ESE 570: Digital Integrated Circuits and VLSI Fundamentals

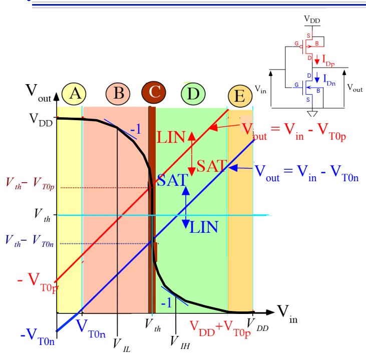
Lec 10: February 14, 2017

MOS Inverter: Dynamic Characteristics

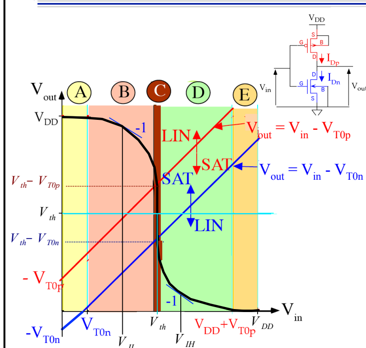
Lecture Outline

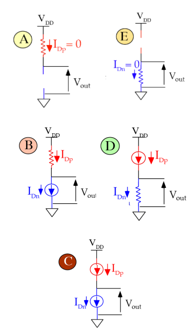
- Review: Symmetric CMOS Inverter Design
- Inverter Power
- Dynamic Characteristics
  - Delay

Review: CMOS Inverter: Visual VTC



Review: CMOS Inverter: Visual VTC





Review: CMOS Inverter: Design/Sizing

$$V_{th} = \frac{V_{T0n} + \sqrt{\frac{1}{k_R}}\left(V_{DD} + V_{T0p}\right)}{1 + \sqrt{\frac{1}{k_R}}} \;\Rightarrow\; k_R = \left(\frac{V_{DD} + V_{T0p} - V_{th}}{V_{th} - V_{T0n}}\right)^2$$

Important design Eq. for CMOS inverter VTC.

If V_{th} is set to ideal case: $V_{th} = \frac{1}{2}V_{DD}$

$$k_R = \left(\frac{V_{DD} + V_{T0p} - 1/2\,V_{DD}}{1/2\,V_{DD} - V_{T0n}}\right)^2 = \left(\frac{1/2\,V_{DD} + V_{T0p}}{1/2\,V_{DD} - V_{T0n}}\right)^2$$

If $V_{T0n} = -V_{T0p} = -V_{T0}$ (symmetric CMOS)

$$k_R = \left(\frac{V_{DD} + V_{T0p} - 1/2\,V_{DD}}{1/2\,V_{DD} - V_{T0n}}\right)^2 = \left(\frac{1/2\,V_{DD} + V_{T0}}{1/2\,V_{DD} + V_{T0}}\right)^2 = 1 \;\Rightarrow\; 1 = \frac{\mu_n W_n}{\mu_p W_p} \Rightarrow \frac{W_p}{W_n} = \frac{\mu_n}{\mu_p}$$

Review: Noise Margin Example

Compute the noise margins for a symmetric CMOS inverter has been designed to achieve $V_{th} = V_{DD}/2$, where $V_{DD} = 5$ V and $V_{T0n} = -V_{T0p} = 1$ V.

$NM_H = V_{OH} - V_{IH}$

$NM_L = V_{IL} - V_{OL}$

$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_{T0})$

$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_{T0})$

$NM_H = NM_L = 2.125$

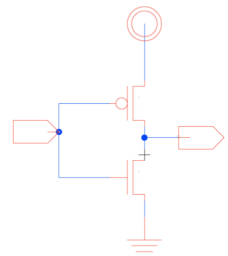
RECALL (with $V_{DD} = 5$ V)

1. Preferred Design => $NM_H, NM_L > V_{DD}/4 = 1.25$ V
2. Ideal NM => $NM_H = NM_L = 2.5\text{ V} > V_{DD}/2$

Inverter Power

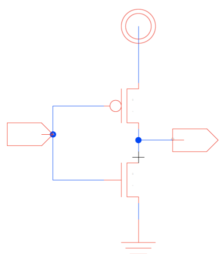
Power

- $P = I \times V$
- Tricky part:
  - Understanding I
  - (pairing with correct V)



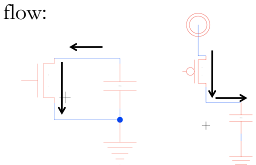
Static Current

- $P = I_{static} \times V_{DD}$

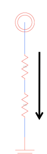


Switching Currents

- Dynamic current flow:

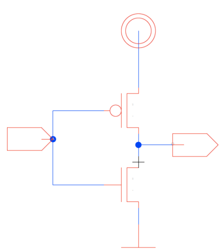


- If both transistor on:
  - Current path from V_{dd} to Gnd
  - Short circuit current



Currents Summary

- I changes over time
- At least two components
  - I_{static} – no switching
  - I_{switch} – when switching
    - I_{dyn} and I_{sc}



