Forecasting for Swap Regret for All Downstream Agents

Aaron Roth, Mirah Shi University of Pennsylvania



Predictions for Decision-Making

Agents take **actions** based on **predictions**

Time 11	gents	Outcome $\mathcal{Y}_t \in [0,1]^d$
	$\bigvee_{t} a_t^1(p_t) -$	$\longrightarrow u^1(a_t^1, y_t)$
$ \begin{array}{c c} \text{Prediction} \\ \hline \\ \text{Alg} \end{array} \xrightarrow{p_t} \in [0,1]^d \end{array} \end{array} $	$\bigvee_{t} \frac{a_t^2(p_t)}{a_t} -$	$\longrightarrow u^2(a_t^2, y_t)$
	$\bigvee_{t} a_t^3(p_t) -$	$\longrightarrow u^3(a_t^3, y_t)$

Question: How do we make online predictions that are simultaneously valuable to any downstream decision-making agent?

Our Results (Informal)

We show how to make predictions so that any downstream agent who best responds has swap regret:

- $\tilde{O}(\sqrt{T})$ for d = 1 (Focus of this poster) - $\tilde{O}(T^{5/8})$ for d = 2

and any agent who smoothly best responds has swap regret:

- $\tilde{O}(T^{2/3})$ for d > 2

no matter what their utility function is. Takeaway: can minimize downstream swap regret without requiring calibrated forecasts!

- Potentially many agents with different utilities
- Learner may not know agents' utility functions

Goal: Guarantee **low swap regret*** for any downstream agent, regardless of utility function

* "Every time I played action *a*, I wouldn't have wanted to play action *b* in hindsight."

Why swap regret? Useful in strategic settings, e.g. :

- Convergence to correlated equilibria
- Strategy-robustness in repeated games

The Story (Prior to This Work)

<u>Calibration</u>* is one solution [Foster and Vohra '98]

* Unbiasedness conditional on value of prediction itself

Major Drawbacks...

Key Idea

For a fixed agent, enough for predictions to be unbiased estimates conditional on their **best response regions*** [NRRX '23]

* Predictions *p* inducing a best response of *a*

But we don't know agents' best response regions...

Structural property: Best response regions are convex!

In 1 dimension: convex sets correspond to subintervals of [0,1]

- Not too many after discretizing predictions

d = 1 :

Best response region



Swap regret rates degrade poorly with d- $\Omega(T^{0.528})$ lower bound on 1-d calibration error [Qiao & Valiant '21] but can achieve $O(\sqrt{T})$ swap regret for a single agent [Blum & Mansour '07]

Other previously known solutions for:

- a weaker benchmark: no *external* regret [KPST '23]
- a *fixed* collection of agents [NRRX '23] -

Q: Can we circumvent calibration to achieve no swap regret for any agent, regardless of their utility? Our work: Yes!





m predictions m^2 possible best response regions

Algorithm: Make predictions that are unbiased conditional on lying in all sub-intervals - Use unbiased prediction algorithm of NRRX '23

Beyond Low Dimensions

Complexity scales in higher dimensions... **New approach:** Consider best response regions of a discretized set of utility functions

- Challenge: best response function is discontinuous \longrightarrow require smooth approx.