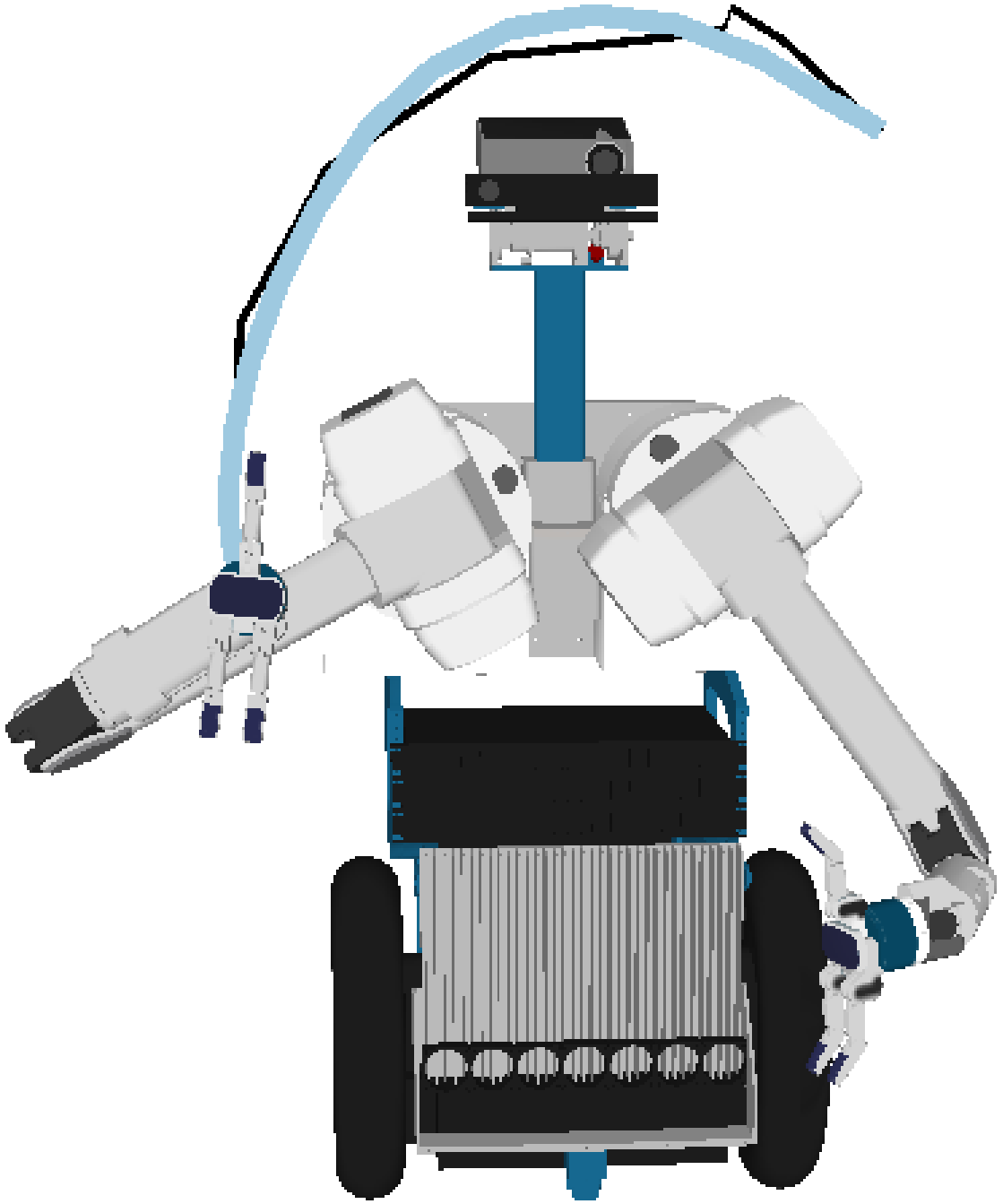
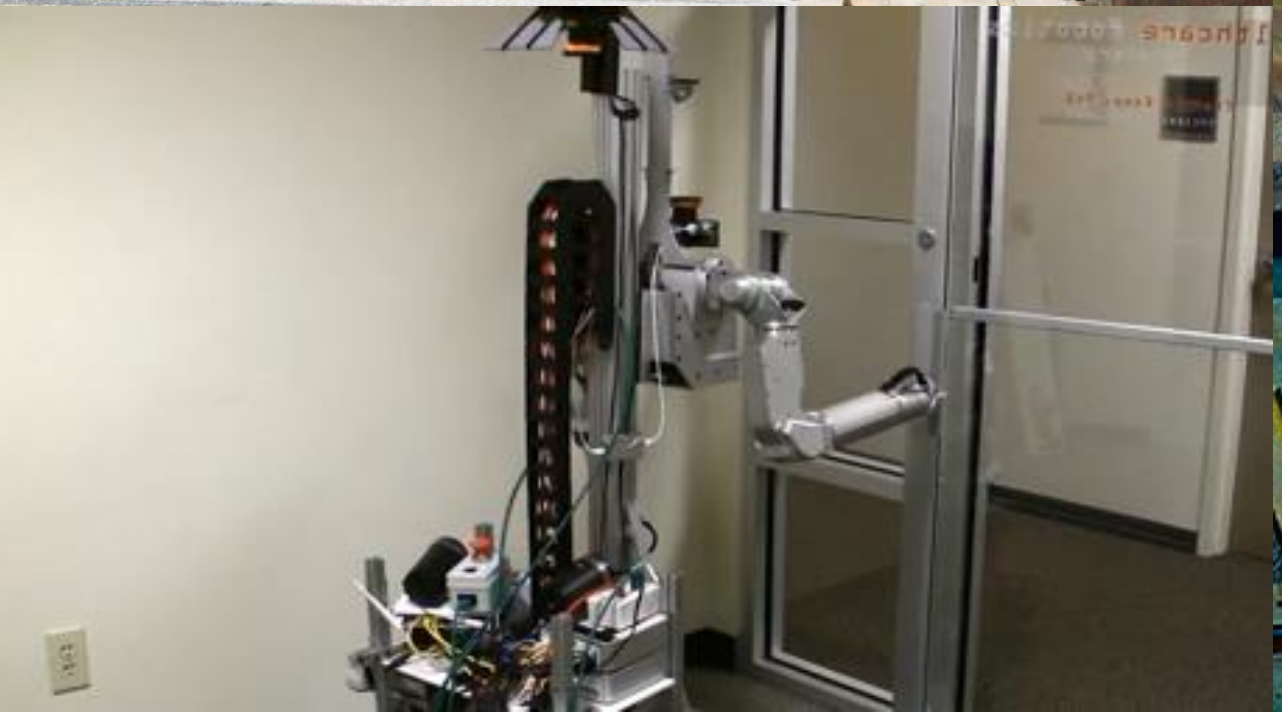


