

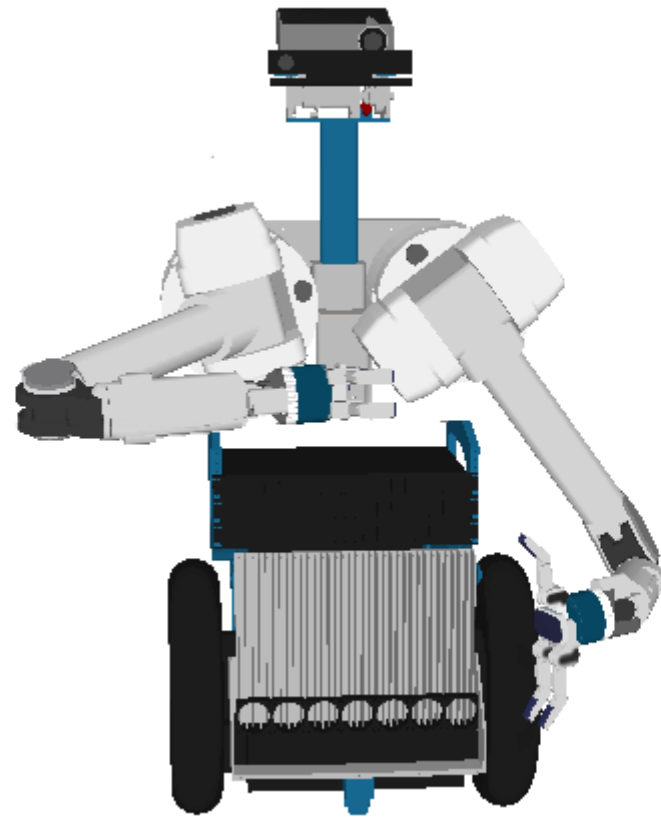
# Distance Metrics and Algorithms for Task Space Path Optimization

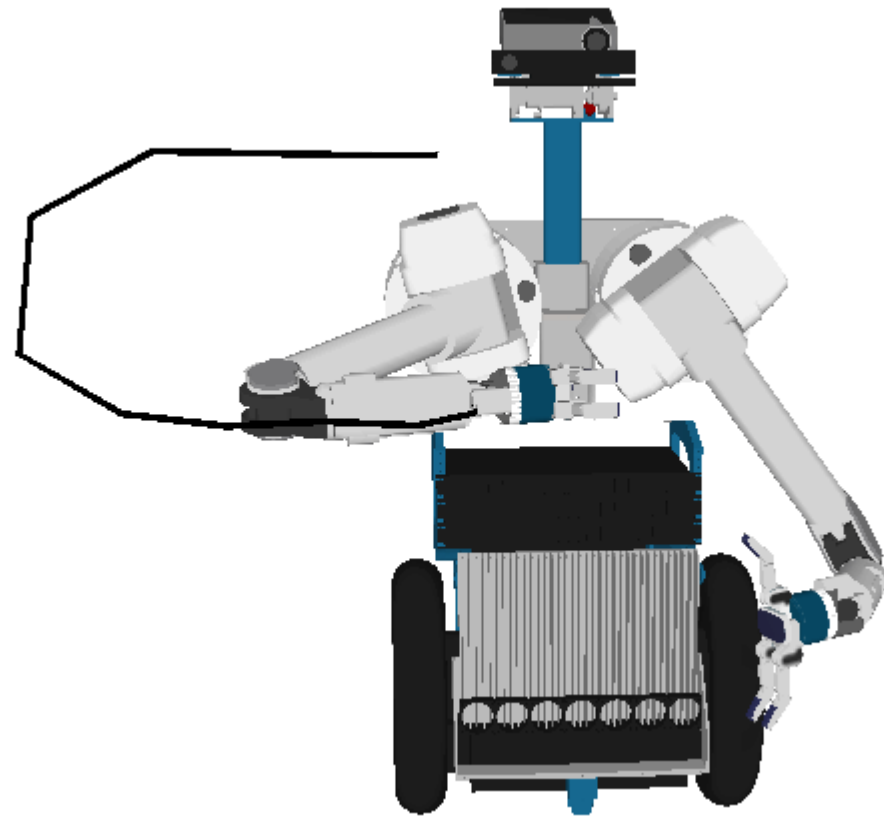
Rachel Holladay

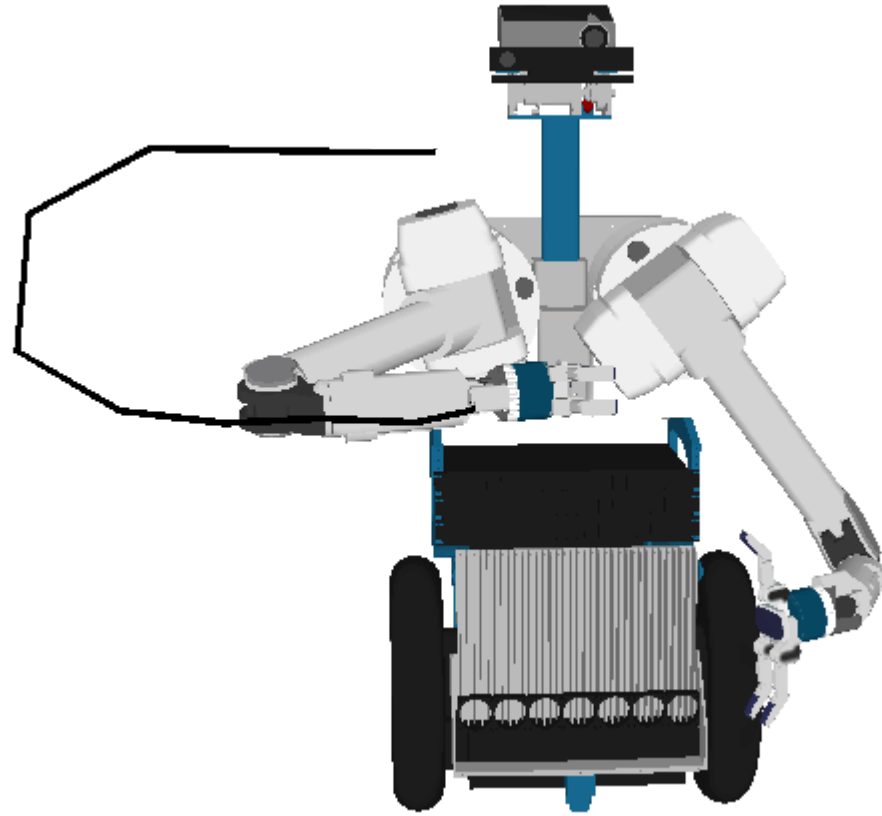
Siddhartha Srinivasa

The Robotics Institute  
Carnegie Mellon University

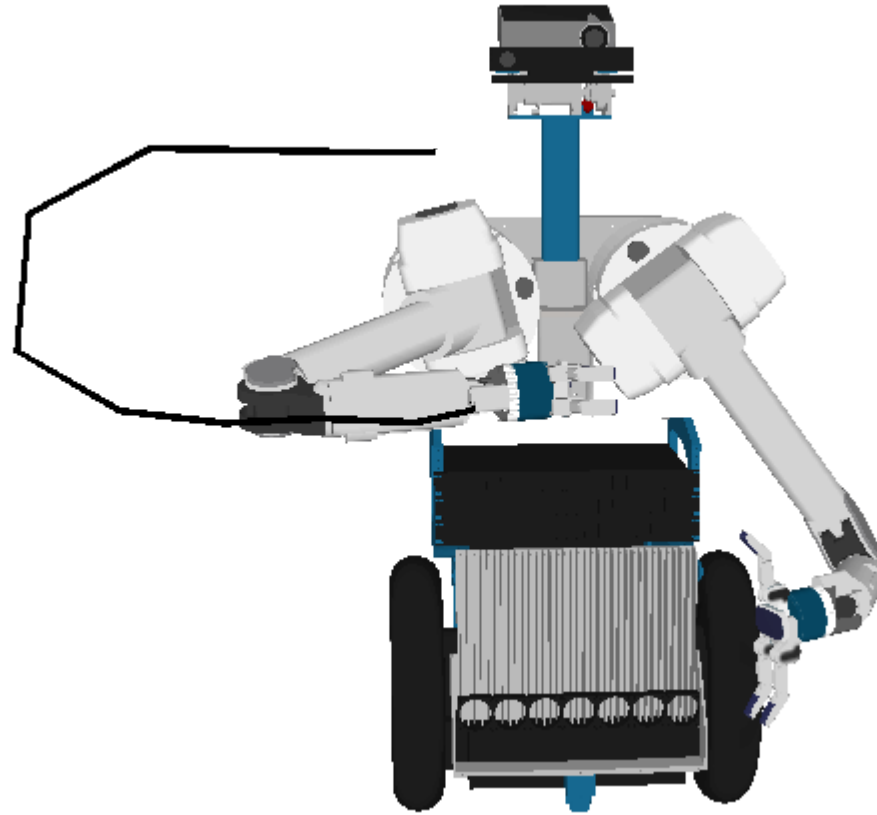








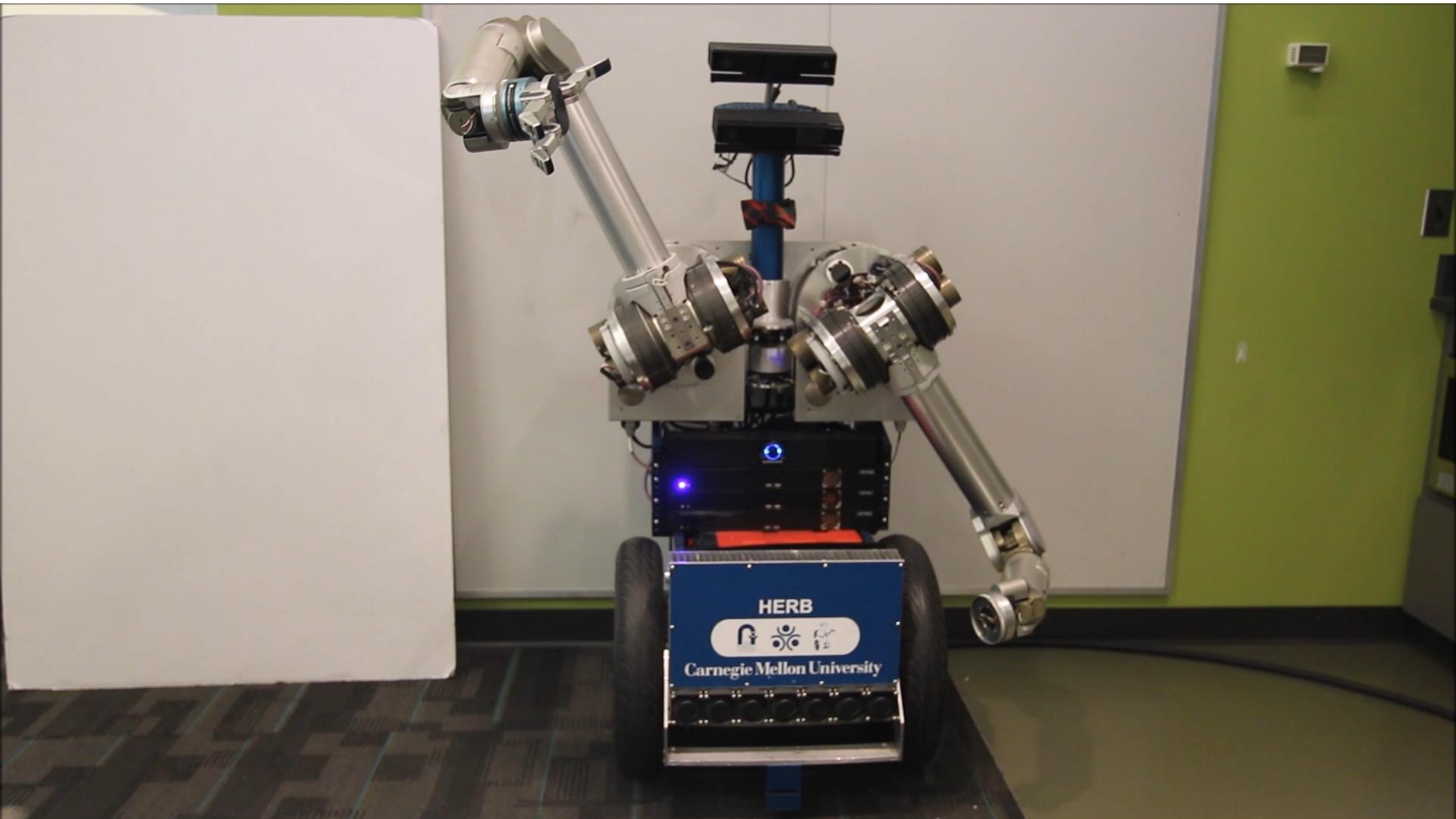
Goal: Follow End-Effector Path  
in Task Space



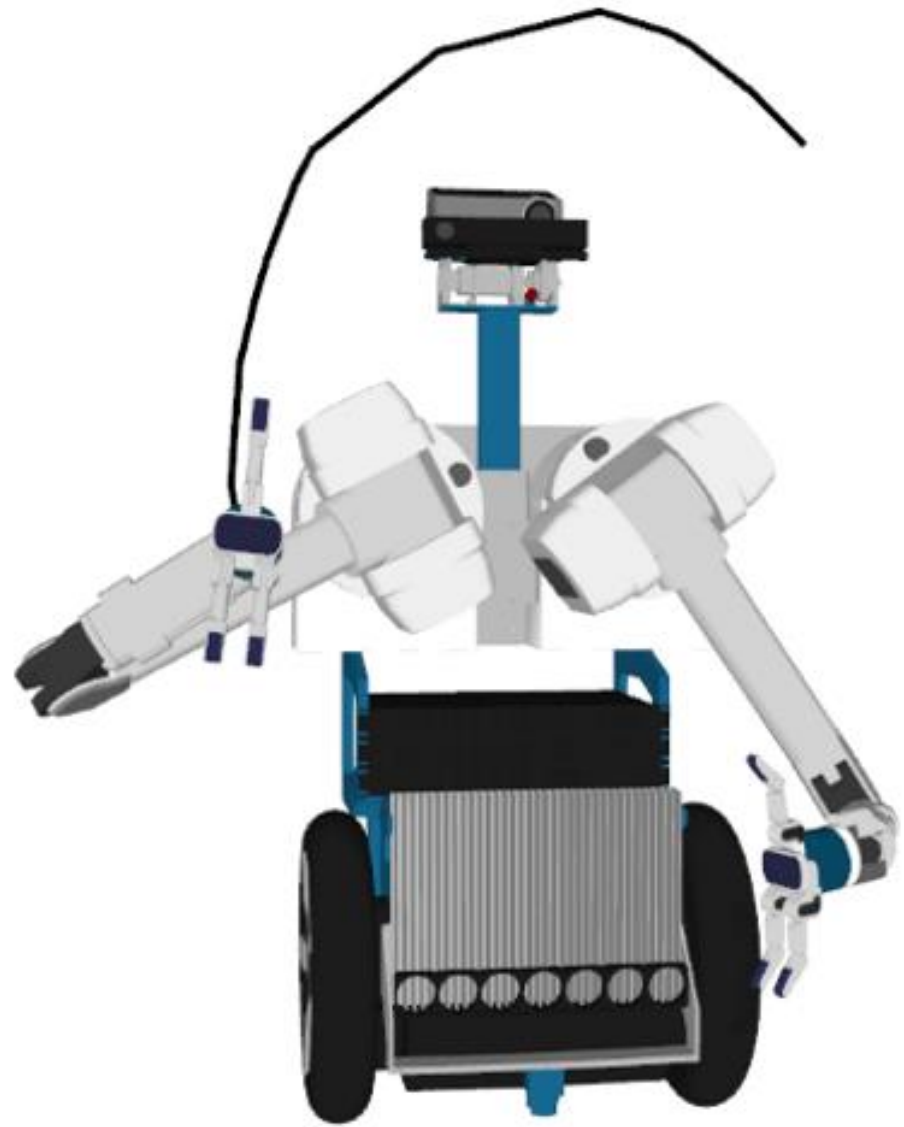
Goal: Follow End-Effector Path  
in Task Space  
*subject to constraints.*