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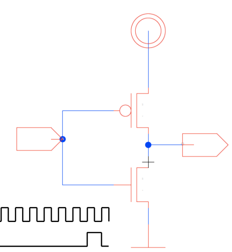
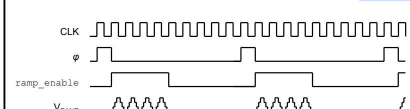


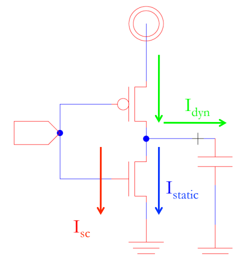


Switching

Dynamic Power

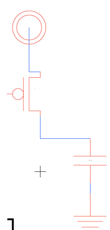
Switching Currents

- $I_{total}(t) = I_{static}(t) + I_{switch}(t)$
- $I_{switch}(t) = I_{sc}(t) + I_{dyn}(t)$



Charging

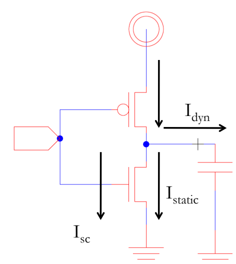
- $I_{dyn}(t)$ – why is it changing?
  - $I_{ds} = f(V_{ds}, V_{gs})$
  - and V_{gs}, V_{ds} changing



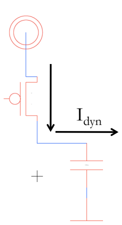
$$I_{DS} \approx v_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$I_{DS} = \mu_n C_{OX} \left(\frac{W}{L} \right) \left[\left(V_{GS} - V_T \right) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Switching Energy – focus on $I_{dyn}(t)$



Switching Energy – focus on $I_{dyn}(t)$

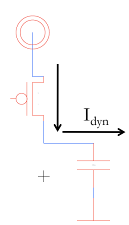


$$E = \int P(t)\,dt$$
$$= \int I(t) V_{dd}\,dt$$
$$= V_{dd} \int I(t)\,dt$$

Switching Energy

- Do we know what this is?

$$\int I_{dyn}(t)\,dt$$

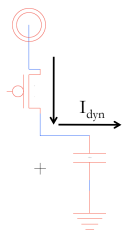


$$E = \int P(t)\,dt$$
$$= \int I(t) V_{dd}\,dt$$
$$= V_{dd} \int I(t)\,dt$$

Switching Energy

- Do we know what this is?

$$Q = \int I_{dyn}(t)dt$$

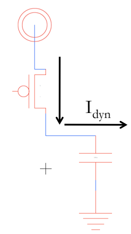


$$E = \int P(t)dt$$
$$= \int I(t)V_{dd}\,dt$$
$$= V_{dd}\int I(t)dt$$

Switching Energy

- Do we know what this is?

$$Q = \int I_{dyn}(t)dt$$



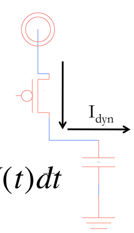
- What is Q?

$$E = \int P(t)dt$$
$$= \int I(t)V_{dd}\,dt$$
$$= V_{dd}\int I(t)dt$$

Switching Energy

- Do we know what this is?

$$Q = \int I_{dyn}(t)dt$$



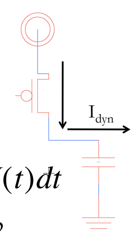
- What is Q?

$$E = \int P(t)dt \qquad Q = CV = \int I(t)dt$$
$$= \int I(t)V_{dd}\,dt$$
$$= V_{dd}\int I(t)dt$$

Switching Energy

- Do we know what this is?

$$Q = \int I_{dyn}(t)dt$$



- What is Q?

$$E = \int P(t)dt \qquad Q = CV = \int I(t)dt$$
$$= \int I(t)V_{dd}\,dt$$
$$= V_{dd}\int I(t)dt \qquad E = CV_{dd}^2$$

Capacitor charging energy

Switching Power

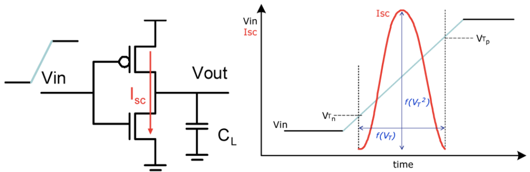
- Every time output switches 0→1 pay:
  - $E = CV^2$
- P_{dyn} = (# 0→1 trans) × CV^2 / time
- # 0→1 trans = ½ # of transitions
- P_{dyn} = (# trans) × ½CV^2 / time

Switching

Short Circuit Power

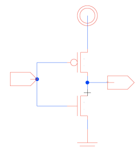
Short Circuit Power

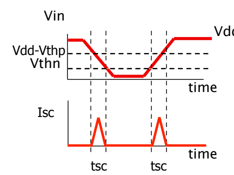
- Between V_{TN} and V_{dd} - V_{TP}
  - Both N and P devices conducting