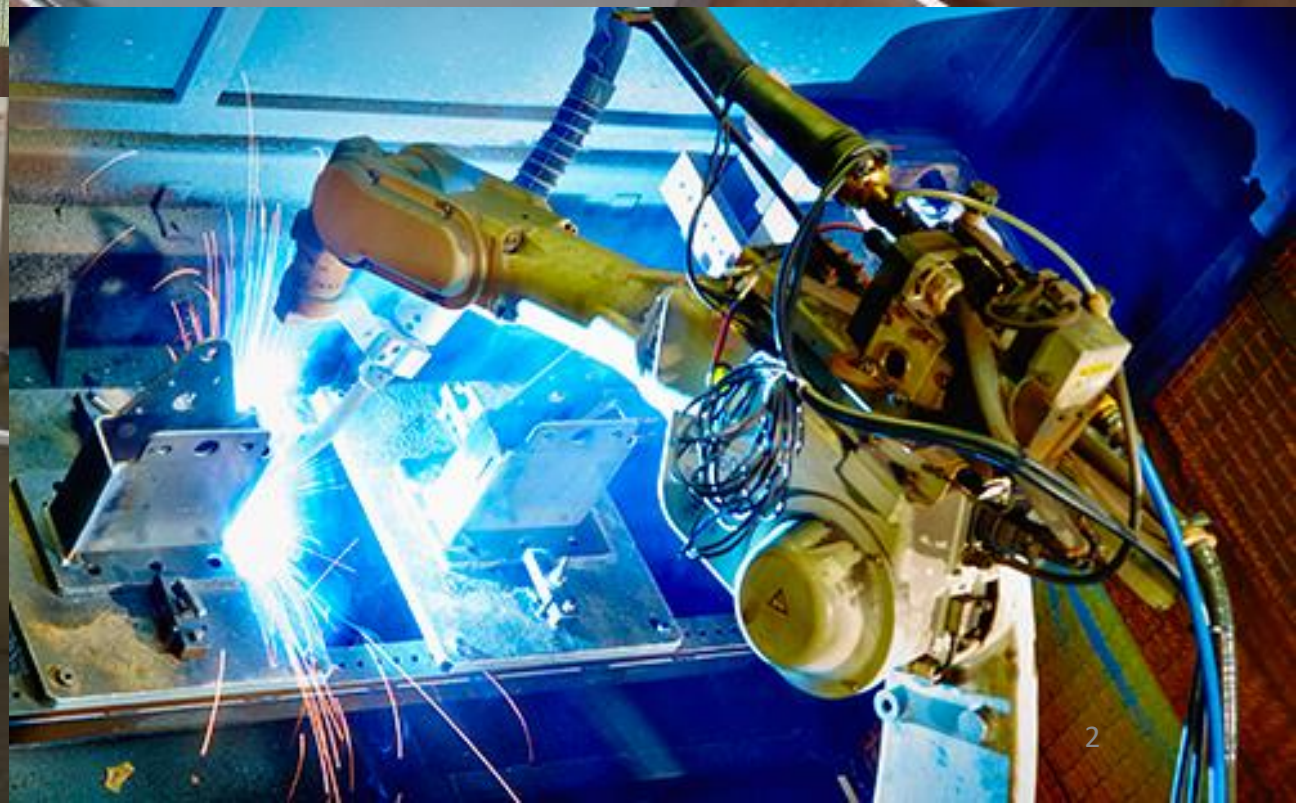
Following Paths in Task Space: Distance Metrics and Planning Algorithms