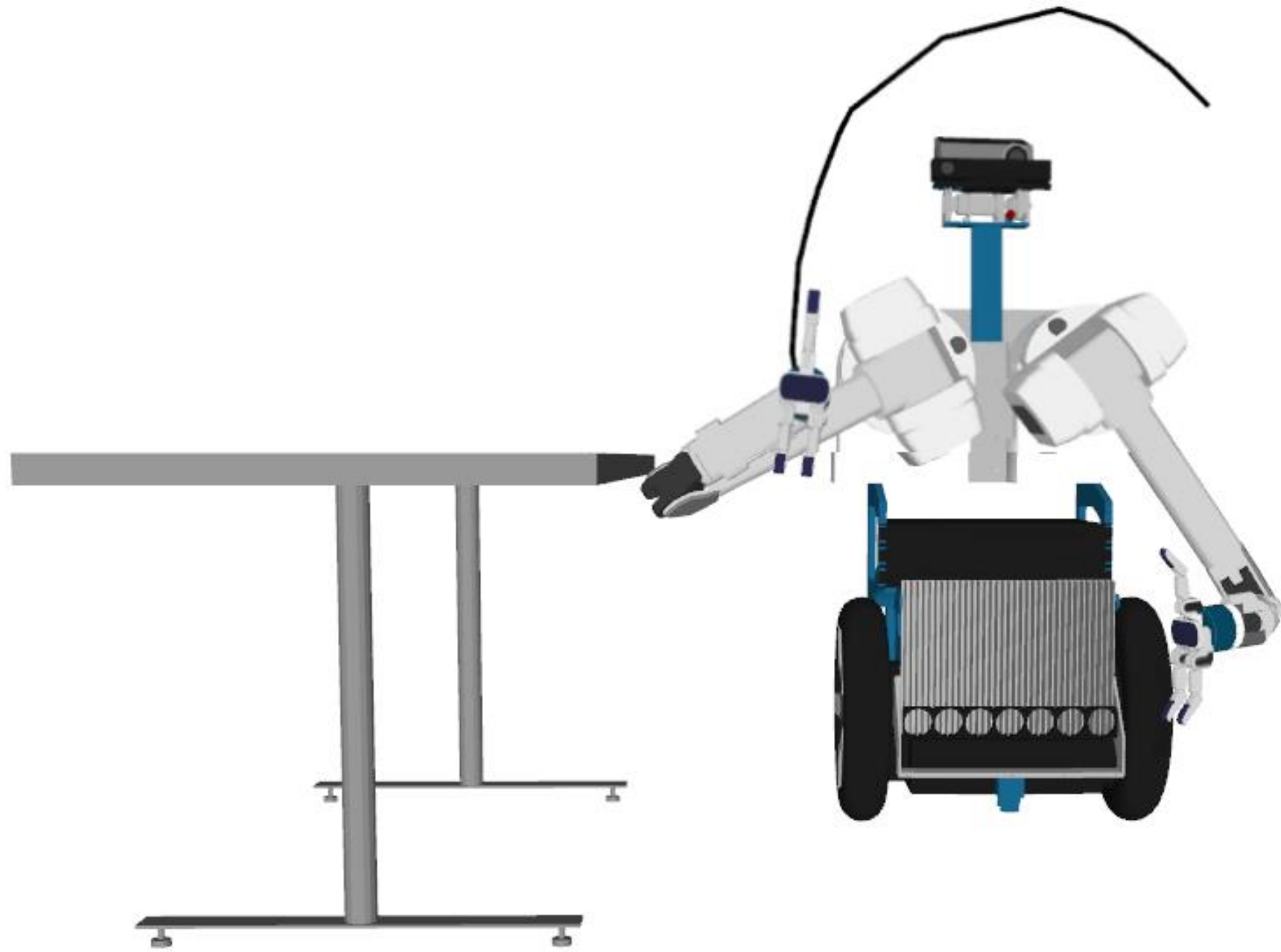
HERB  
Carnegie Mellon University

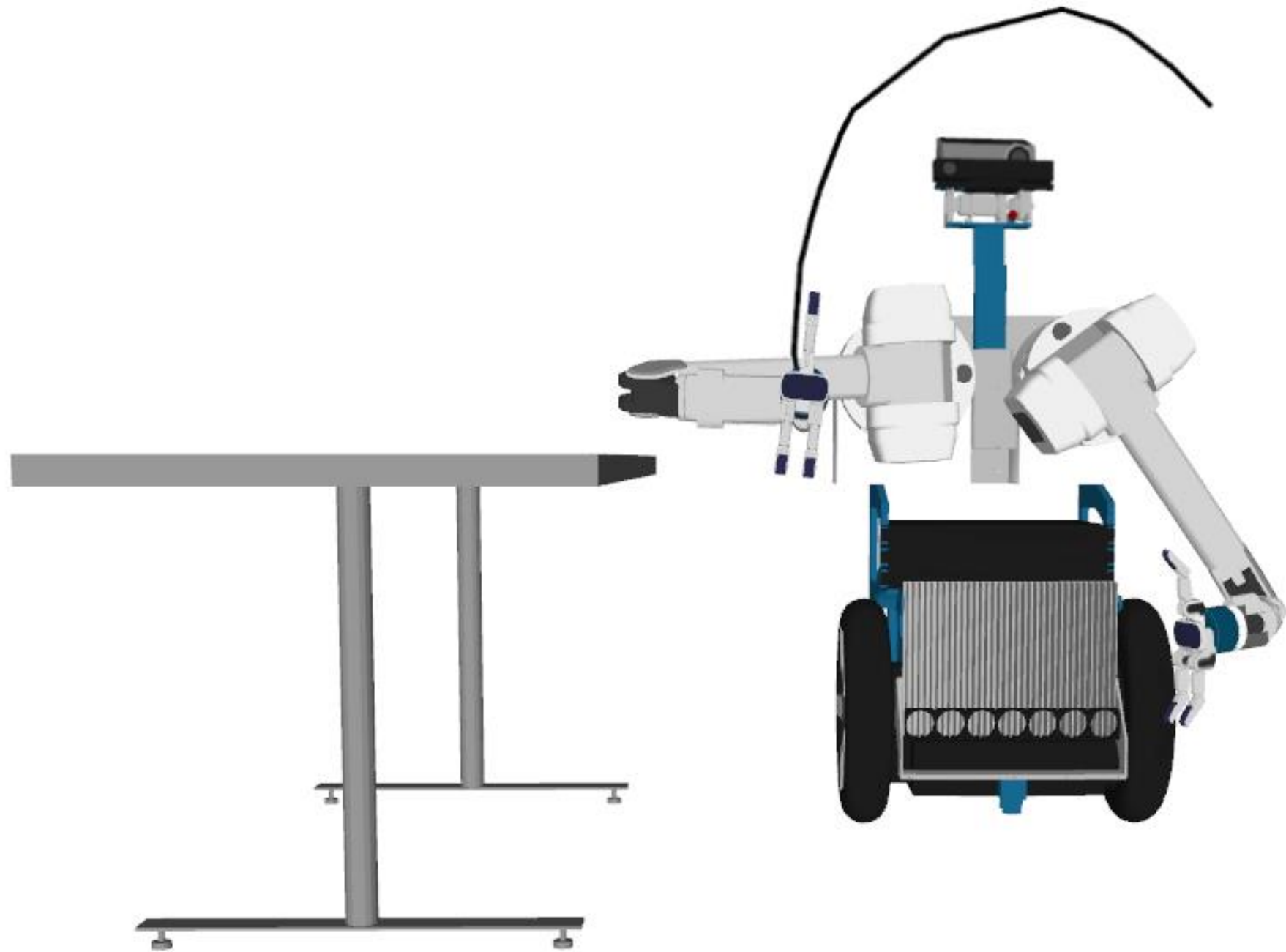


HERB  
Carnegie Mellon University









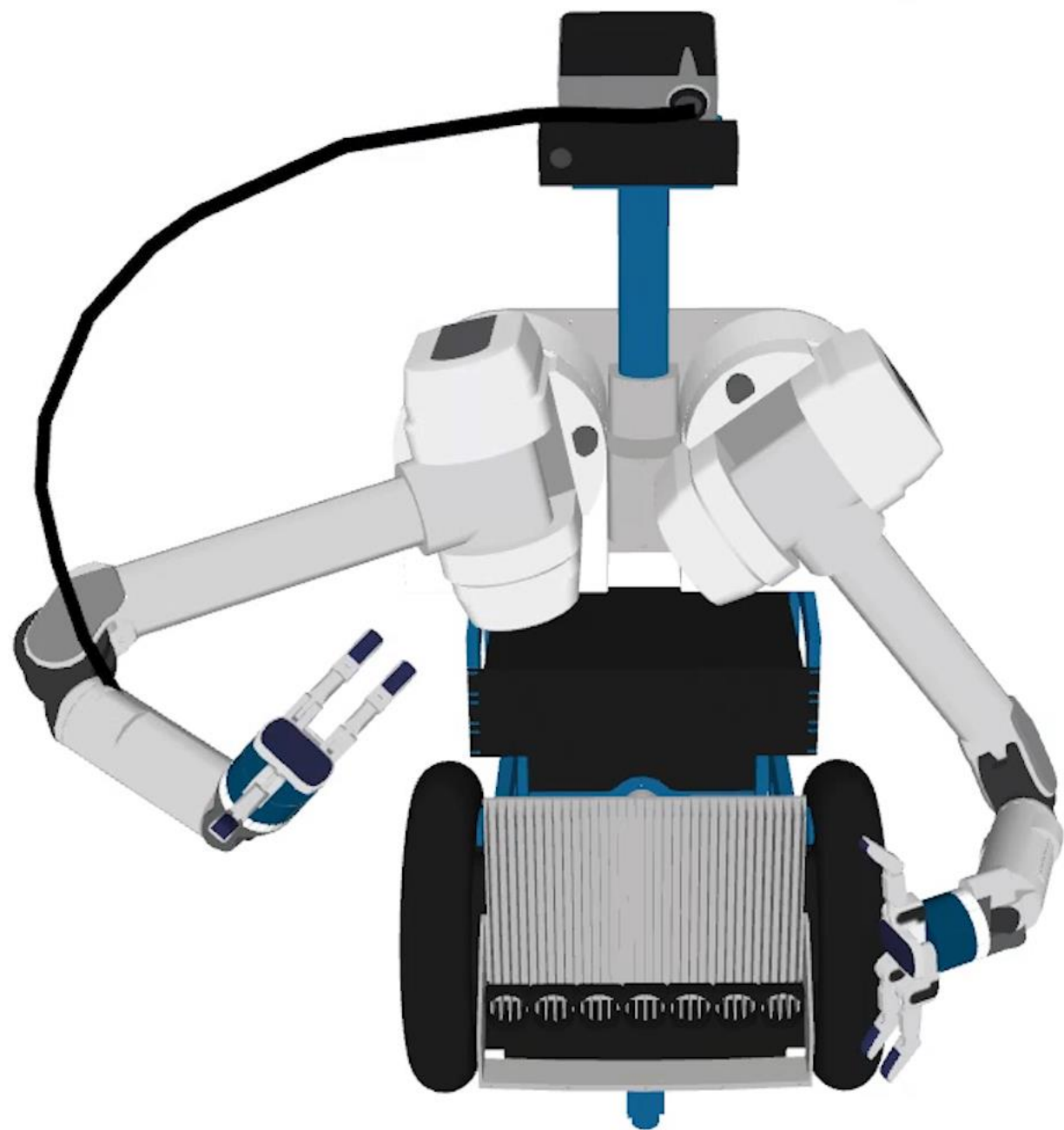
Goal:  
Follow a Reference Path

Goal:

Follow a Reference Path  
*by leveraging motion planning.*







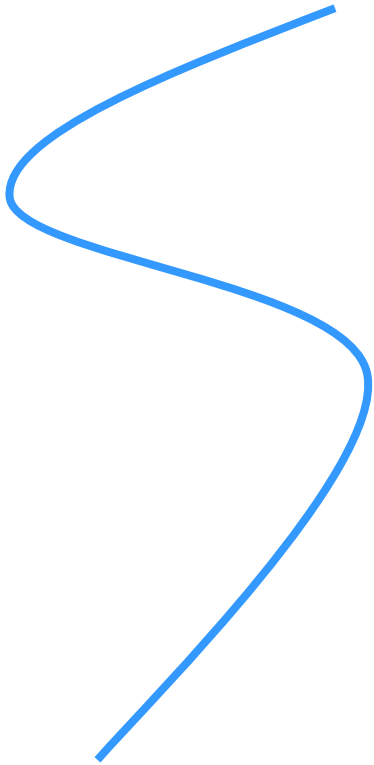
## *Key Insight*

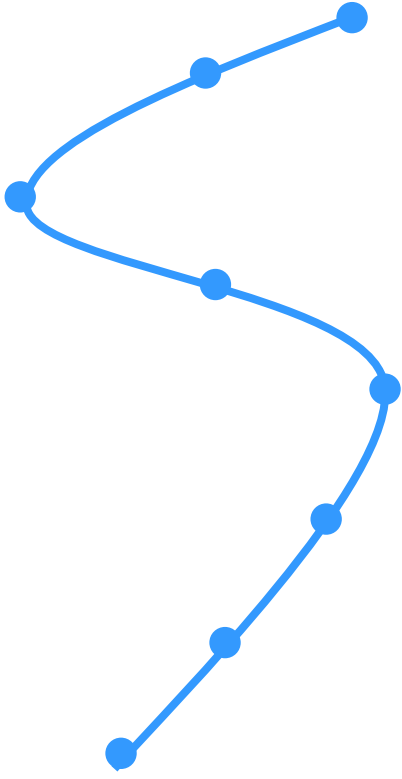
*Use trajectory optimization to optimize our path to be close to our reference path.*

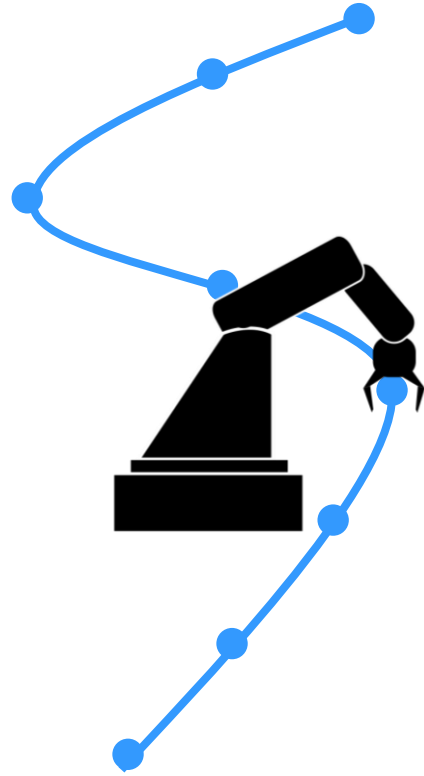


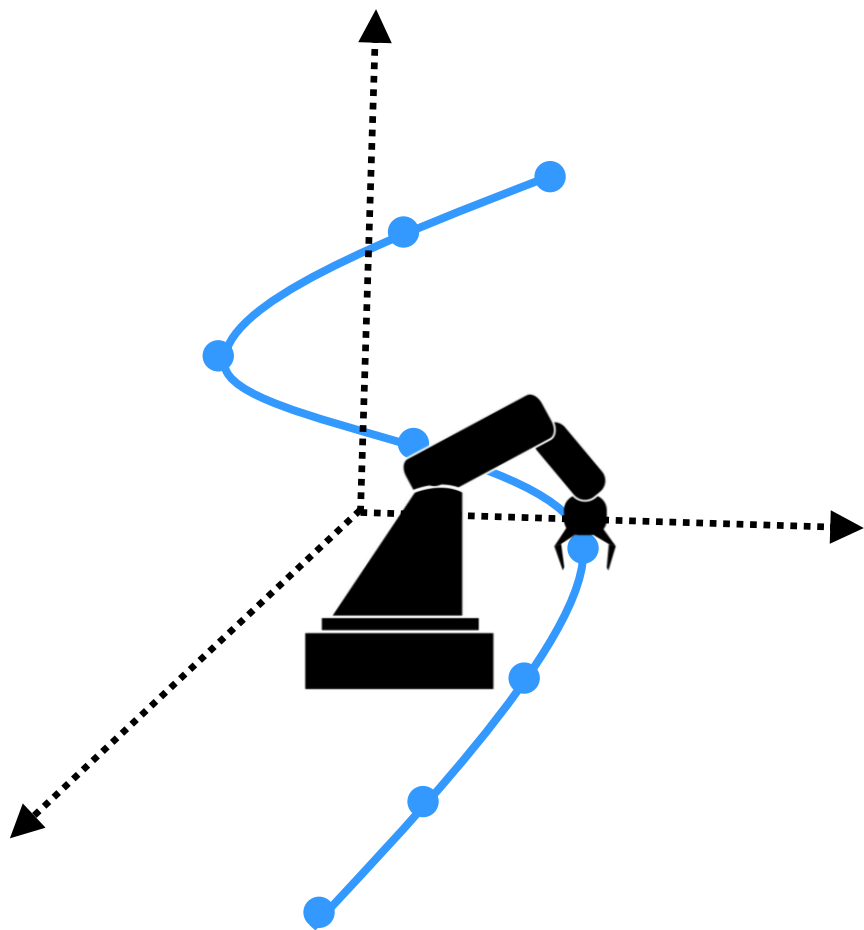
## *Key Insight*

*Use trajectory optimization to optimize our path to be **close** to our reference path.*