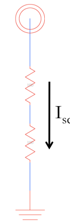


Short Circuit Power

- Between V_{TN} and V_{dd} - V_{TP}
  - Both N and P devices conducting
- Roughly:



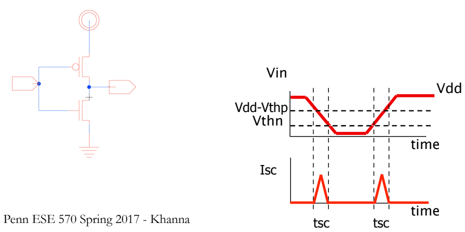




Peak Current

- I_{peak} around $V_{dd}/2$
  - If $|V_{TN}|=|V_{TP}|$ and sized equal rise/Spring

$$I_{DS} \approx v_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$



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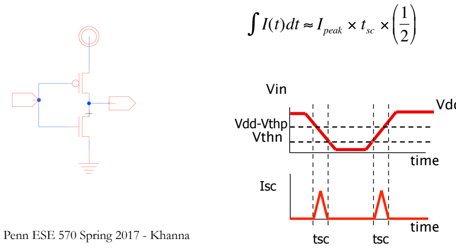
$$I_{DS} \approx v_{sat} C_{OX} W \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

$$\int I(t)dt \approx I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right)$$



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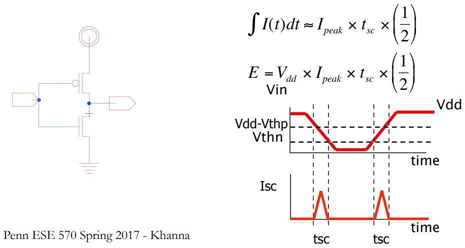
$$\int I(t)dt \approx I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right)$$

$$E = V_{dd} \times I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right)$$



Short Circuit Energy

- Make it look like a capacitance, C_{SC}
  - $Q=I \times t$
  - $Q=CV$

$$E = V_{dd} \times \left(I_{peak} \times t_{sc} \times \left(\frac{1}{2} \right) \right)$$

$$E = V_{dd} \times Q_{SC}$$

$$E = V_{dd} \times (C_{SC} V_{dd}) = C_{SC} V_{dd}^2$$

Short Circuit Energy

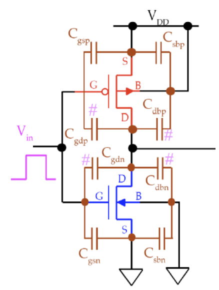
- Every time switch (0→1 and 1→0)
  - Also dissipate short-circuit energy: $E = CV^2$
  - Different $C = C_{sc}$
  - C_{cs} "fake" capacitance (for accounting)

Dynamic Characteristics

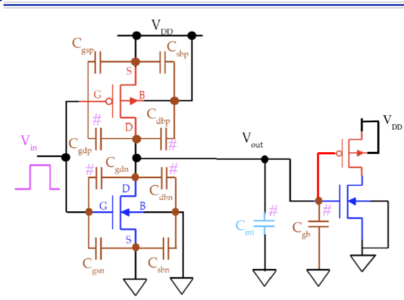
Inverter Delay

- Caused by charging and discharging the capacitive load
  - What is the load?

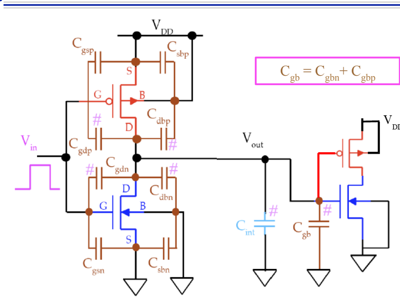
Inverter Delay



Inverter Delay

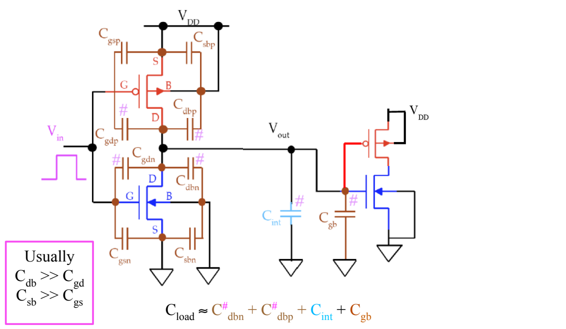


Inverter Delay



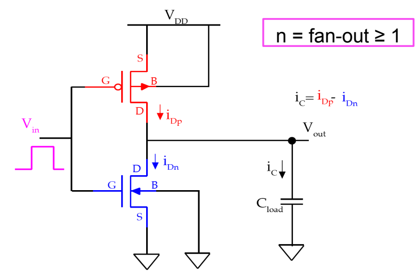
$$C_{load} = C^{\#}_{dbn} + C^{\#}_{dbp} + C^{\#}_{gdn} + C^{\#}_{gdp} + C_{int} + C_{gb}$$