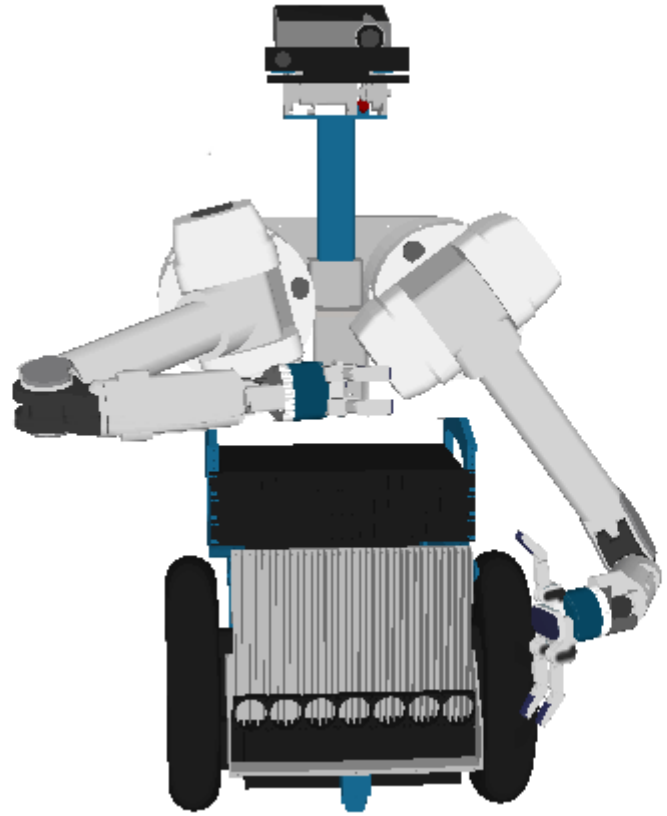
Rachel Holladay

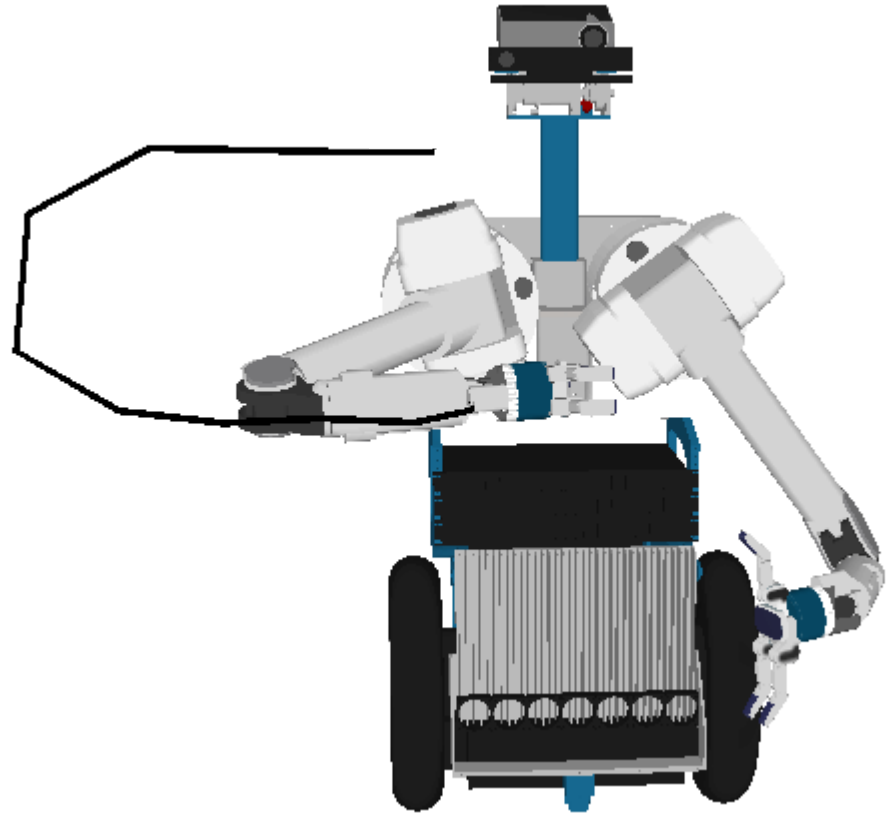
SCS Undergraduate Senior Thesis
Advised by Siddhartha Srinivasa
Robotics Institute
Carnegie Mellon University