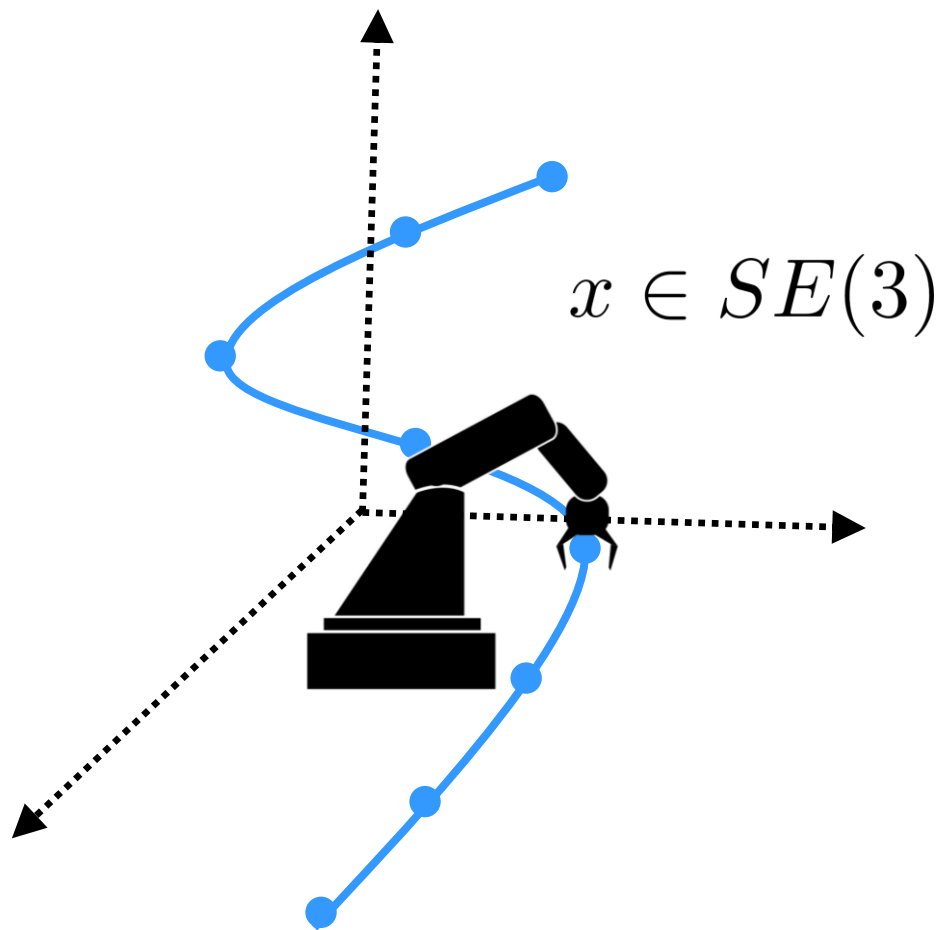




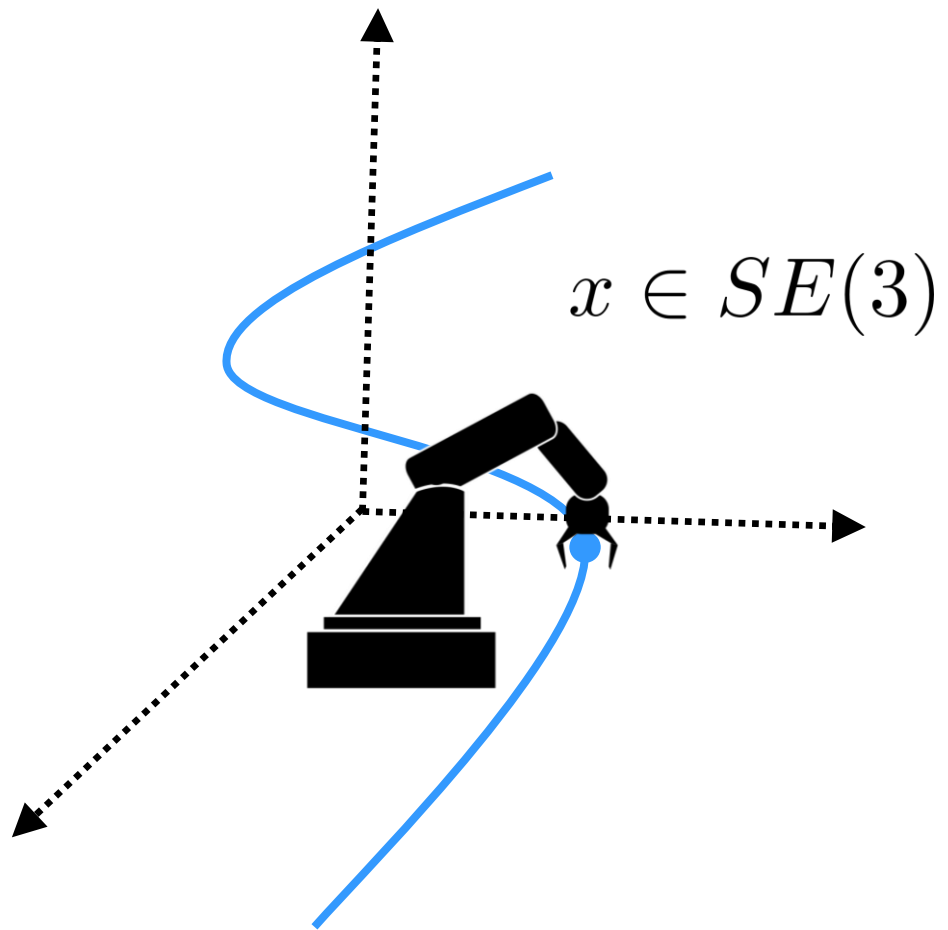




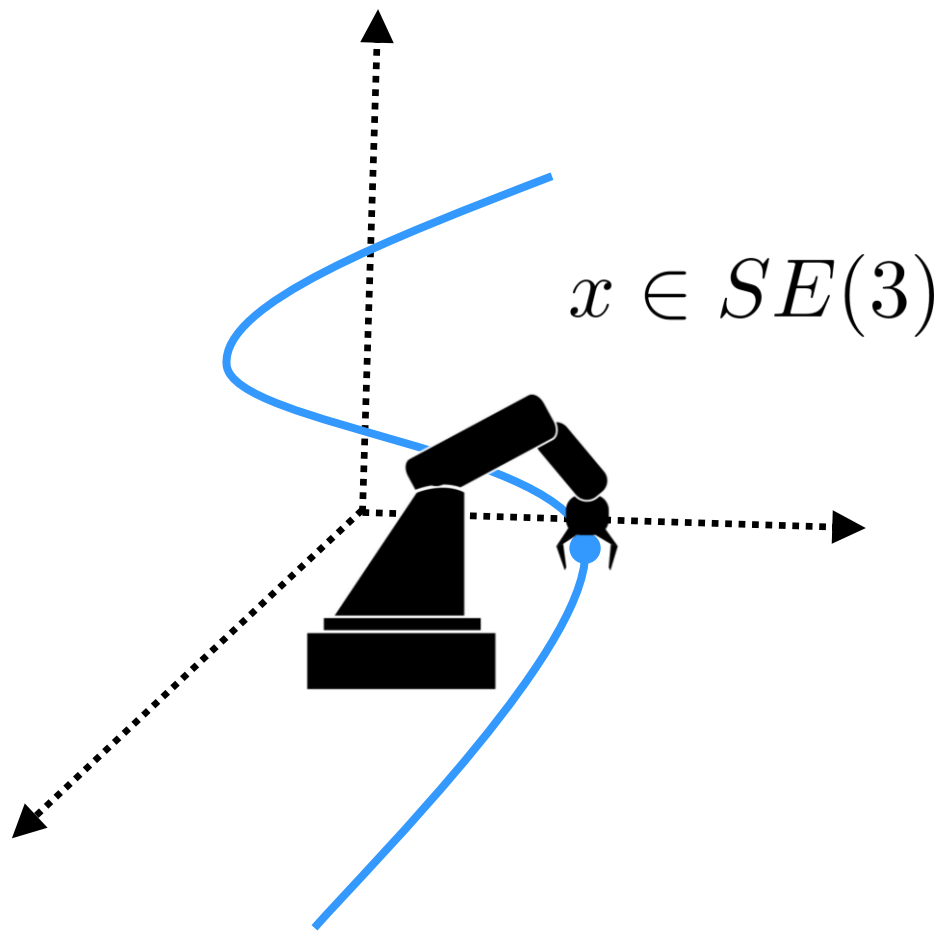
Task Space



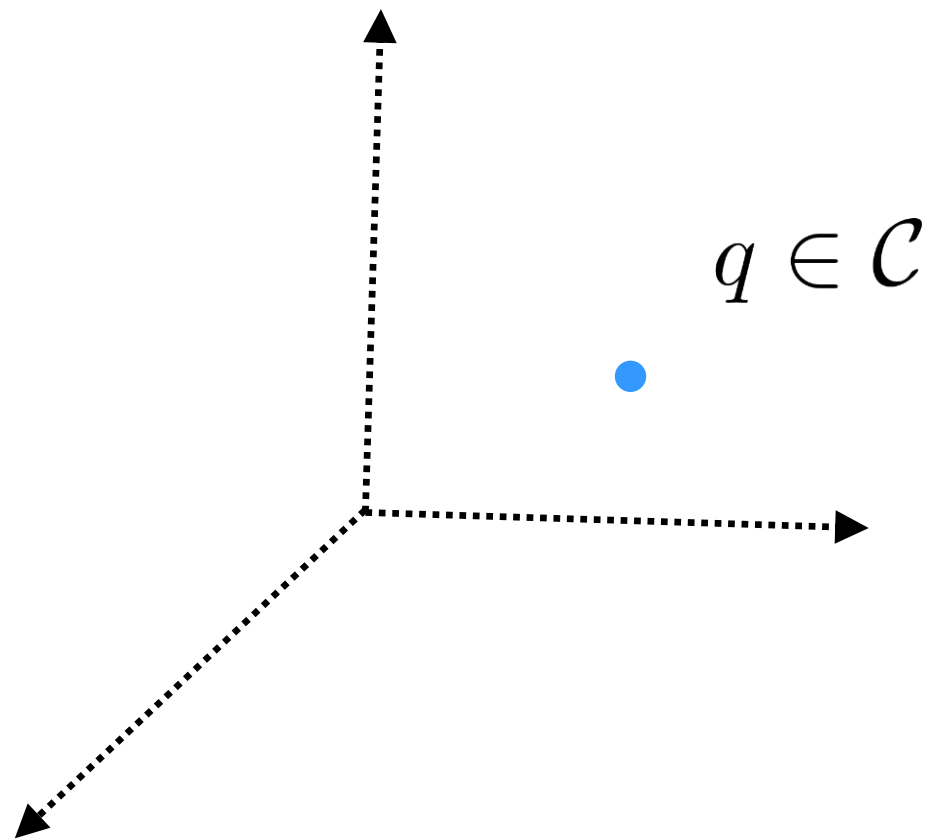
Task Space



Task Space

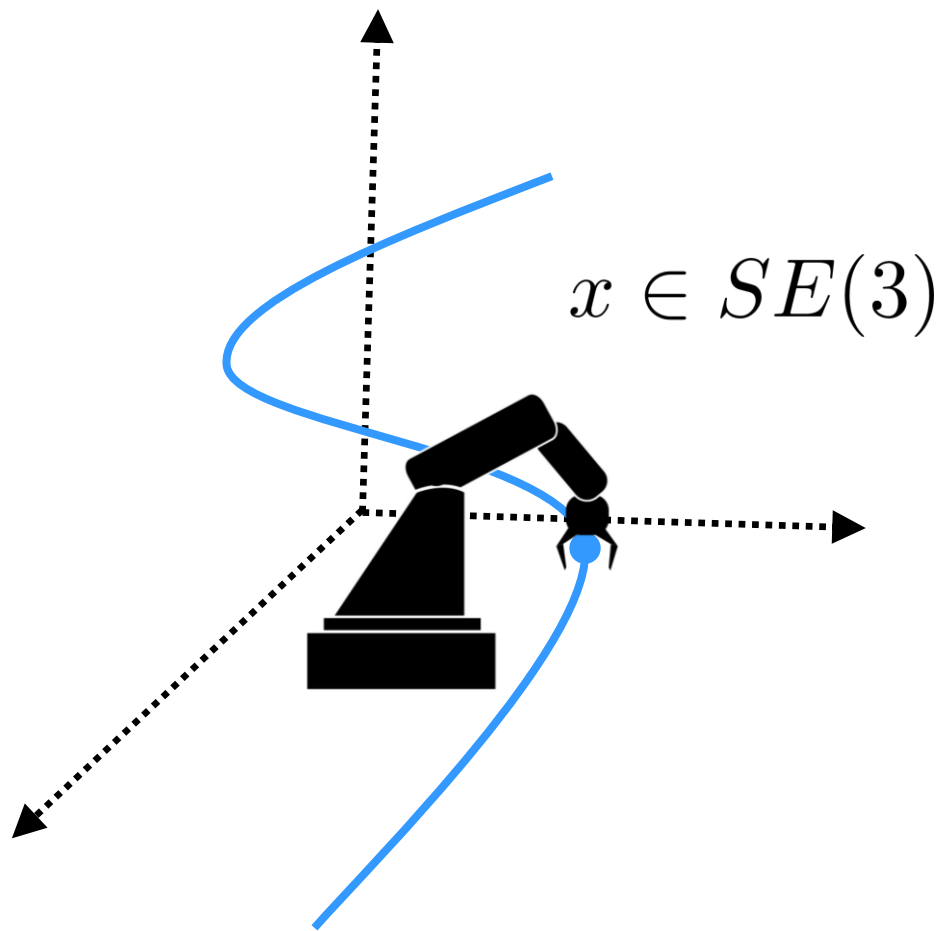


Task Space

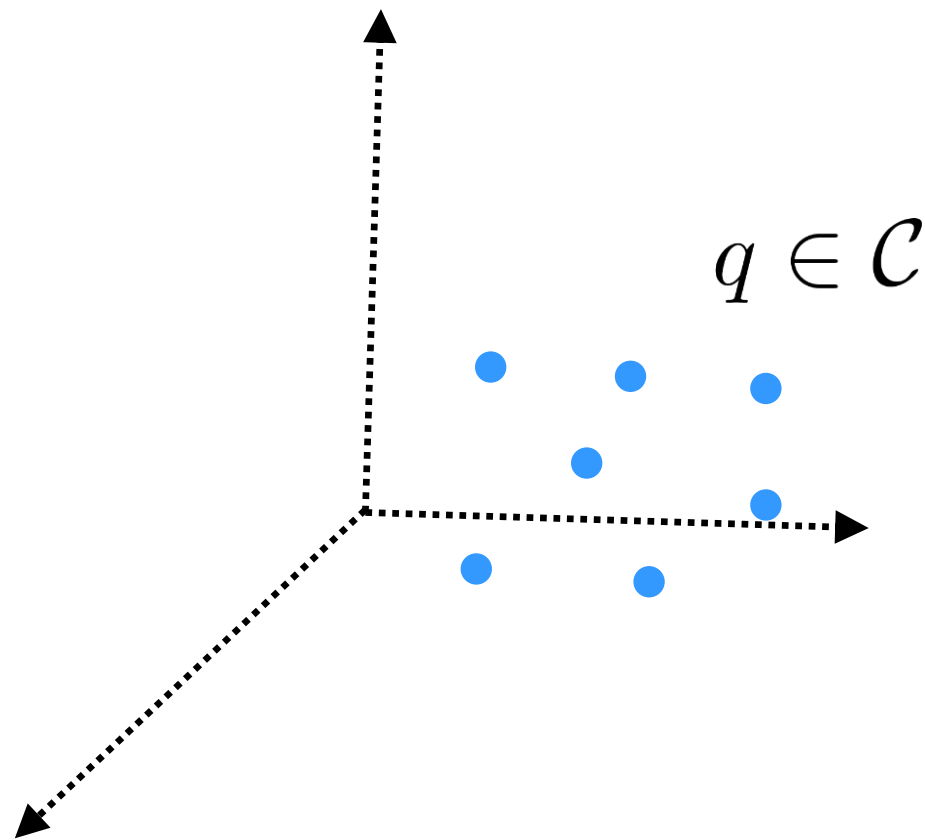


Configuration Space