Inverter Delay



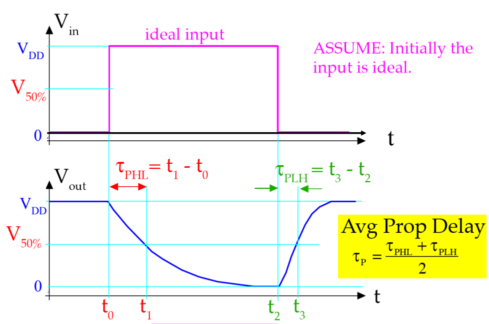
$C_{load} \approx C^{\#}_{dbn} + C^{\#}_{dbp} + C_{int} + C_{gb}$

Inverter Delay



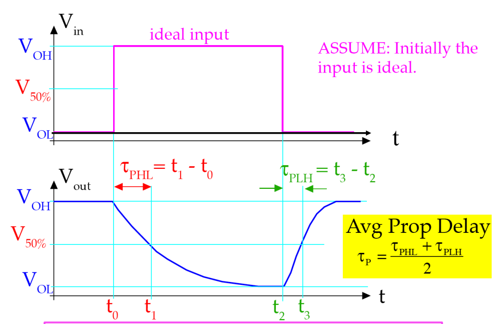
$C_{load} \approx C_{dbn} + C_{dbp} + C_{int} + nC_{gb}$

Propogation Delay Definitions



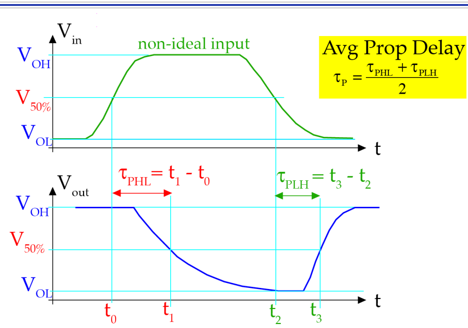
$V_{50\%} = V_{DD}/2$

Propogation Delay Definitions



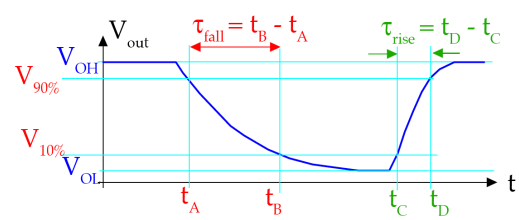
$V_{50\%} = V_{OL} + 0.5\,[V_{OH} - V_{OL}] = 0.5\,[V_{OL} + V_{OH}]$

Propogation Delay Definitions



$V_{50\%} = V_{OL} + 0.5\,[V_{OH} - V_{OL}] = 0.5\,[V_{OL} + V_{OH}]$

Rise/Spring Times



$V_{10\%} = V_{OL} + 0.1\,[V_{OH} - V_{OL}]$

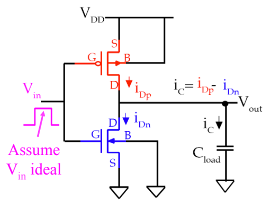
$V_{90\%} = V_{OL} + 0.9\,[V_{OH} - V_{OL}]$

MOS Inverter Dynamic Performance

- ANALYSIS (OR SIMULATION): For a given MOS inverter schematic and C_{load}, estimate (or measure) the propagation delays
- DESIGN: For given specs for the propagation delays and C_{load}^*, determine the MOS inverter schematic

METHODS:
1. Average Current Model

$$\tau_{PHL} \approx C_{load}\frac{\Delta V_{HL}}{I_{avg,HL}} = C_{load}\frac{V_{OH}-V_{50\%}}{I_{avg,HL}}$$
$$\tau_{PLH} \approx C_{load}\frac{\Delta V_{LH}}{I_{avg,LH}} = C_{load}\frac{V_{50\%}-V_{OL}}{I_{avg,LH}}$$



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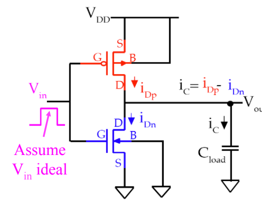
METHODS:
2. Differential Equation Model

$$i_C = C_{load}\frac{dV_{out}}{dt} \Rightarrow \int dt = C_{load}\int\frac{dV_{out}}{i_C}$$
$$dt \approx \tau_{PHL} \text{ or } \tau_{PLH}$$



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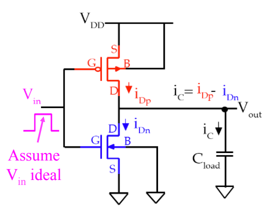
- DESIGN: For given specs for the propagation delays and C_{load}^*, determine the MOS inverter schematic

METHODS:
3. 1st Order RC delay Model

$$\tau_{PHL} \approx 0.69 \cdot C_{load} \cdot R_n$$
$$\tau_{PLH} \approx 0.69 \cdot C_{load} \cdot R_p$$