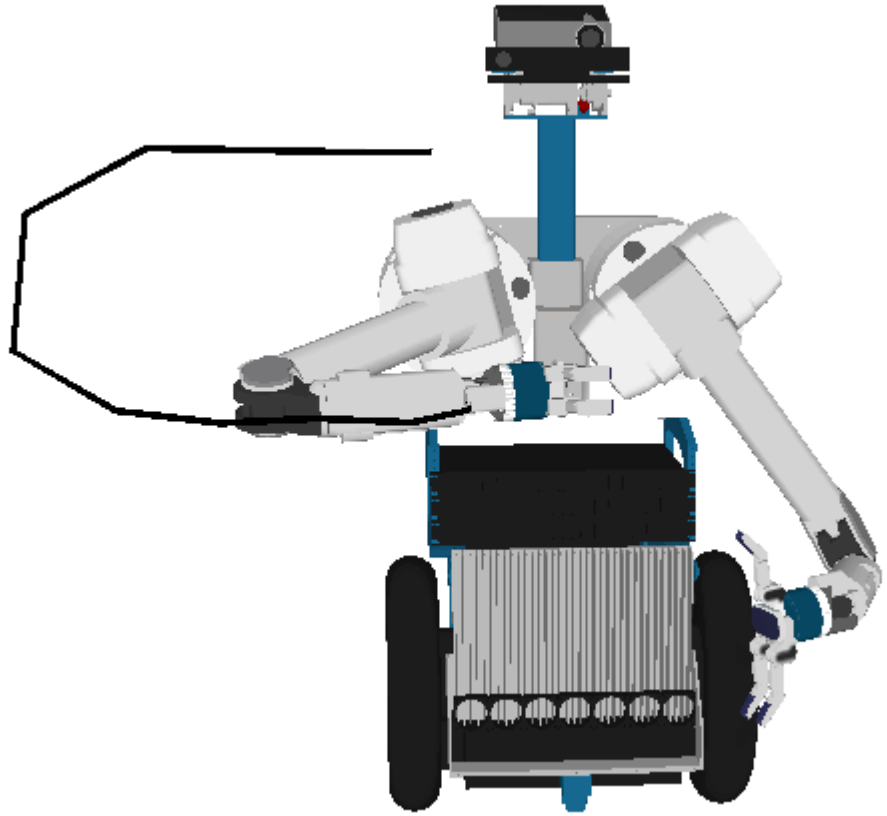
THE ROBOTICS INSTITUTE



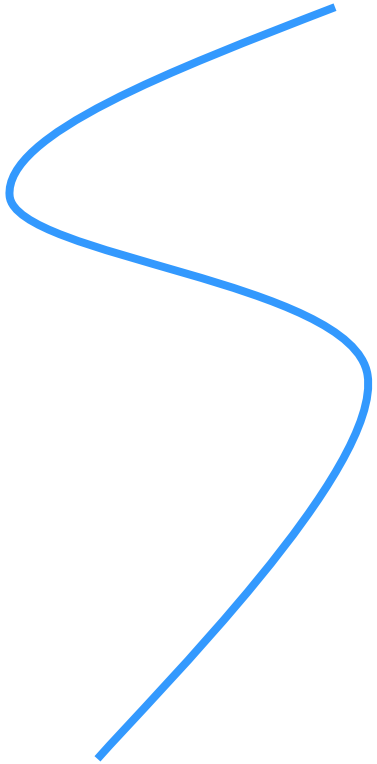