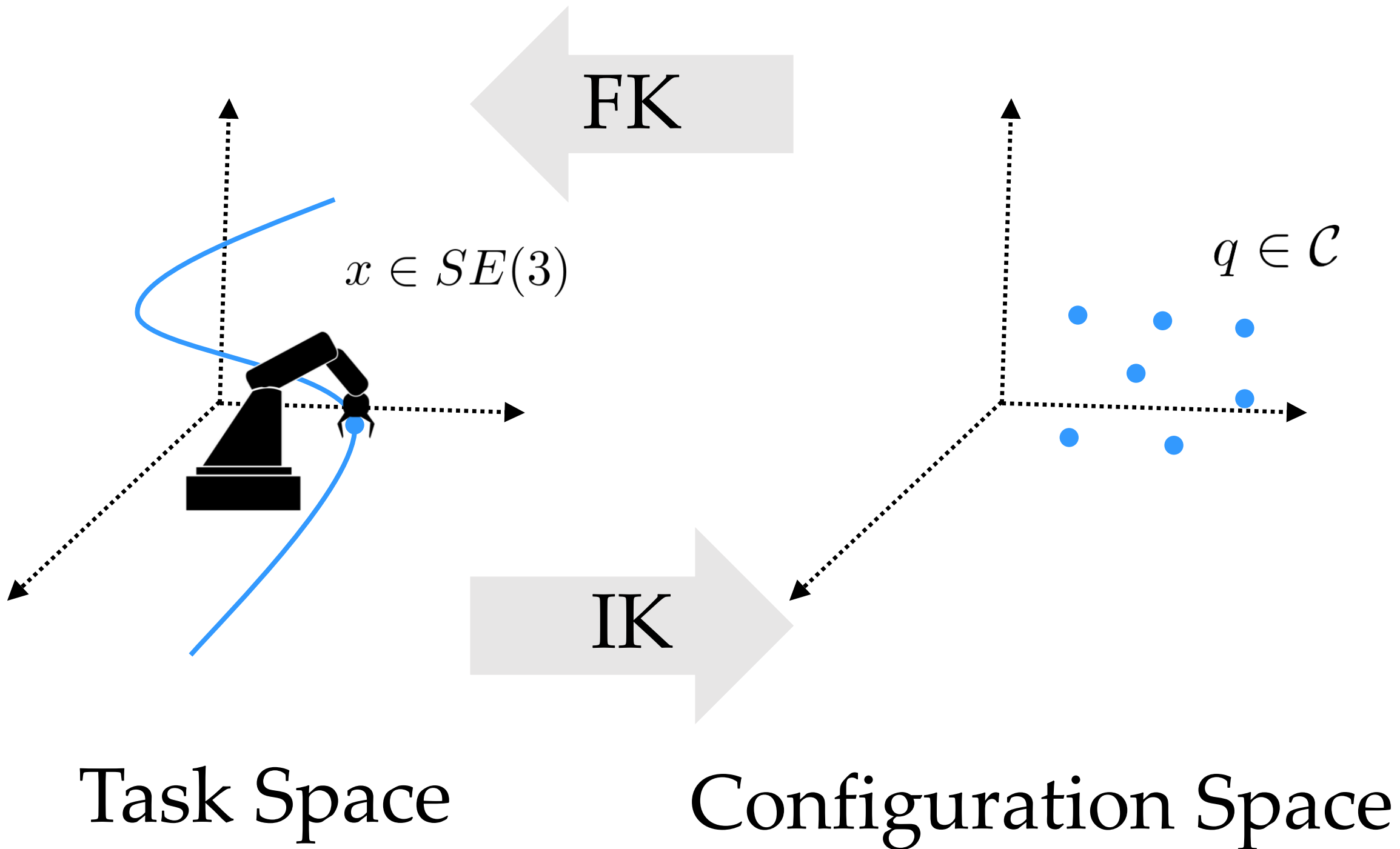




Task Space



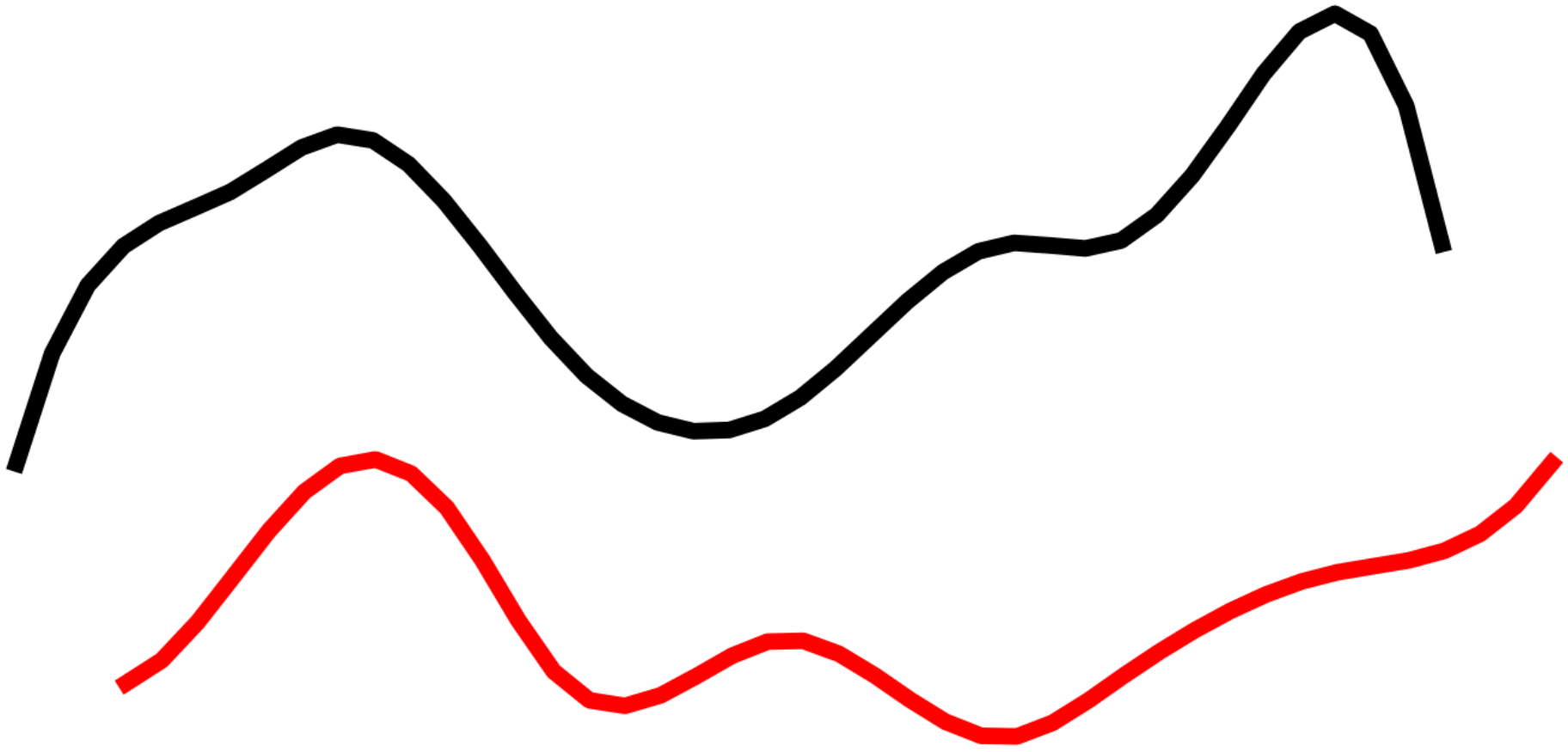
Configuration Space

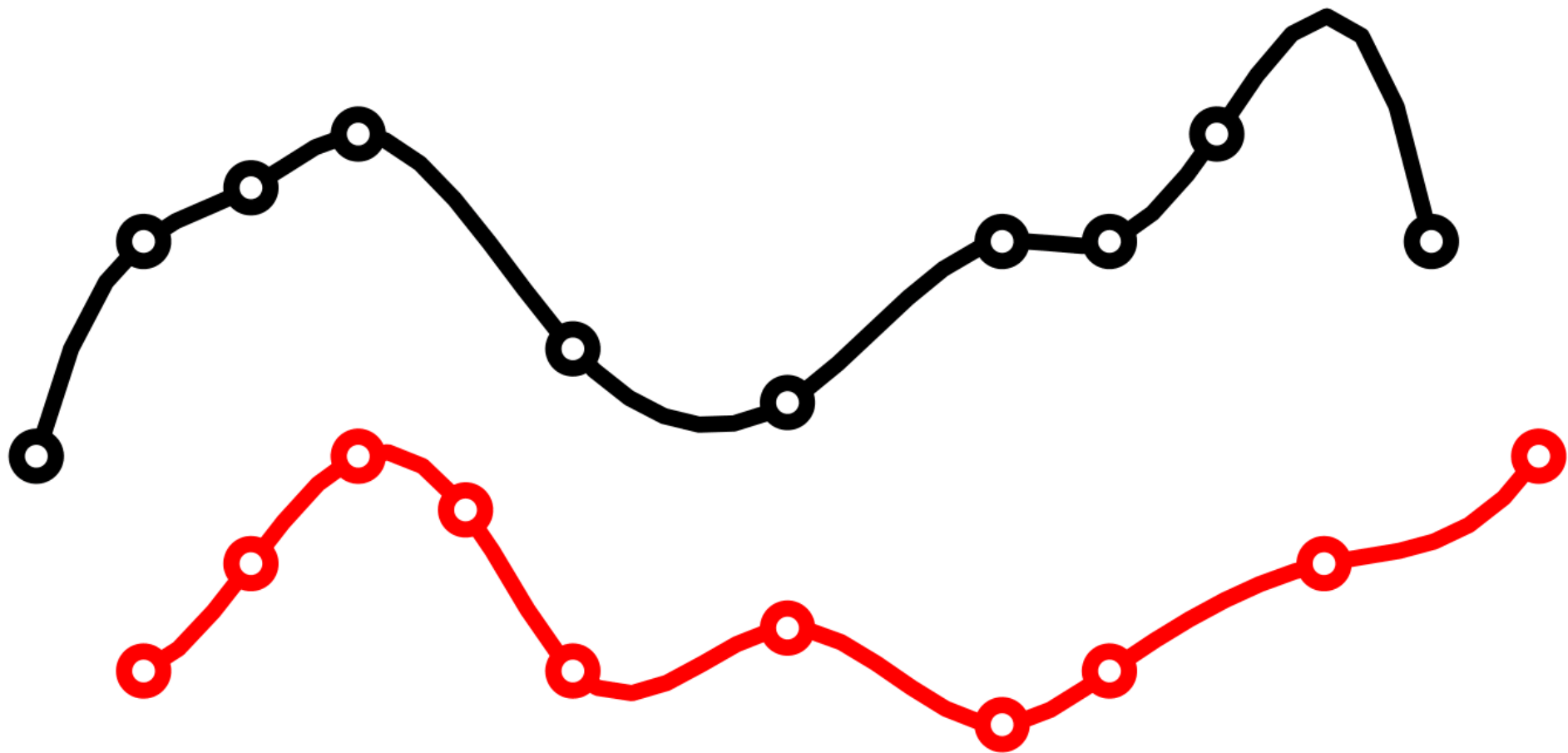


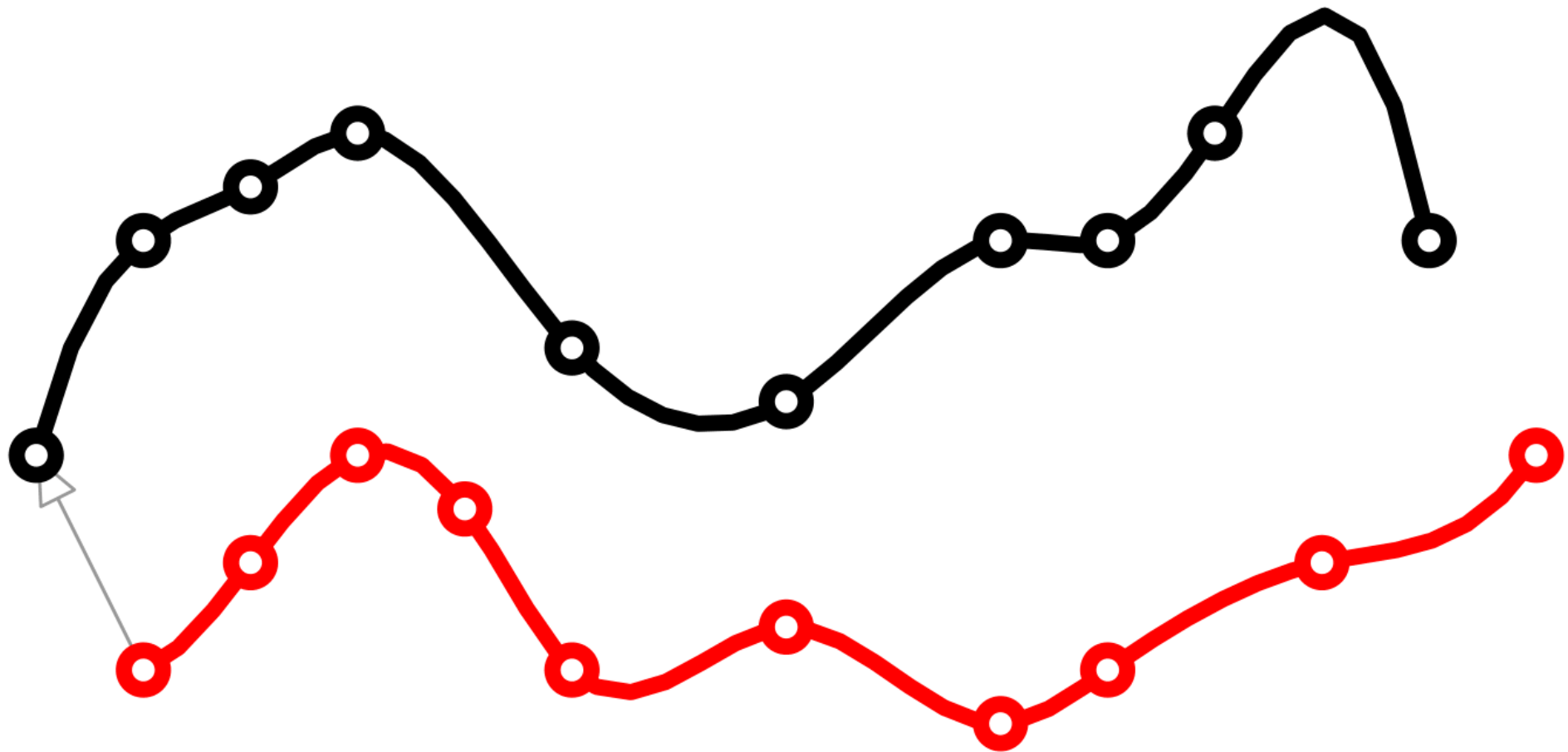
How to compare the distance  
between task space paths?

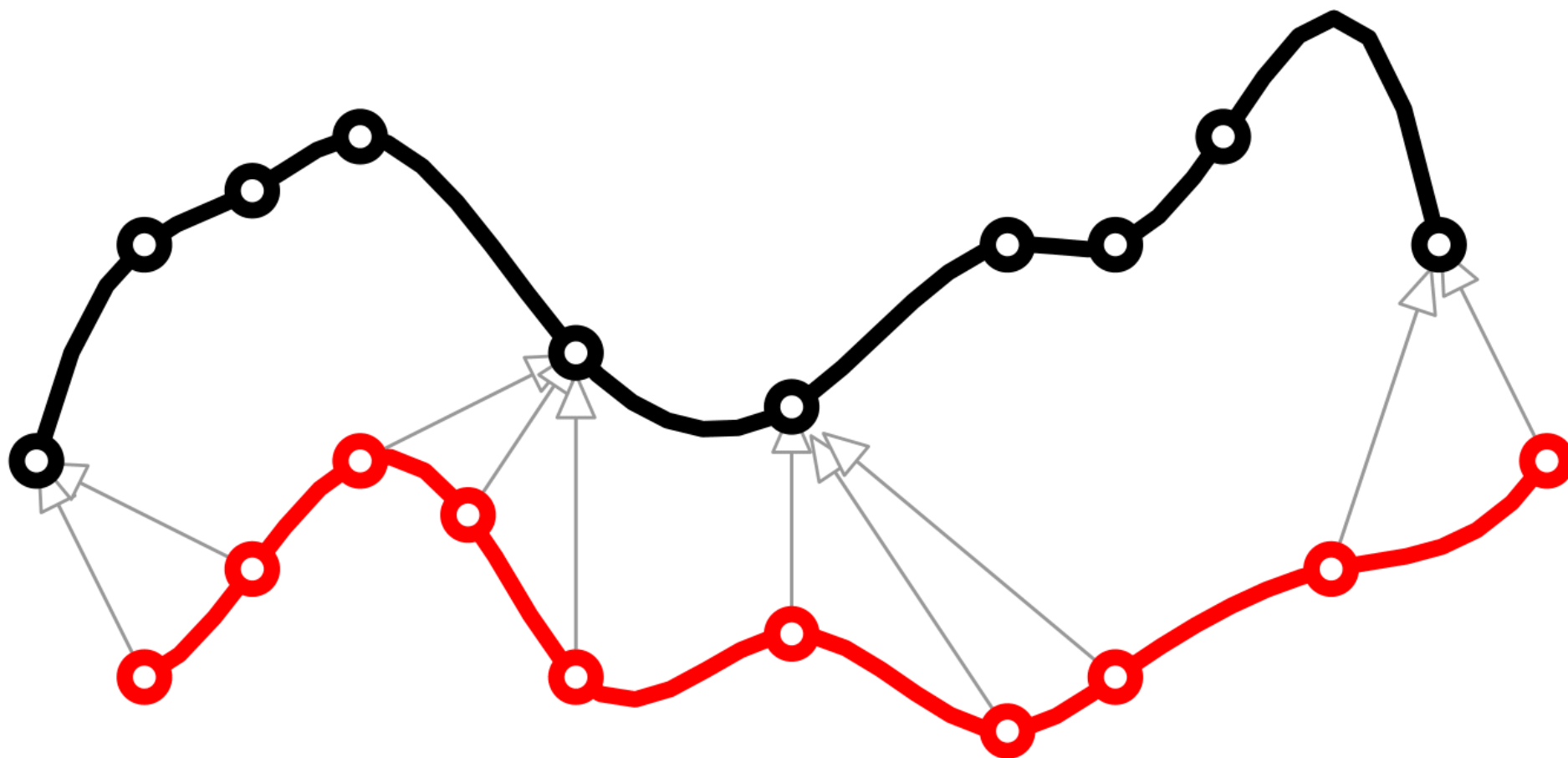
How to compare the distance  
between task space paths?