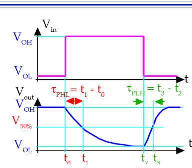


Method 1

Average Current Model

Calculation of Propagation Delays

$$\tau_{PHL} \approx C_{load}\frac{\Delta V_{HL}}{I_{avg,HL}} = C_{load}\frac{V_{OH}-V_{50\%}}{I_{avg,HL}}$$
$$\tau_{PLH} \approx C_{load}\frac{\Delta V_{LH}}{I_{avg,LH}} = C_{load}\frac{V_{50\%}-V_{OL}}{I_{avg,LH}}$$



Calculation of Propagation Delays

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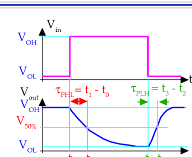


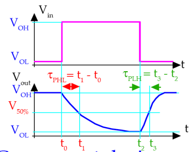
$I_{avg,HL}$ -> approximate average C_{load} current during high-to-low V_{out} transition

$$I_{avg,HL} = \frac{1}{2}\left[i_C(V_{in} = V_{OH}, V_{out} = V_{OH}) + i_C(V_{in} = V_{OH}, V_{out} = V_{50\%})\right]$$

Calculation of Propagation Delays

$$\tau_{PHL} \approx C_{load}\frac{\Delta V_{HL}}{I_{avg,HL}} = C_{load}\frac{V_{OH}-V_{50\%}}{I_{avg,HL}}$$

$$\tau_{PLH} \approx C_{load}\frac{\Delta V_{LH}}{I_{avg,LH}} = C_{load}\frac{V_{50\%}-V_{OL}}{I_{avg,LH}}$$



$I_{avg,HL}$ -> approximate average C_{load} current during high-to-low V_{out} transition

$$I_{avg,HL} = \frac{1}{2}\left[i_C(V_{in} = V_{OH}, V_{out} = V_{OH}) + i_C(V_{in} = V_{OH}, V_{out} = V_{50\%})\right]$$

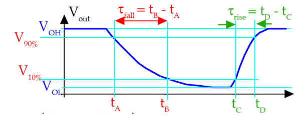
$I_{avg,LH}$ -> approximate average C_{load} current during low-to-high V_{out} transition

$$I_{avg,LH} = \frac{1}{2}\left[i_C(V_{in} = V_{OL}, V_{out} = V_{OL}) + i_C(V_{in} = V_{OL}, V_{out} = V_{50\%})\right]$$

Calculation of Rise/Spring Times

$$\tau_{fall} \approx C_{load}\frac{\Delta V_{90\%-10\%}}{I_{avg,90\%-10\%}} = C_{load}\frac{V_{90\%}-V_{10\%}}{I_{avg,90\%-10\%}}$$

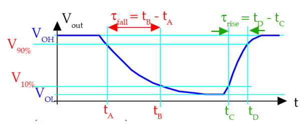
$$\tau_{rise} \approx C_{load}\frac{\Delta V_{10\%-90\%}}{I_{avg,10\%-90\%}} = C_{load}\frac{V_{90\%}-V_{10\%}}{I_{avg,10\%-90\%}}$$



Calculation of Rise/Spring Times

$$\tau_{fall} \approx C_{load}\frac{\Delta V_{90\%-10\%}}{I_{avg,90\%-10\%}} = C_{load}\frac{V_{90\%}-V_{10\%}}{I_{avg,90\%-10\%}}$$

$$\tau_{rise} \approx C_{load}\frac{\Delta V_{10\%-90\%}}{I_{avg,10\%-90\%}} = C_{load}\frac{V_{90\%}-V_{10\%}}{I_{avg,10\%-90\%}}$$



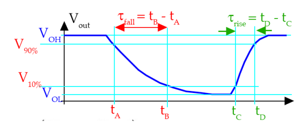
$I_{avg,90\text{-to-}10\%}$ -> approximate average C_{load} current during 90%-to-10% V_{out} transition

$$I_{avg,90-to-10\%} = \frac{1}{2}\left[i_C(V_{in} = V_{OH}, V_{out} = V_{90\%}) + i_C(V_{in} = V_{OH}, V_{out} = V_{10\%})\right]$$

Calculation of Rise/Spring Times

$$\tau_{fall} \approx C_{load}\frac{\Delta V_{90\%-10\%}}{I_{avg,90\%-10\%}} = C_{load}\frac{V_{90\%}-V_{10\%}}{I_{avg,90\%-10\%}}$$

$$\tau_{rise} \approx C_{load}\frac{\Delta V_{10\%-90\%}}{I_{avg,10\%-90\%}} = C_{load}\frac{V_{90\%}-V_{10\%}}{I_{avg,10\%-90\%}}$$



$I_{avg,90\text{-to-}10\%}$ -> approximate average C_{load} current during 90%-to-10% V_{out} transition

$$I_{avg,90-to-10\%} = \frac{1}{2}\left[i_C(V_{in} = V_{OH}, V_{out} = V_{90\%}) + i_C(V_{in} = V_{OH}, V_{out} = V_{10\%})\right]$$