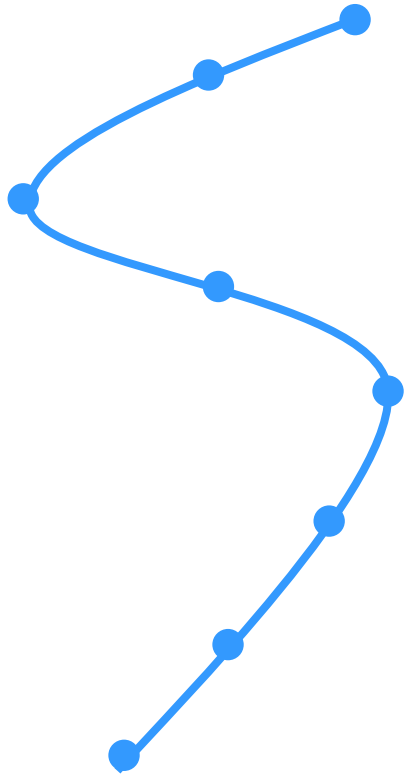


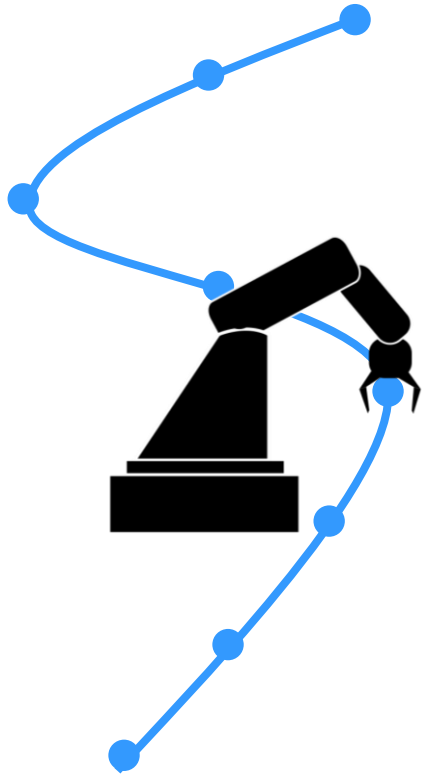


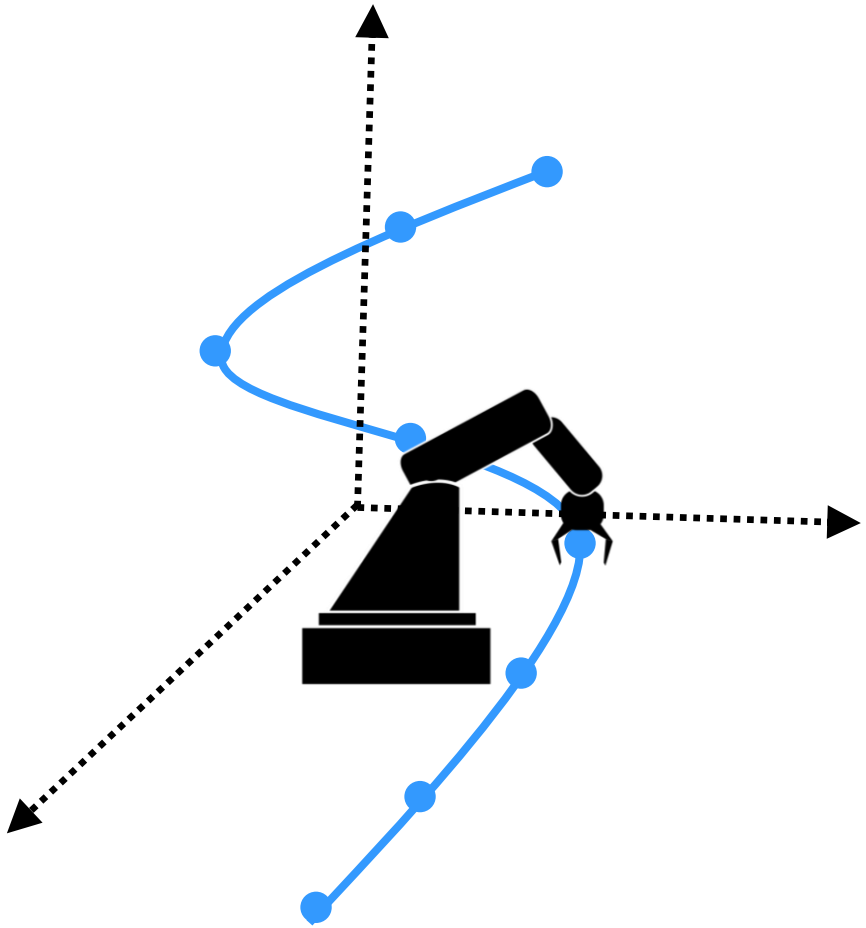


Goal: Follow Reference Path