*Borrow from computational geometry.*



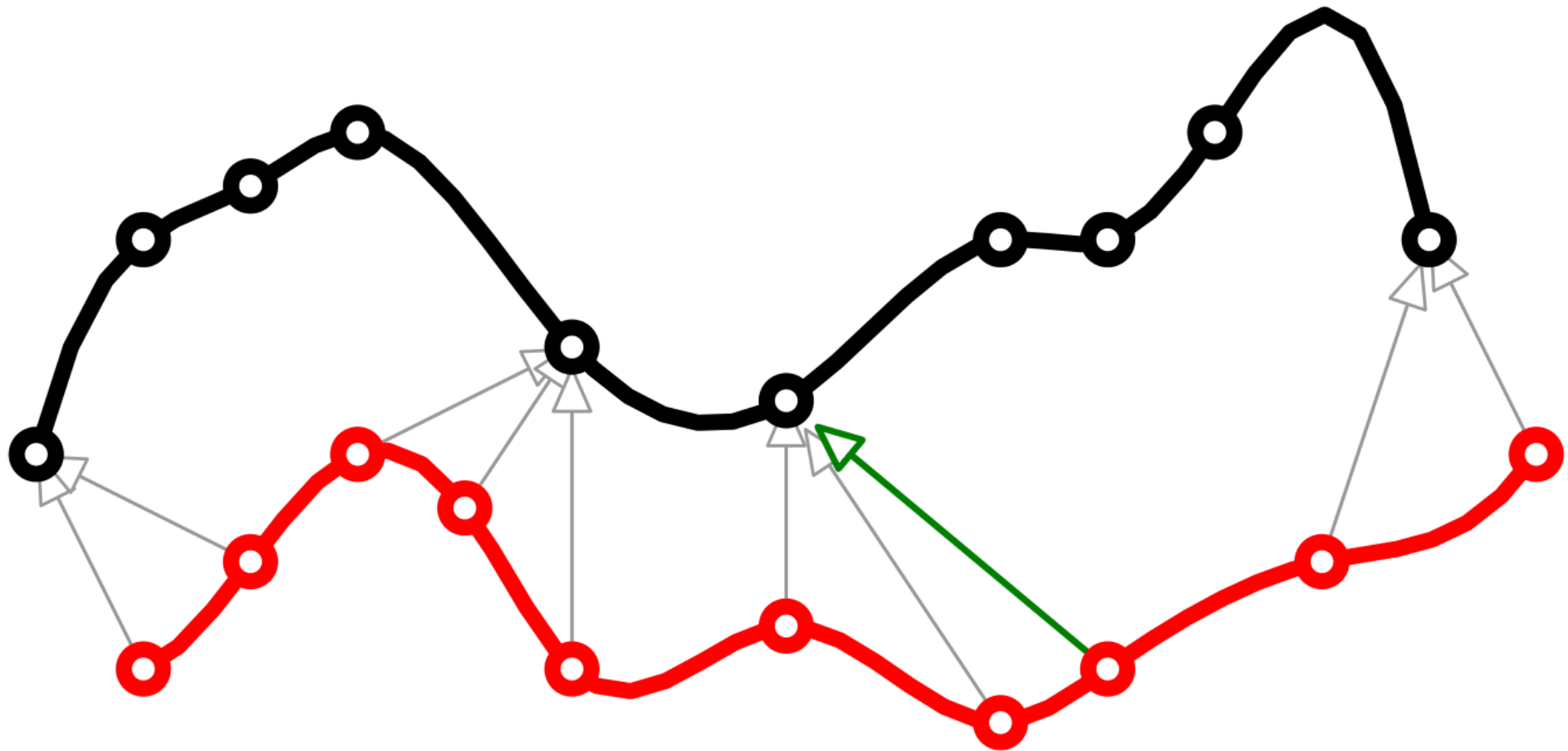


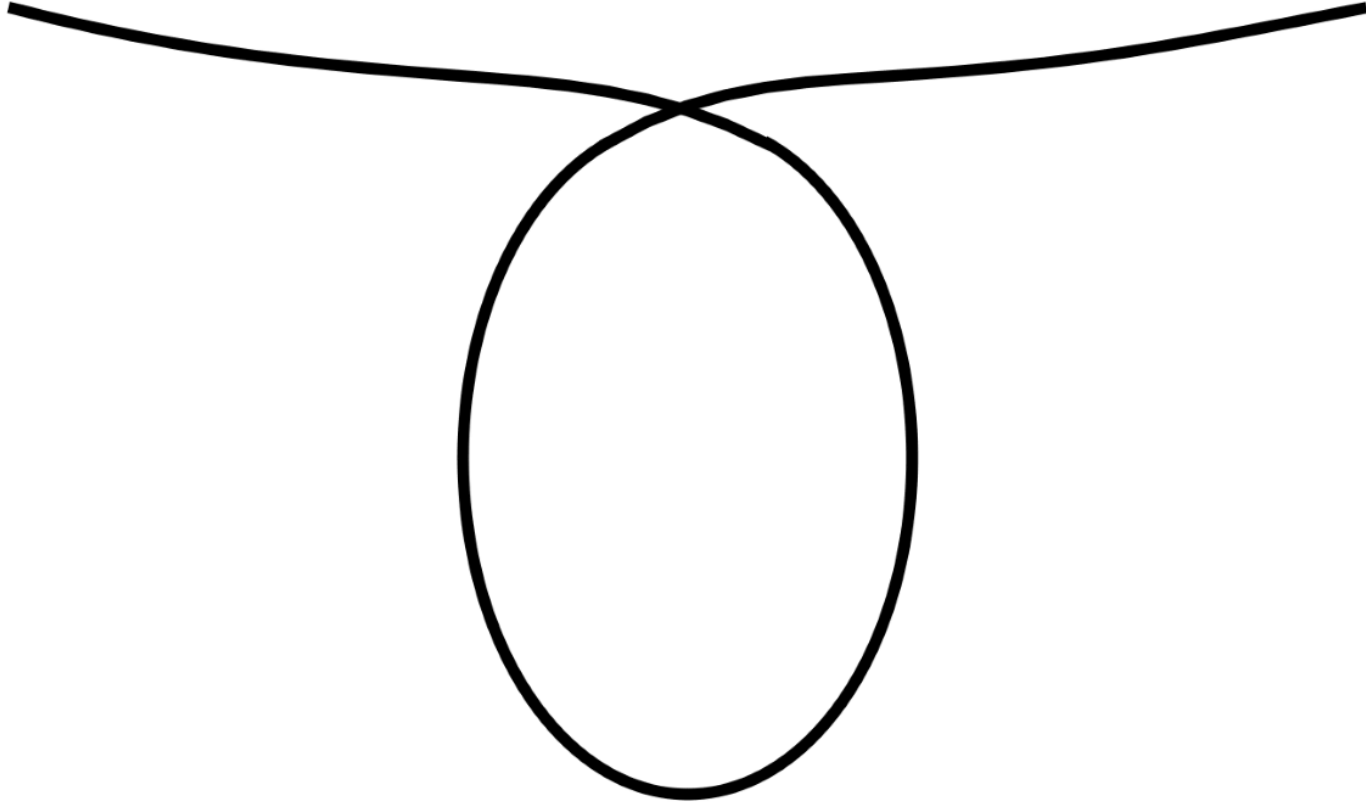


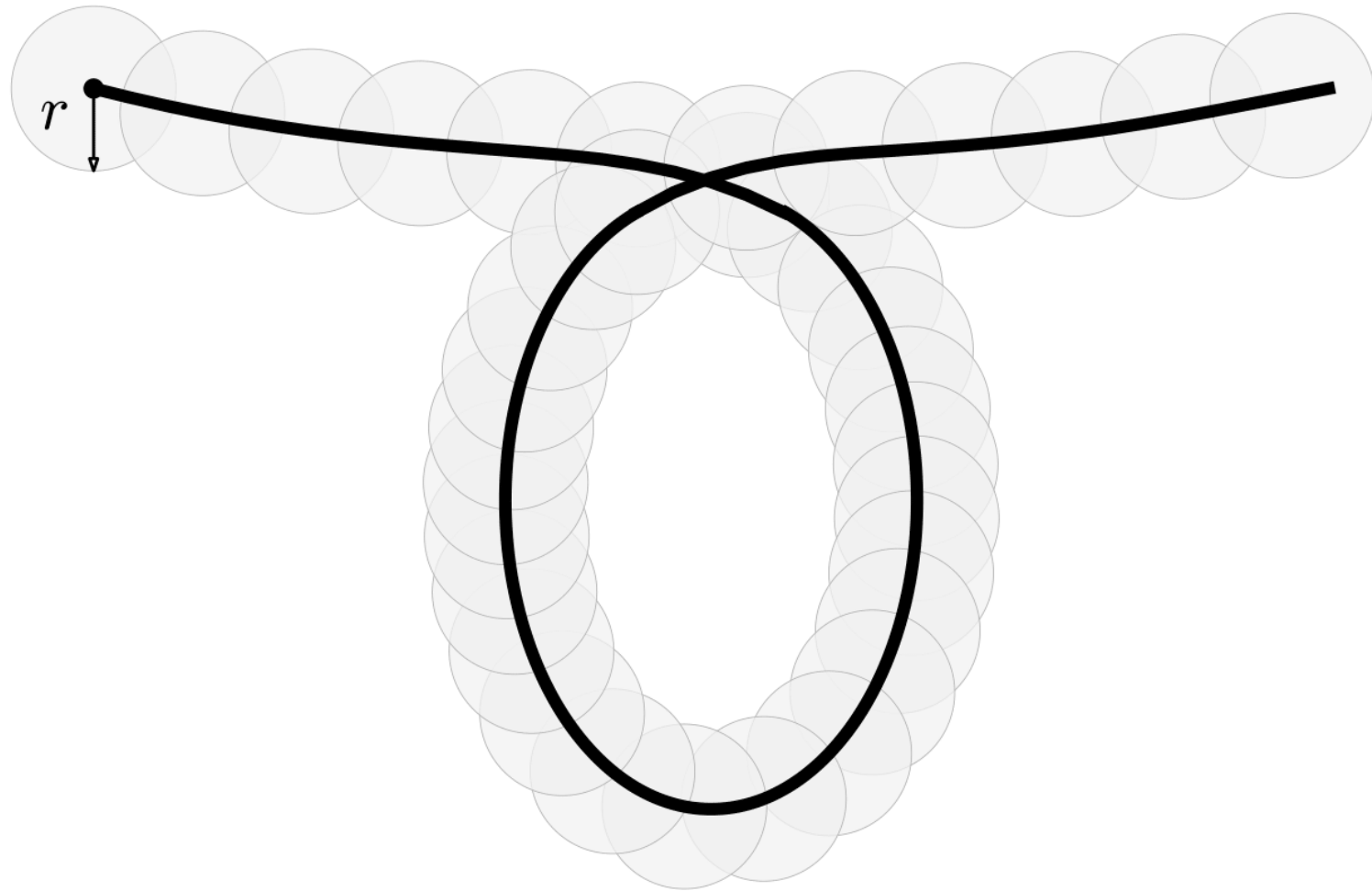




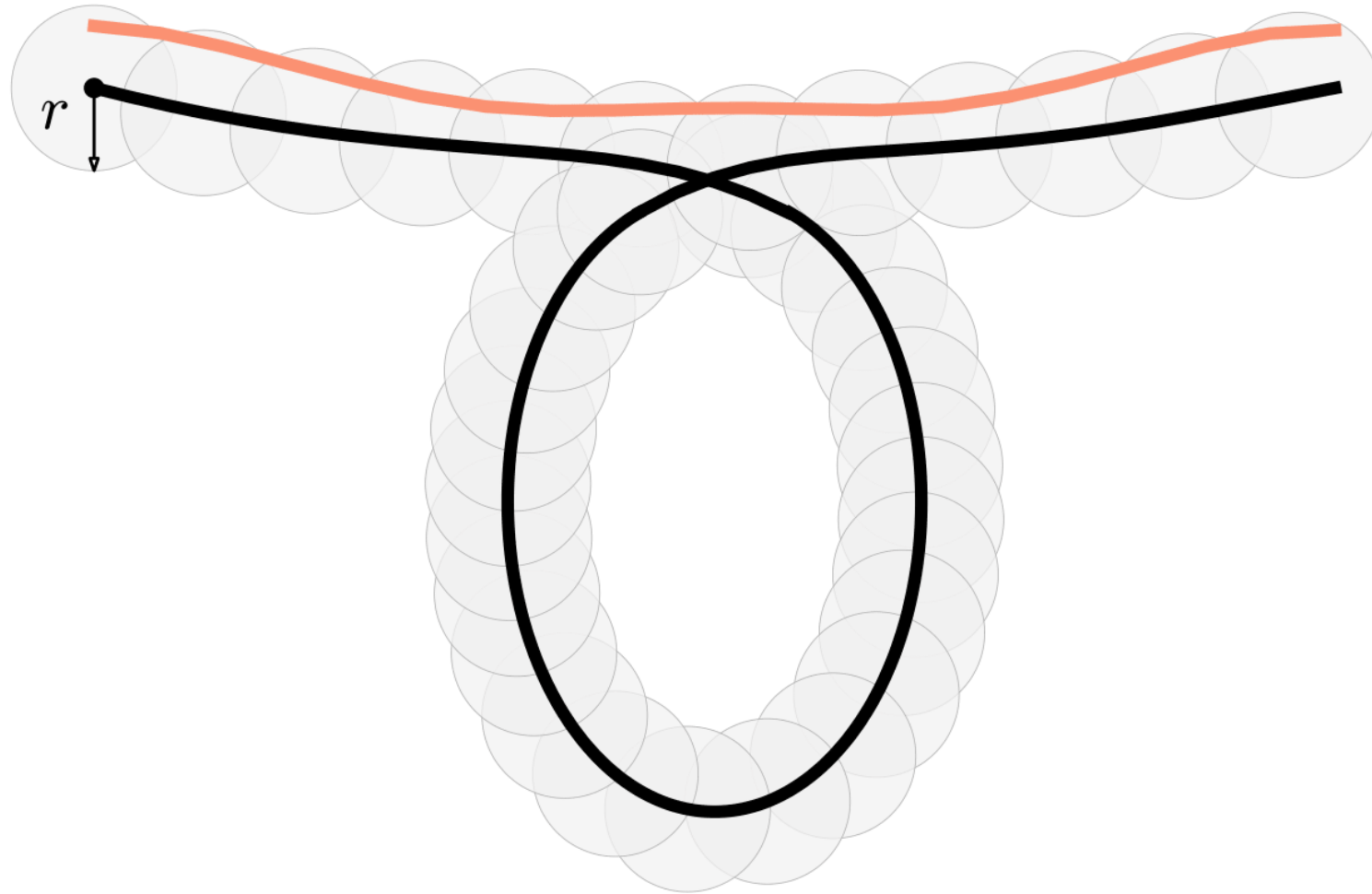
# One-way Hausdorff Distance



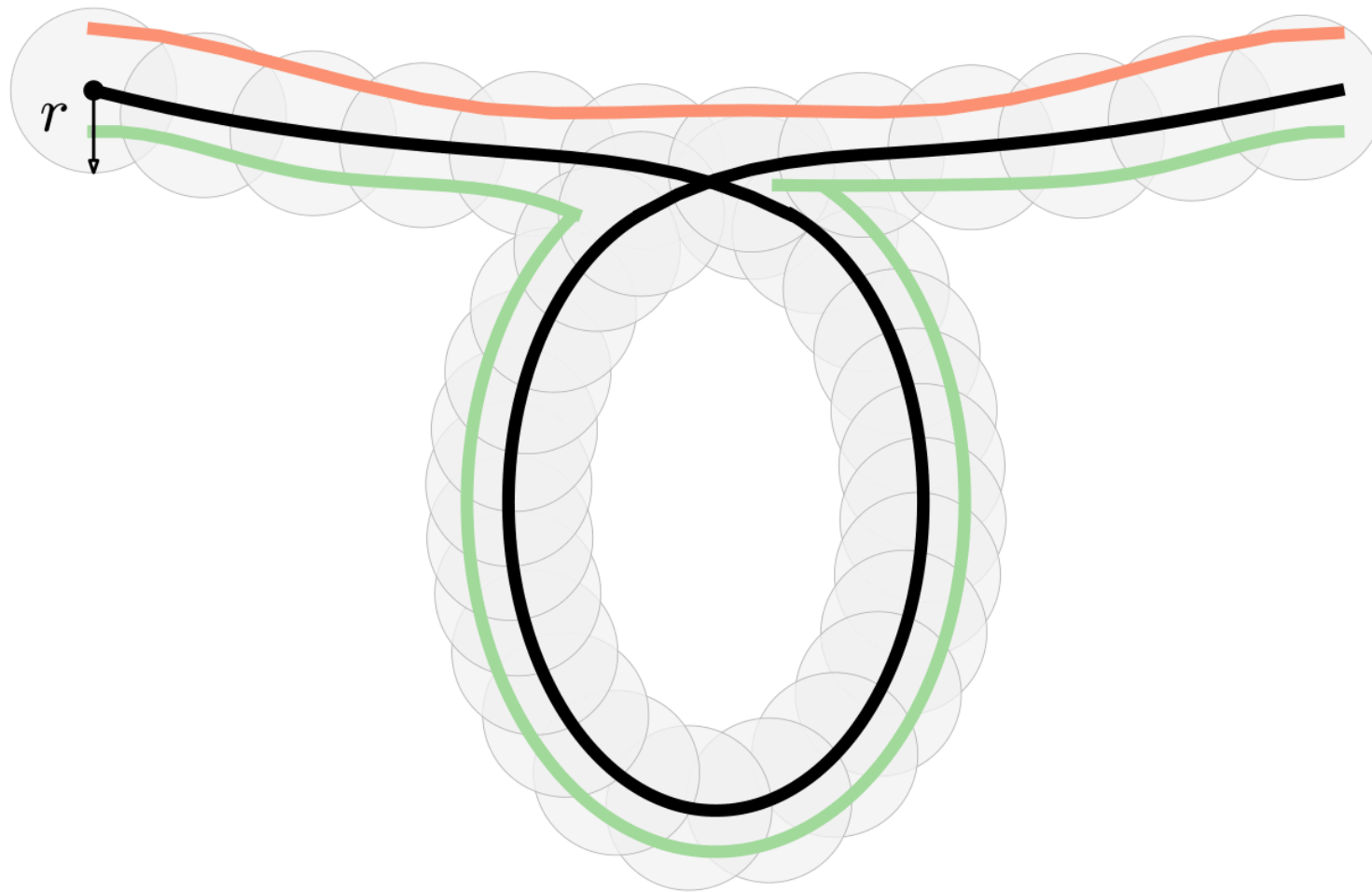




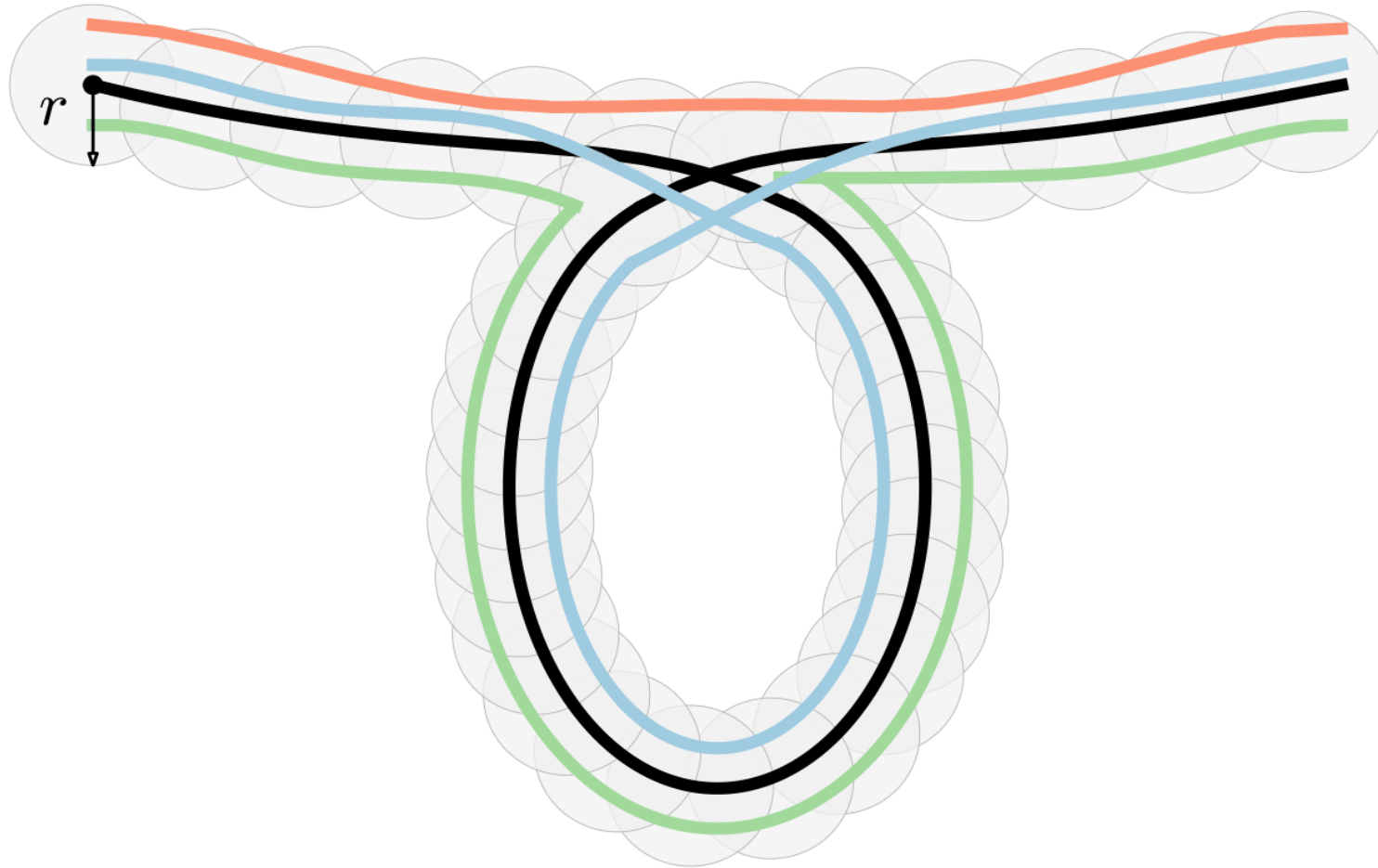
# One-way Hausdorff Distance



# Two-way Hausdorff Distance



Follow Balls *in order*.