$I_{avg,10\text{-to}90\%}$ -> approximate average C_{load} current during 10%-to-90% V_{out} transition

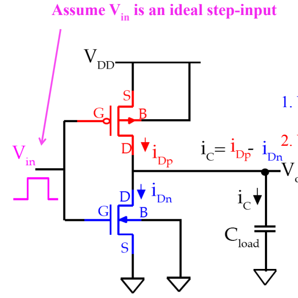
$$I_{avg,10-to-90\%} = \frac{1}{2}\left[i_C(V_{in} = V_{OL}, V_{out} = V_{10\%}) + i_C(V_{in} = V_{OL}, V_{out} = V_{90\%})\right]$$

Method 2

Differential Equation Model

Calculating Propagation Delays

Assume V_{in} is an ideal step-input



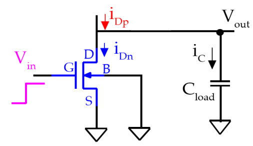
Two Cases

1. V_{in} abruptly rises => V_{out} Springs => τ_{PHL}
2. V_{in} abruptly Springs => V_{out} rises => τ_{PLH}

$$i_C = C_{load}\frac{dV_{out}}{dt} = i_{DP} - i_{Dn}$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

Case 1: V_{in} Abruptly Rises - τ_{PHL}

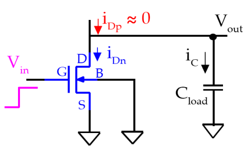


Case 1: V_{in} Abruptly Rises - τ_{PHL}

IC: $V_{out} = V_{DD}$, $V_{in} = 0 \text{ -> } V_{DD}$

nMOS - ON

p-MOS OFF



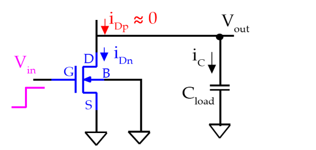
$$C_{load}\frac{dV_{out}}{dt} \approx -i_{Dn}$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

IC: $V_{out} = V_{DD}$, $V_{in} = 0 \text{ -> } V_{DD}$

nMOS - ON SAT $V_{out} \geqslant V_{DD} - V_{T0n}$

p-MOS OFF LIN $0 \leqslant V_{out} < V_{DD} - V_{T0n}$



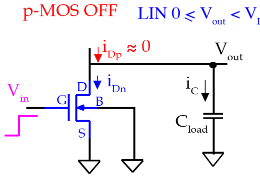
$$C_{load}\frac{dV_{out}}{dt} \approx -i_{Dn}$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

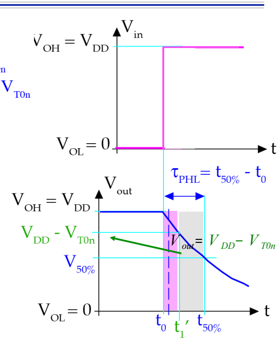
IC: $V_{out} = V_{DD}$, $V_{in} = 0 \text{ -> } V_{DD}$

nMOS - ON SAT $V_{out} \geqslant V_{DD} - V_{T0n}$

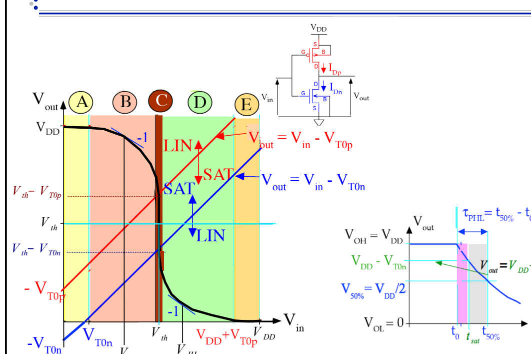
p-MOS OFF LIN $0 \leqslant V_{out} < V_{DD} - V_{T0n}$



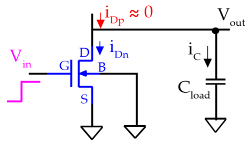
$$C_{load}\frac{dV_{out}}{dt} \approx -i_{Dn}$$

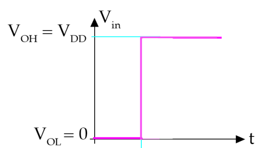


Recall: CMOS Inverter: Visual VTC



Case 1: V_{in} Abruptly Rises - τ_{PHL}





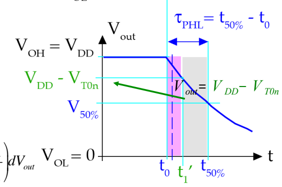
$$C_{load}\frac{dV_{out}}{dt} \approx -i_{Dn}$$

$$dt = C_{load}\frac{-dV_{out}}{i_{Dn}}$$

$$\tau_{PHL} = \int_{t=t_0}^{t=t_{50\%}} dt = C_{load}\int_{V_{out}=V_{DD}}^{V_{out}=V_{DD}/2}\left(\frac{-1}{i_{Dn}}\right)dV_{out}$$

$$\tau_{PHL} = C_{load}\int_{V_{DD}}^{V_{DD}-V_{T0n}}\left(\frac{-1}{i_{Dn}}\right)dV_{out} + C_{load}\int_{V_{DD}-V_{T0n}}^{V_{DD}/2}\left(\frac{-1}{i_{Dn}}\right)dV_{out}$$

