etc.
  - Opamp topologies considered today include telescopic, folded cascode, and modified two stage (cascoded input/output)
    - Each carries different tradeoffs on the above specifications

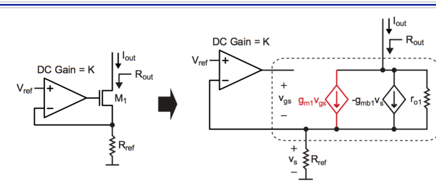
Admin

- HW 2
  - Due Friday
- HW 3
  - posted on Friday
    - Design an opamp in Cadence!

More Advanced Techniques (Extras)

- Gain boosting technique
- Nested Miller technique

Gain Boosting of Current Sources

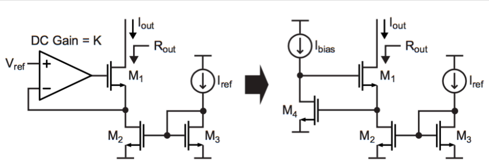


- We can achieve increased output impedance of a current source with an amplifier
  - The amplifier essentially increases g_{m1} by factor K:

$$R_{out} = (K g_{m1} r_{o1})\, R_{ref}$$

- Key issue: what is a convenient implementation of the above circuit?

A Simple Gain Boosting Amplifier



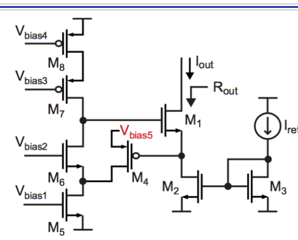
- Common source amplifier utilized

$$K = g_{m4} r_{o4},\; R_{ref} = r_{o2}$$

$$\Rightarrow\; R_{out} = (g_{m4} r_{o4})\,(g_{m1} r_{o1})\, r_{o2} \approx (g_m r_o)^2 r_{o2}$$

- Issue: current source requires significant headroom due to the fact that $V_{ds2} = V_{gs4}$

Folded Cascode Gain Boosting Amplifier



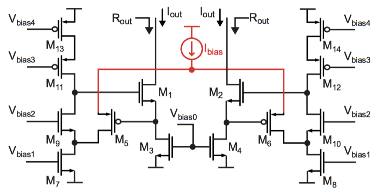
- Folded cascode yields

$$K = g_{m4}\,(((g_{m6} r_{o6}) r_{o5}) || ((g_{m7} r_{o7}) r_{o8}))$$

$$\Rightarrow\; R_{out} \approx (g_m r_o)^3 r_{o2}$$

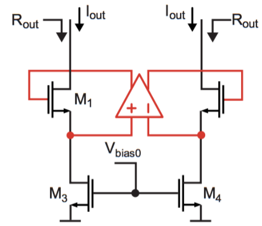
  - Improved headroom and higher gain!

Differential Version of Gain Boosting Amp



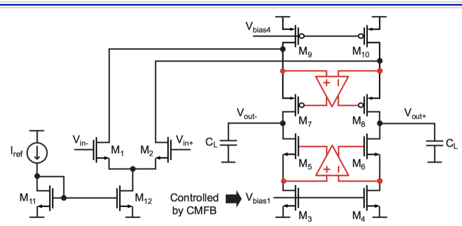
- Leverage fully differential nature of current sources within the opamp
  - PMOS gain devices are now part of a differential pair
  - Need CMFB to set common-mode gate voltages of M_1 and M_2 (I.e. to set V_{bias0})

Symbolic View of Folded Cascode Gain Boosting Amp



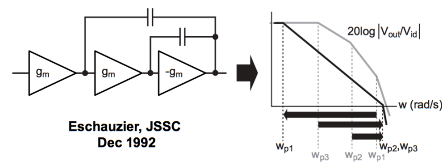
- We can apply this to the overall folded cascode opamp

Folded Cascode with Gain Boosting



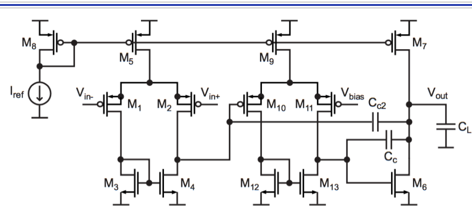
- Pro: Gain boosting provides substantial increase of DC gain while maintaining good input and output swing
  - Gain is on the order of $(g_m r_o)^4$
- Con: very complex!

Nested Miller Compensation



- Advantage: increased DC gain with high input and output swing
- Issue: more parasitic poles to deal with
  - Leads to lower unity gain bandwidth for reasonable phase margin

Nested Miller Example



- Intermediate gain stages must be non-inverting in order to achieve stable feedback
- Compensation resistors should also be included to eliminate the impact of RHP zeros
  - Not shown for simplicity