# Fréchet Distance





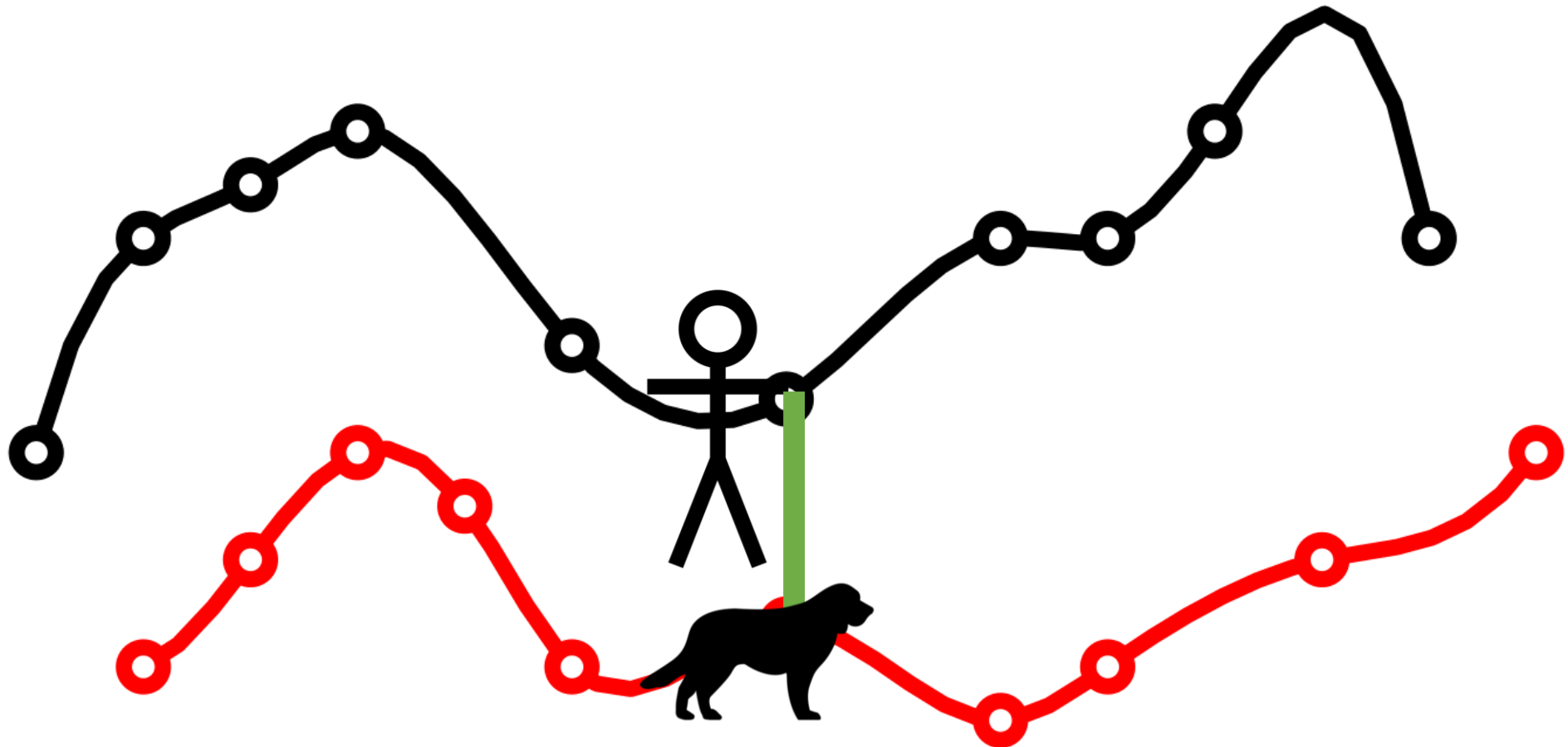
# Fréchet Distance



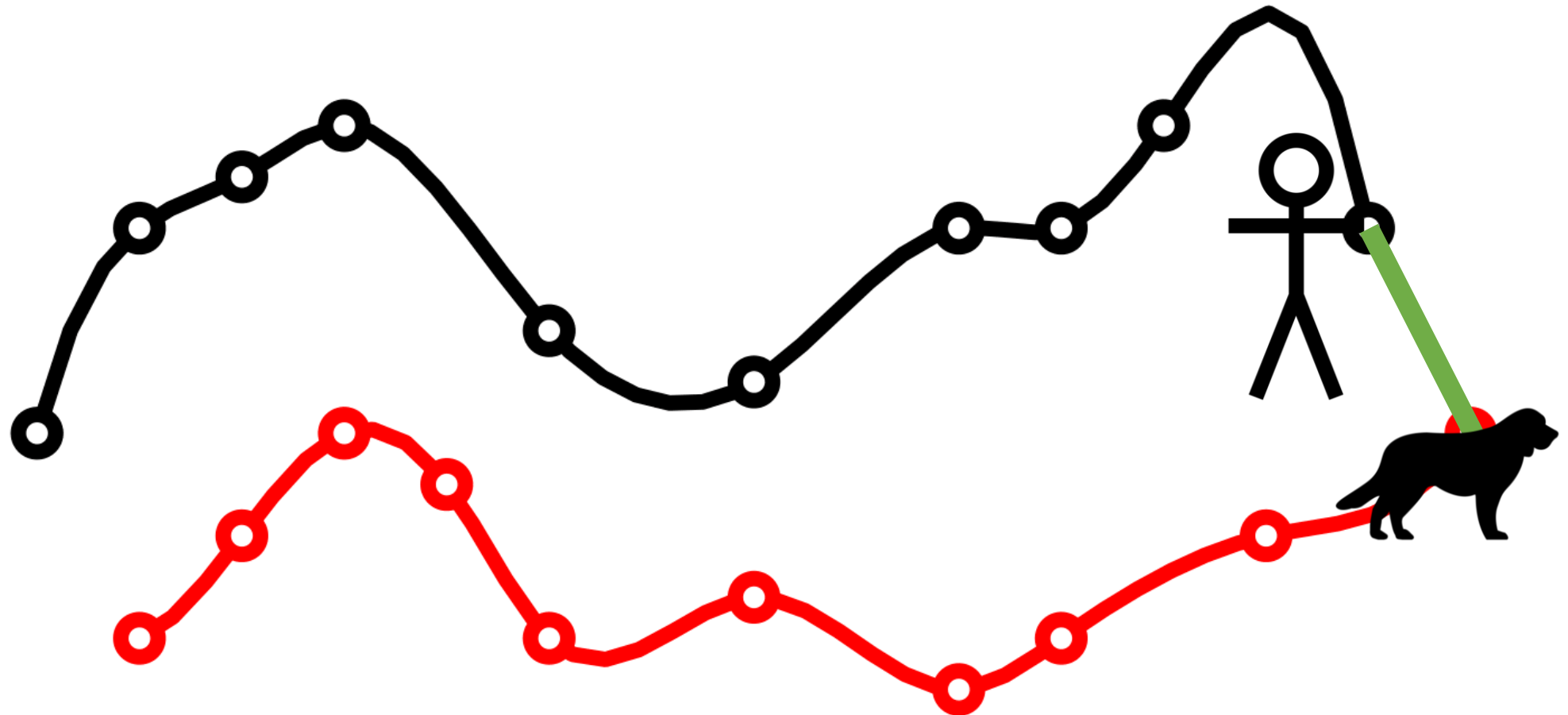
# Fréchet Distance



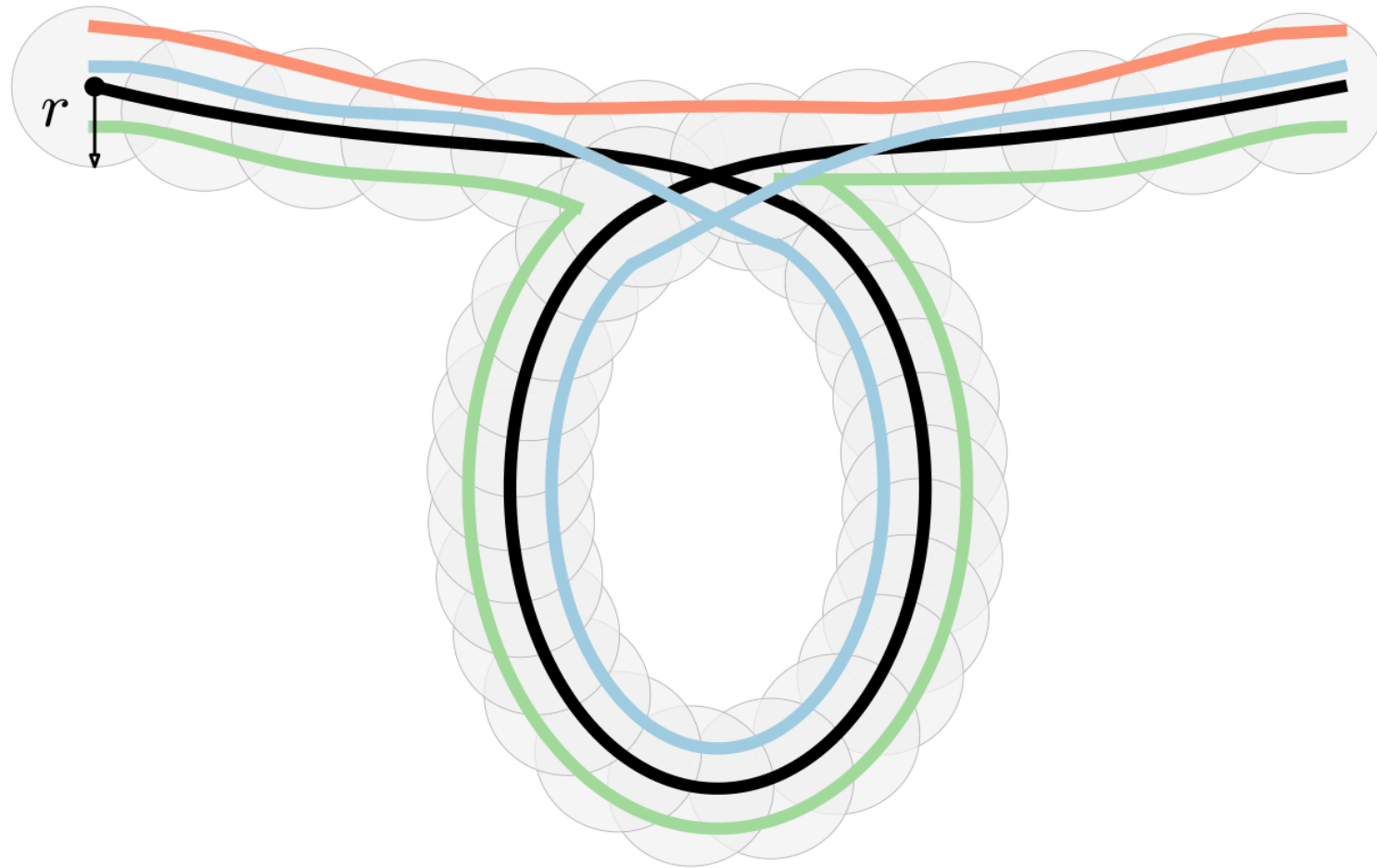
# Fréchet Distance



# Fréchet Distance

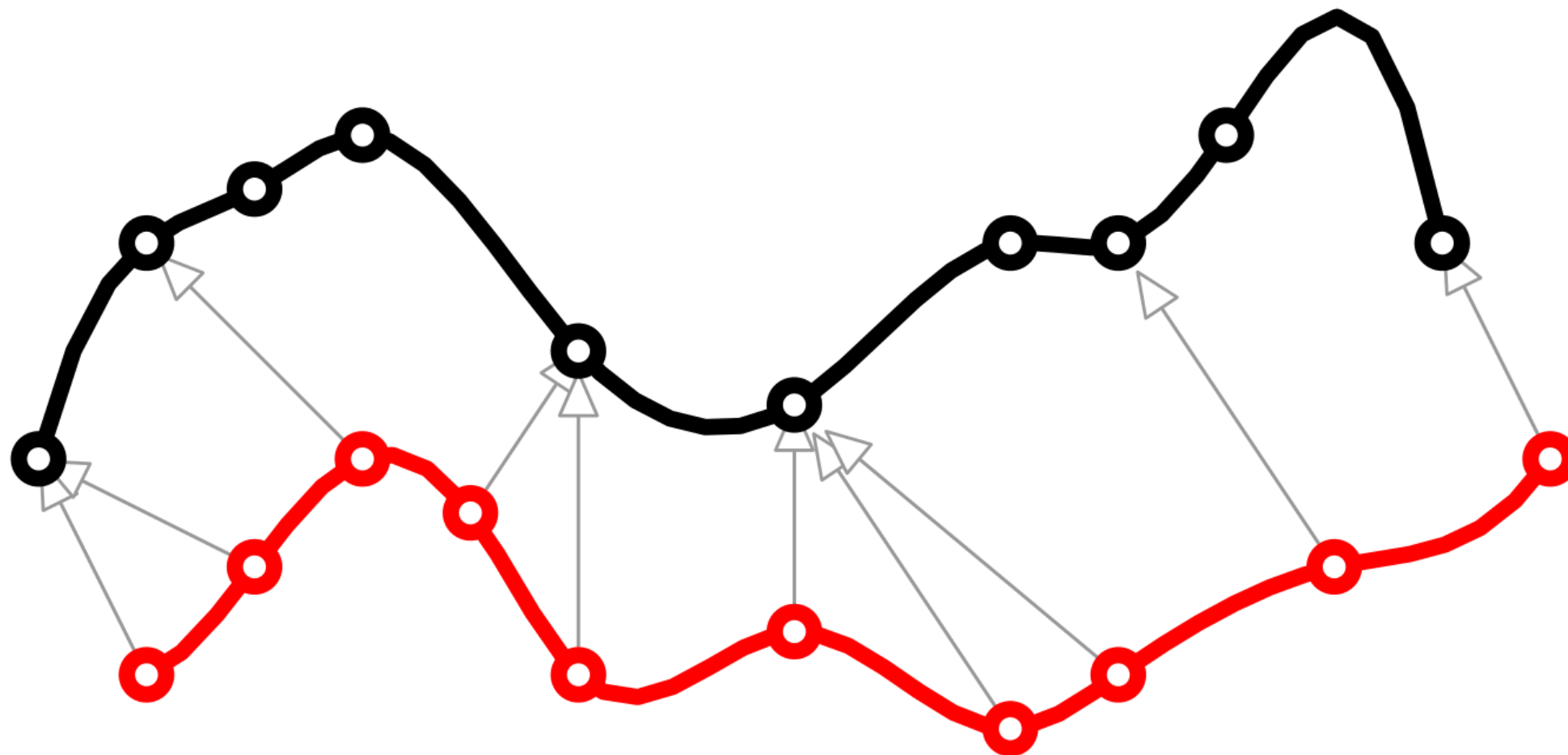


# Fréchet Distance

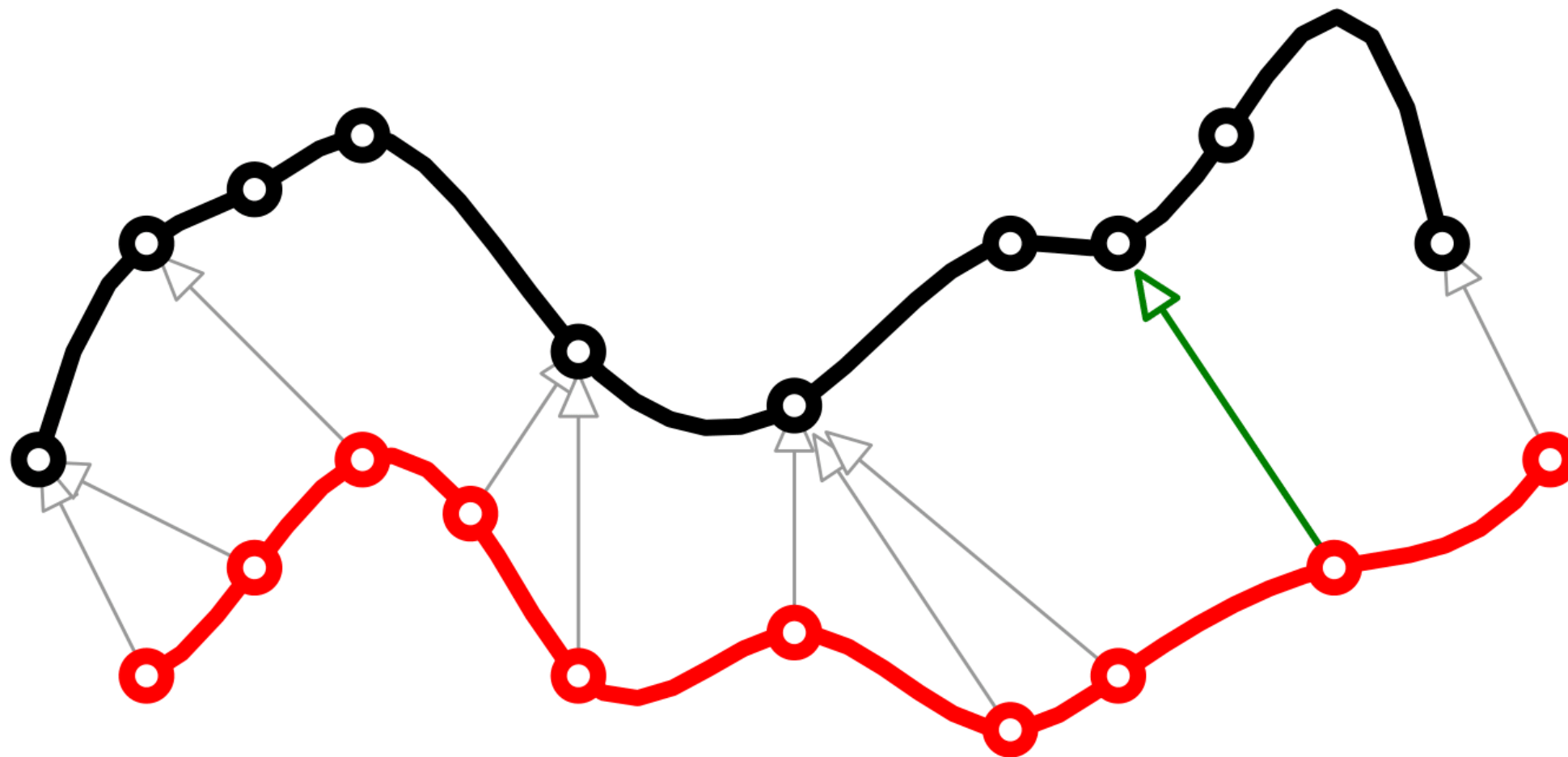


Use the Fréchet Distance to capture the task space distance between paths.