$t_0 \rightarrow t_1$ $t_1 \rightarrow t_{50\%}$

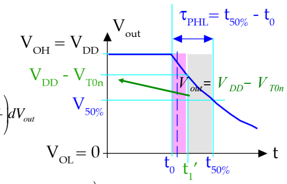


Case 1: V_{in} Abruptly Rises - τ_{PHL}

saturation   linear

$t_0 \rightarrow t_1$ $t_1 \rightarrow t_{50\%}$

$$\tau_{PHL} = C_{load}\int_{V_{DD}}^{V_{DD}-V_{T0n}}\left(\frac{-1}{i_{Dn}}\right)dV_{out} + C_{load}\int_{V_{DD}-V_{T0n}}^{V_{DD}/2}\left(\frac{-1}{i_{Dn}}\right)dV_{out}$$



saturation: $i_{Dn} = \frac{k_n}{2}(V_{in} - V_{T0n})^2$

$$\tau_{PHL,sat} = C_{load}\int_{V_{DD}}^{V_{DD}-V_{T0n}}\left(\frac{-1}{\frac{k_n}{2}(V_{DD}-V_{T0n})^2}\right)dV_{out}$$

$$\tau_{PHL,sat} = \frac{-C_{load}}{\frac{k_n}{2}(V_{DD}-V_{T0n})^2}\int_{V_{DD}}^{V_{DD}-V_{T0n}} dV_{out}$$

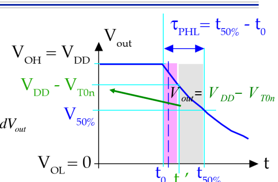
$$\boxed{\tau_{PHL,sat} = \frac{2C_{load}V_{T0n}}{k_n(V_{DD}-V_{T0n})^2}}$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

saturation   linear

$t_0 \rightarrow t_1$ $t_1 \rightarrow t_{50\%}$

$$\tau_{PHL} = C_{load}\int_{V_{DD}}^{V_{DD}-V_{T0n}}\left(\frac{-1}{i_{Dn}}\right)dV_{out} + C_{load}\int_{V_{DD}-V_{T0n}}^{V_{DD}/2}\left(\frac{-1}{i_{Dn}}\right)dV_{out}$$



linear: $i_{Dn} = \frac{k_n}{2}\left((V_{in} - V_{T0n})V_{out} - V_{out}^2\right)$

$$\tau_{PHL,lin} = C_{load}\int_{V_{DD}-V_{T0n}}^{V_{DD}/2}\left(\frac{-1}{\frac{k_n}{2}\left(2(V_{DD}-V_{T0n})V_{out}-V_{out}^2\right)}\right)dV_{out}$$

$$\tau_{PHL,lin} = \frac{2C_{load}}{k_n}\int_{V_{DD}-V_{T0n}}^{V_{DD}/2}\left(\frac{-1}{\left(2(V_{DD}-V_{T0n})V_{out}-V_{out}^2\right)}\right)dV_{out}$$

$$\tau_{PHL,lin} = \frac{2C_{load}}{k_n}\cdot\frac{-1}{2(V_{DD}-V_{T0n})}\ln\left(\frac{V_{out}}{(2(V_{DD}-V_{T0n})-V_{out})}\right)\Bigg|_{V_{out}=V_{DD}-V_{T0n}}^{V_{out}=V_{DD}/2}$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

$$\tau_{PHL,lin} = \frac{2C_{load}}{k_n}\cdot\frac{-1}{2(V_{DD}-V_{T0n})}\ln\left(\frac{V_{out}}{(2(V_{DD}-V_{T0n})-V_{out})}\right)\Bigg|_{V_{out}=V_{DD}-V_{T0n}}^{V_{out}=V_{DD}/2}$$

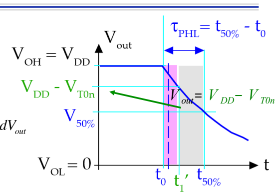
$$\boxed{\tau_{PHL,lin} = \frac{C_{load}}{k_n(V_{DD}-V_{T0n})}\ln\left(\frac{2(V_{DD}-V_{T0n})-V_{DD}/2}{V_{DD}/2}\right)}$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

saturation   linear

$t_0 \rightarrow t_1$ $t_1 \rightarrow t_{50\%}$

$$\tau_{PHL} = C_{load}\int_{V_{DD}}^{V_{DD}-V_{T0n}}\left(\frac{-1}{i_{Dn}}\right)dV_{out} + C_{load}\int_{V_{DD}-V_{T0n}}^{V_{DD}/2}\left(\frac{-1}{i_{Dn}}\right)dV_{out}$$



$$\tau_{PHL,sat} = \frac{2C_{load}V_{T0n}}{k_n(V_{DD}-V_{T0n})^2} \qquad \tau_{PHL,lin} = \frac{C_{load}}{k_n(V_{DD}-V_{T0n})}\ln\left(\frac{2(V_{DD}-V_{T0n})-V_{DD}/2}{V_{DD}/2}\right)$$