# Gradient of Distance Function

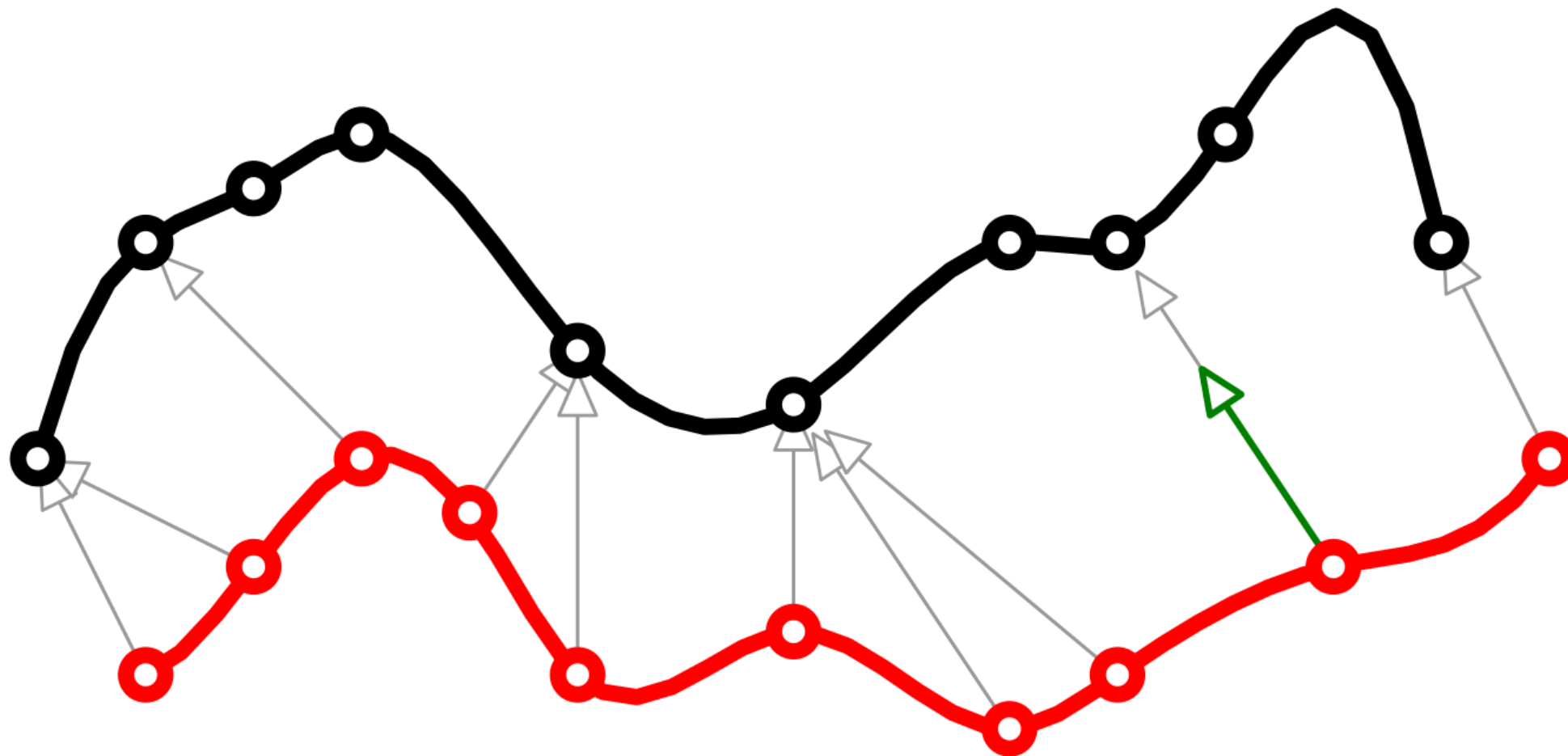


# Gradient of Distance Function





# Gradient of Distance Function

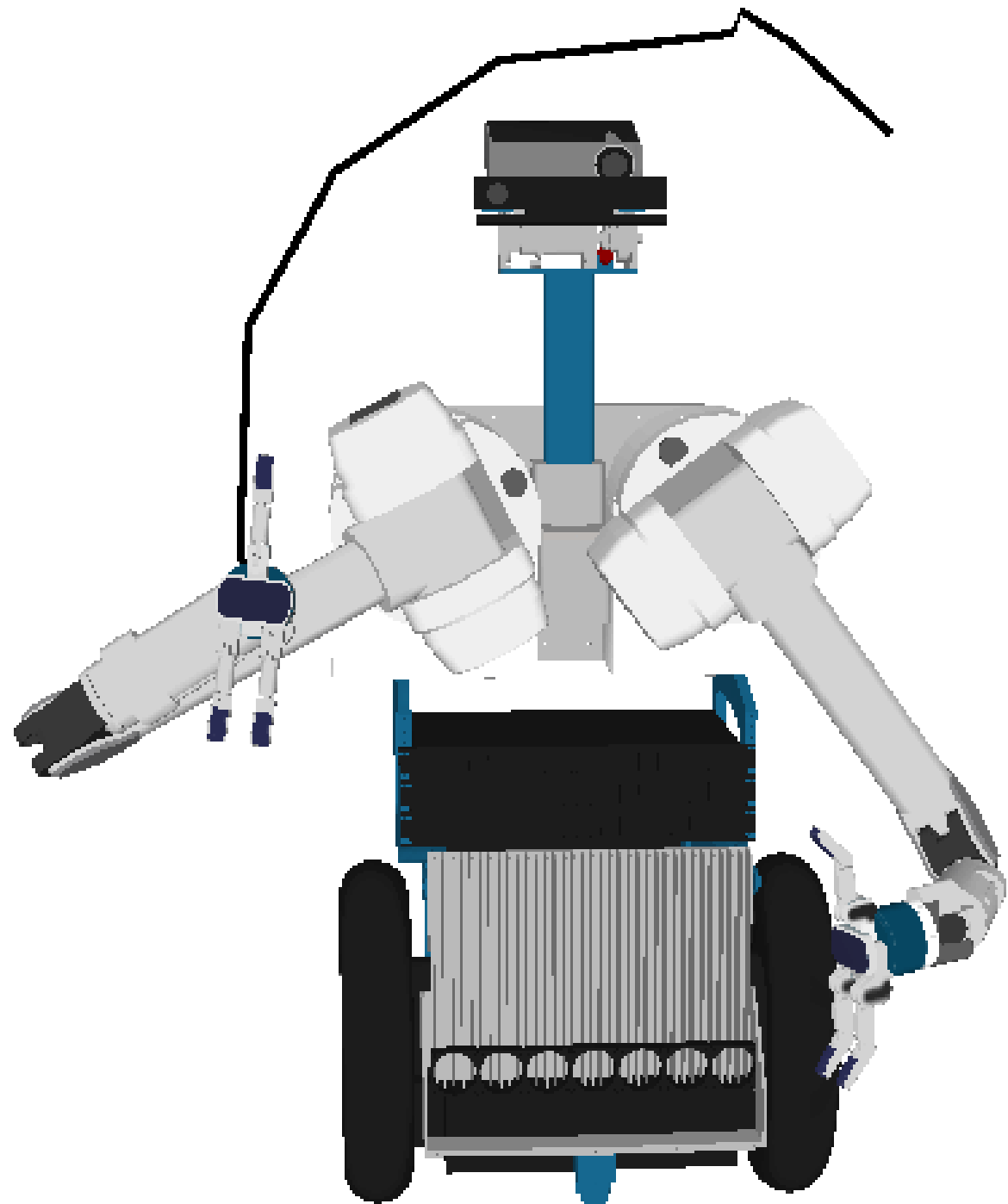


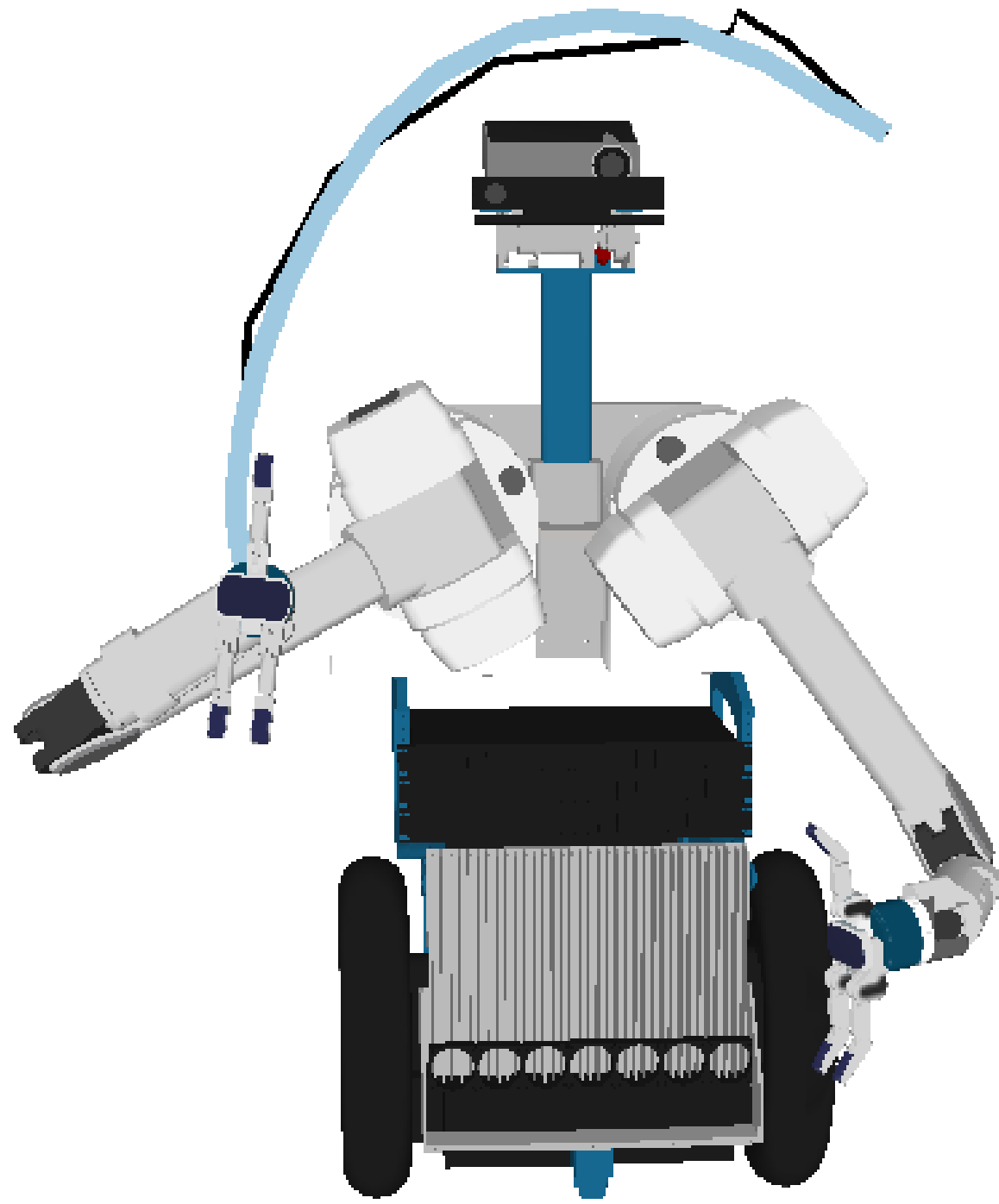
$$\xi^* = \arg \min_{\xi \in \Xi} \|\xi - \bar{\xi}\| \quad \text{s.t. constraints}$$

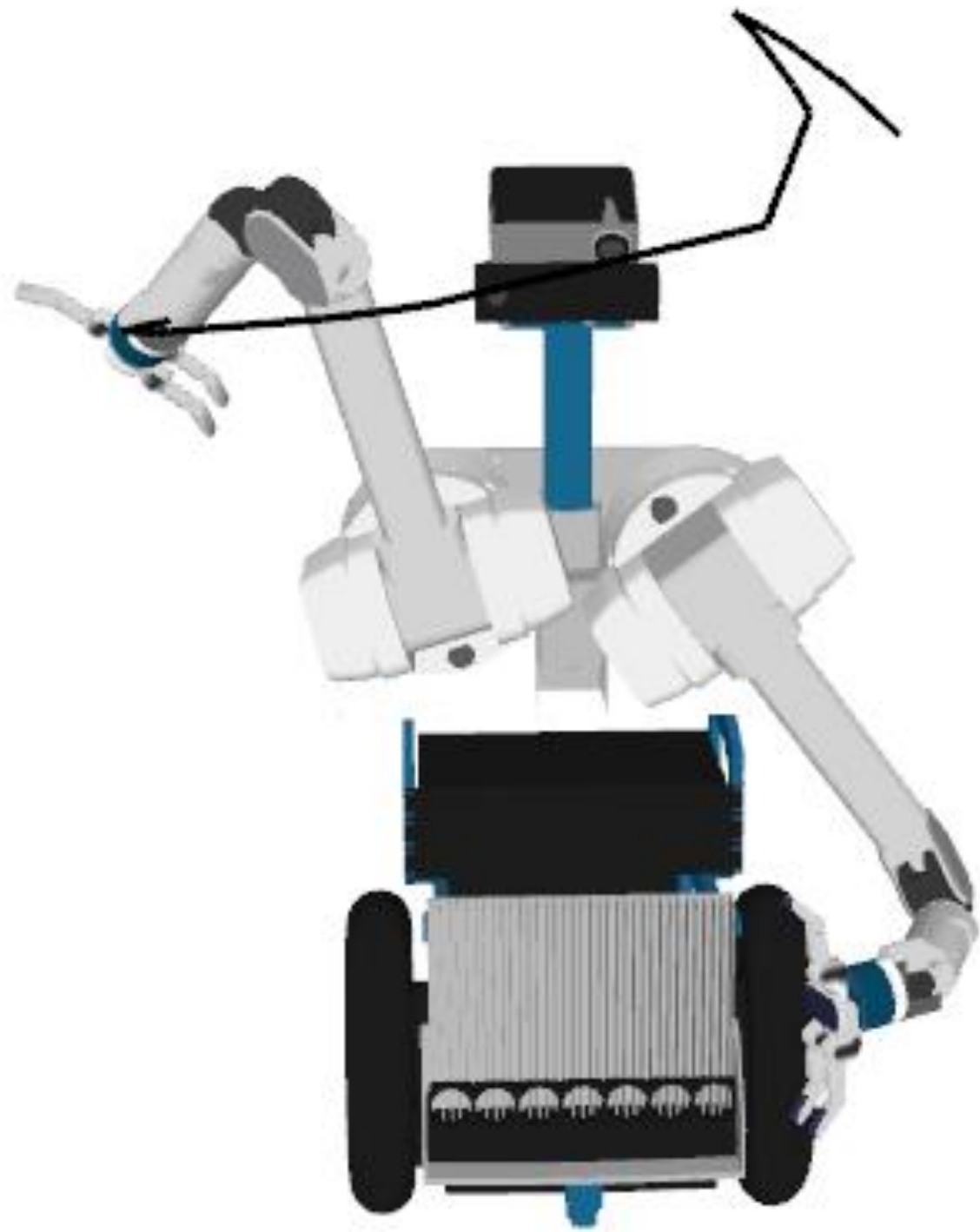
$$\xi^* = \arg \min_{\xi \in \Xi} \|\xi - \bar{\xi}\| \quad \text{s.t. constraints}$$

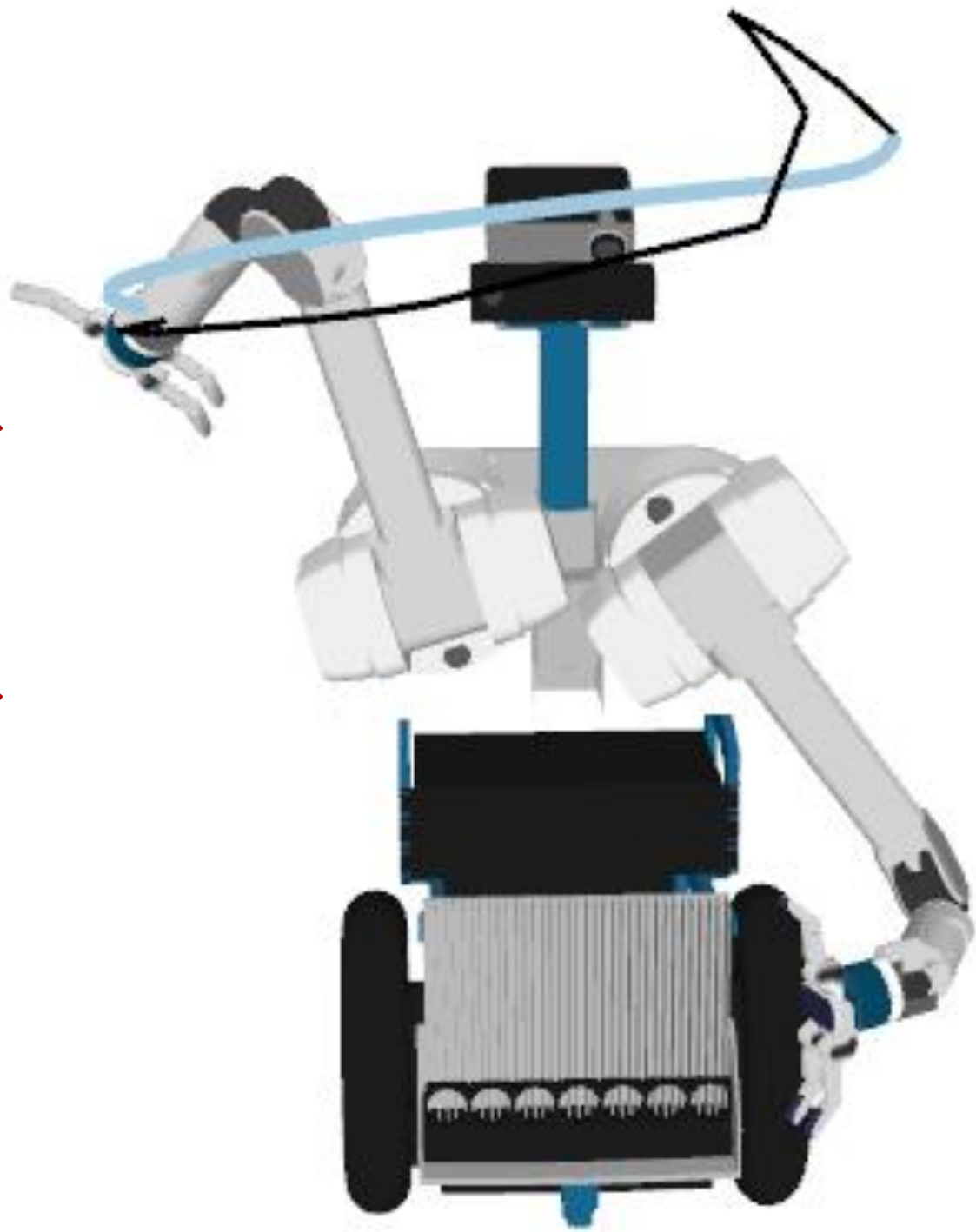
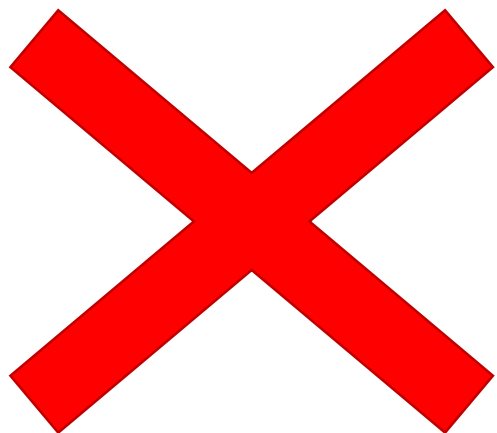


Discrete  
Fréchet



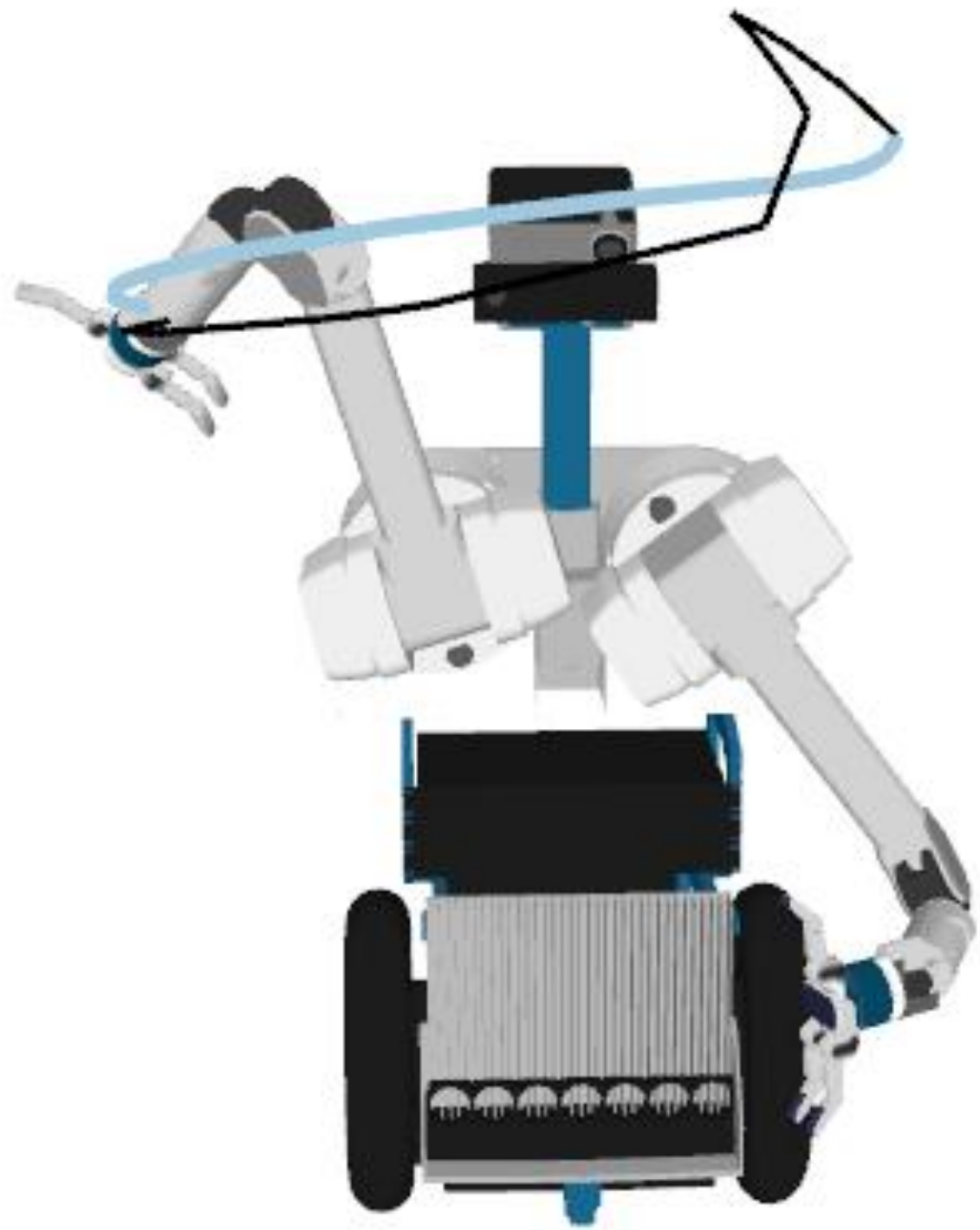


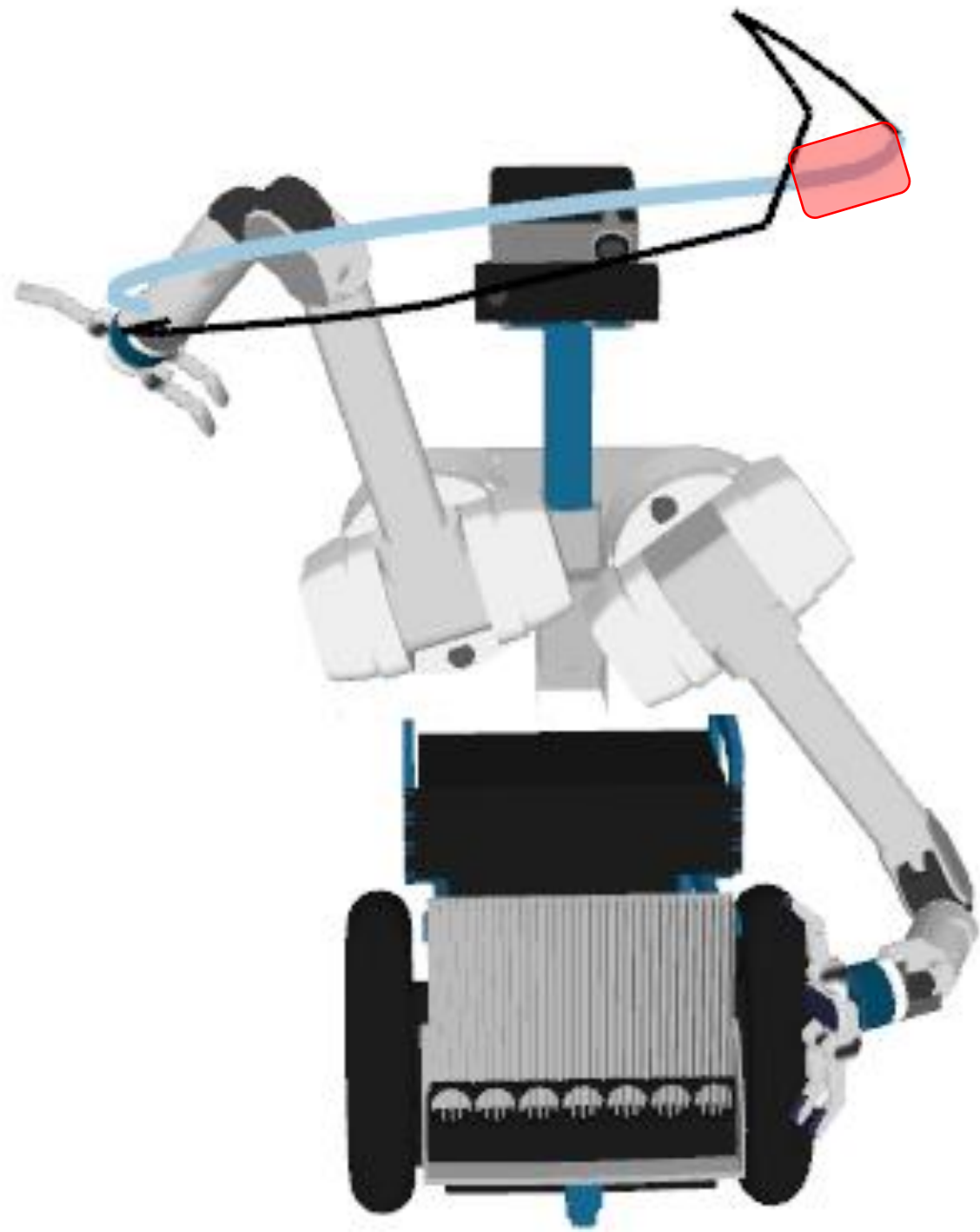




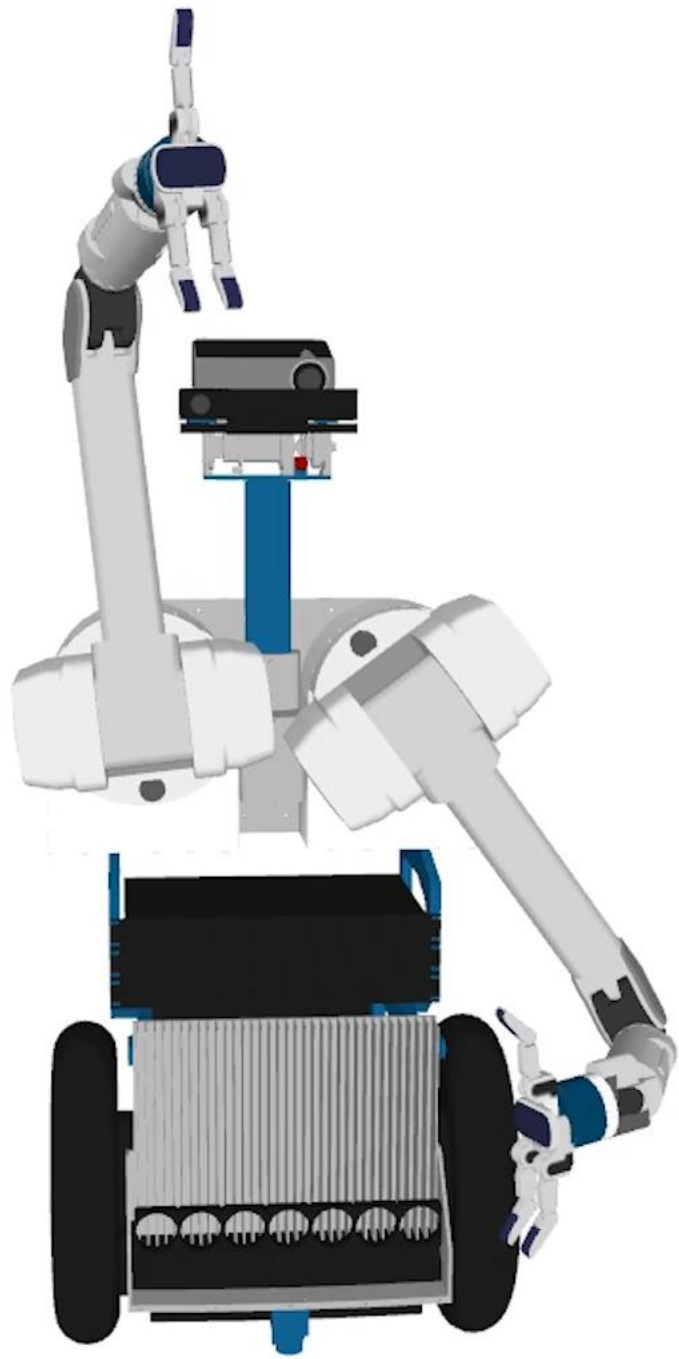
How can we assist our optimizer?

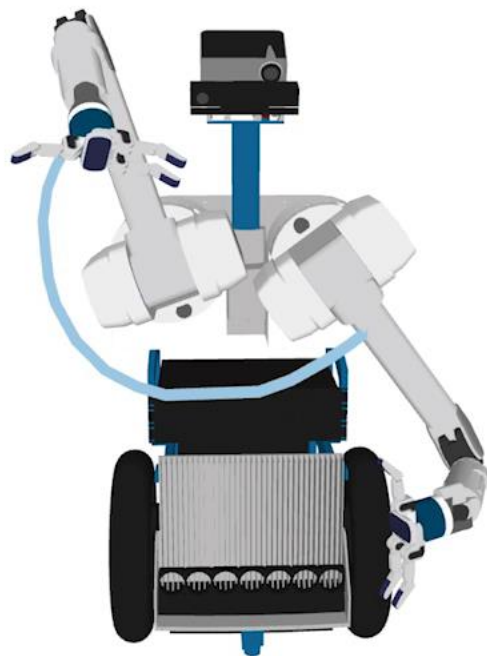
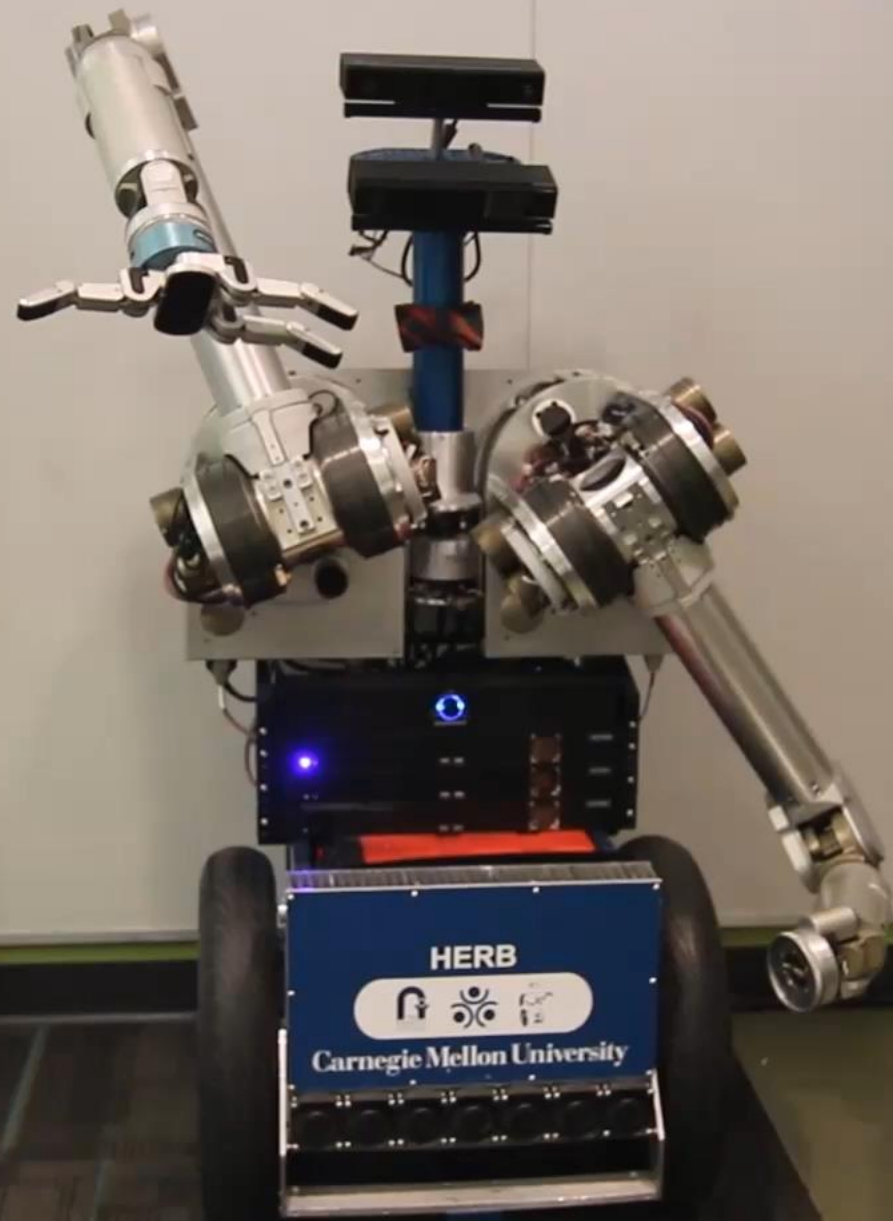






Use our *distance metrics* to find the areas that need additional constraints.



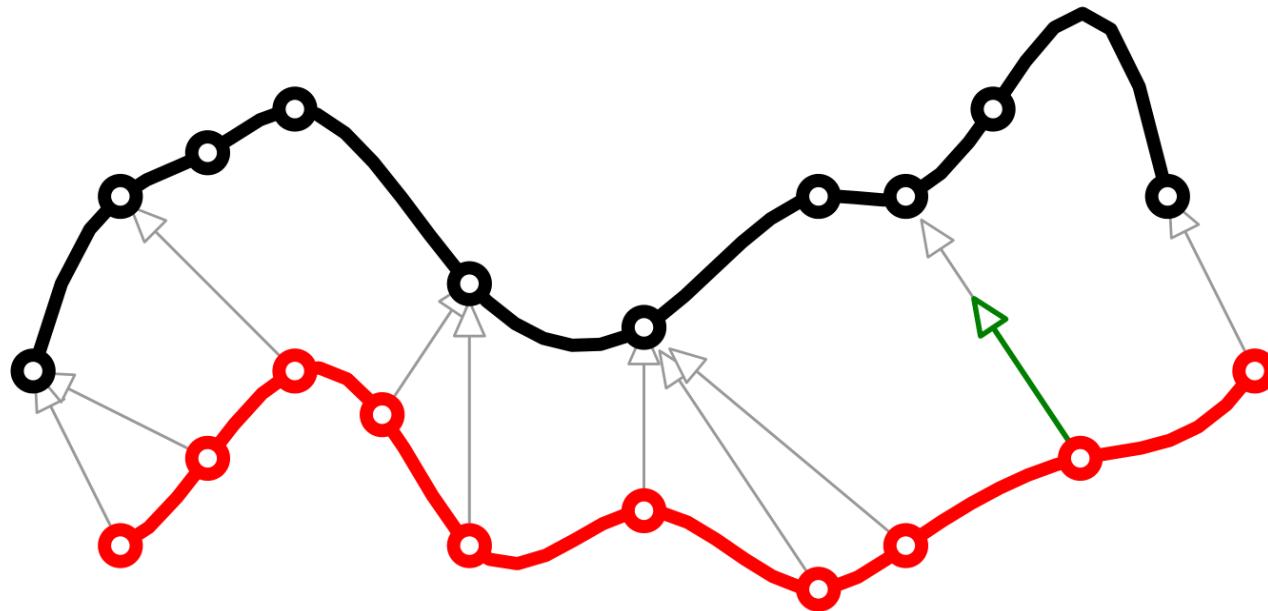


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## *Key Insight*

*Use trajectory optimization to optimize our path to be **close** to our reference path.*

*Use computational geometry techniques to measure task space distance between paths.*



# Distance Metrics and Algorithms for Task Space Path Optimization

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Siddhartha Srinivasa

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<http://www.andrew.cmu.edu/user/rmh/>