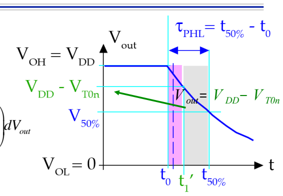
$$\boxed{\tau_{PHL} = \frac{2C_{load}V_{T0n}}{k_n(V_{DD}-V_{T0n})^2} + \frac{C_{load}}{k_n(V_{DD}-V_{T0n})}\ln\left(\frac{2(V_{DD}-V_{T0n})-V_{DD}/2}{V_{DD}/2}\right)}$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

saturation   linear

$t_0 \rightarrow t_1$ $t_1 \rightarrow t_{50\%}$

$$\tau_{PHL} = C_{load}\int_{V_{DD}}^{V_{DD}-V_{T0n}}\left(\frac{-1}{i_{Dn}}\right)dV_{out} + C_{load}\int_{V_{DD}-V_{T0n}}^{V_{DD}/2}\left(\frac{-1}{i_{Dn}}\right)dV_{out}$$



$$\tau_{PHL,sat} = \frac{2C_{load}V_{T0n}}{k_n(V_{DD}-V_{T0n})^2} \qquad \tau_{PHL,lin} = \frac{C_{load}}{k_n(V_{DD}-V_{T0n})}\ln\left(\frac{2(V_{DD}-V_{T0n})-V_{DD}/2}{V_{DD}/2}\right)$$

$$\tau_{PHL} = \frac{2C_{load}V_{T0n}}{k_n(V_{DD}-V_{T0n})^2} + \frac{C_{load}}{k_n(V_{DD}-V_{T0n})}\ln\left(\frac{2(V_{DD}-V_{T0n})-V_{DD}/2}{V_{DD}/2}\right)$$

$\propto R_n$

$$\tau_{PHL} = C_{load}\left(\frac{1}{k_n(V_{DD}-V_{T0n})}\left[\frac{2V_{T0n}}{(V_{DD}-V_{T0n})} + \ln\left(\frac{2(V_{DD}-V_{T0n})}{V_{DD}/2} - 1\right)\right]\right)$$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

$$\tau_{PHL} = C_{load} \frac{1}{k_n (V_{DD} - V_{T0n})} \left[\frac{2V_{T0n}}{(V_{DD} - V_{T0n})} + \ln\left(\frac{2(V_{DD} - V_{T0n})}{V_{DD}/2} - 1 \right) \right]$$

Recall from static CMOS Inverter:

$$V_{th} = \frac{V_{T0n} + \sqrt{\frac{1}{k_R}}(V_{DD} + V_{T0p})}{1 + \sqrt{\frac{1}{k_R}}} \longrightarrow k_R = \left(\frac{V_{DD} + V_{T0p} - V_{th}}{V_{th} - V_{T0n}} \right)^2$$

DESIGN: (1) $V_{th} \rightarrow k_R$; (2) $\tau_{PHL} \rightarrow k_n$; (3) k_R & $k_n \rightarrow k_p$

Case 1: V_{in} Abruptly Rises - τ_{PHL}

$$\tau_{PHL} = C_{load} \frac{1}{k_n (V_{DD} - V_{T0n})} \left[\frac{2V_{T0n}}{(V_{DD} - V_{T0n})} + \ln\left(\frac{2(V_{DD} - V_{T0n})}{V_{DD}/2} - 1 \right) \right]$$

Recall from static CMOS Inverter:

$$V_{th} = \frac{V_{T0n} + \sqrt{\frac{1}{k_R}}(V_{DD} + V_{T0p})}{1 + \sqrt{\frac{1}{k_R}}} \longrightarrow k_R = \left(\frac{V_{DD} + V_{T0p} - V_{th}}{V_{th} - V_{T0n}} \right)^2$$

DESIGN: (1) $V_{th} \rightarrow k_R$; (2) $\tau_{PHL} \rightarrow k_n$; (3) k_R & $k_n \rightarrow k_p$

(1) $V_{th} \rightarrow k_R$; (2) $\tau_{PLH} \rightarrow k_p$; (3) k_R & $k_p \rightarrow k_n$

Idea

- $P_{tot} = P_{static} + P_{dyn} + P_{sc}$
  - Can't ignore Static Power (aka. Leakage power)
- Propogation Delay
  - Average Current Model
  - Differential Equation Model
  - 1st Order Model

Admin

- HW 4 due Thursday, 2/16