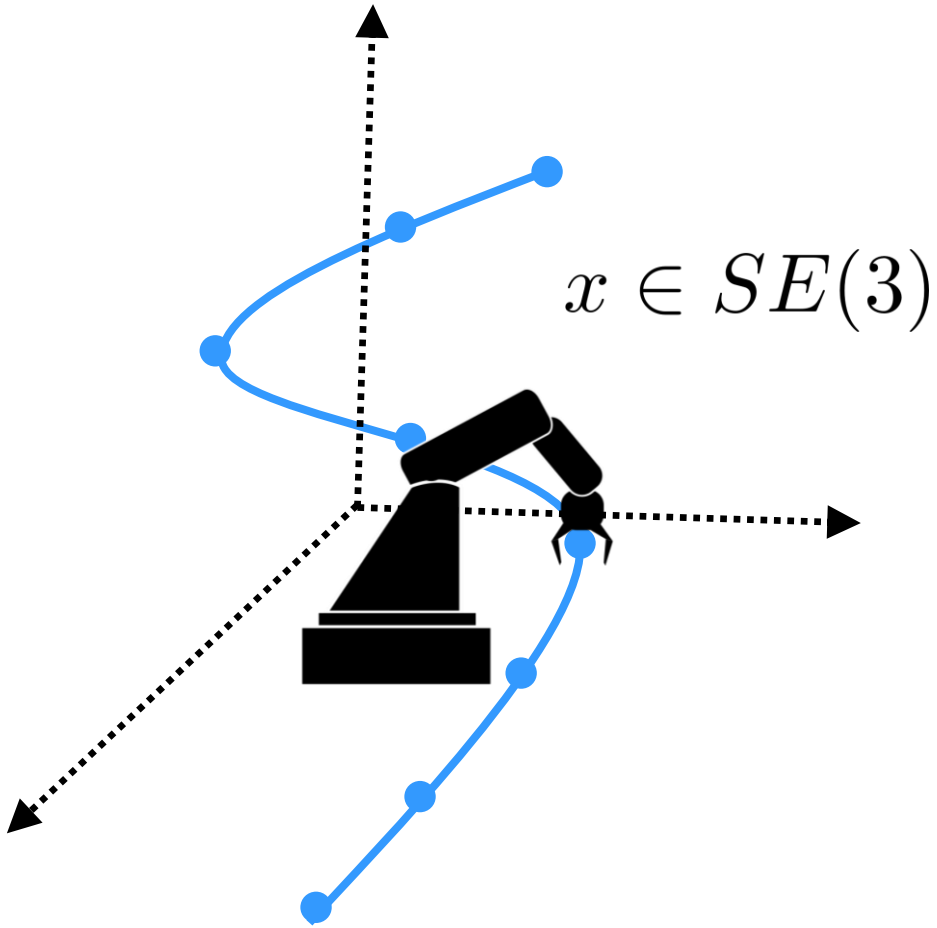








Task Space



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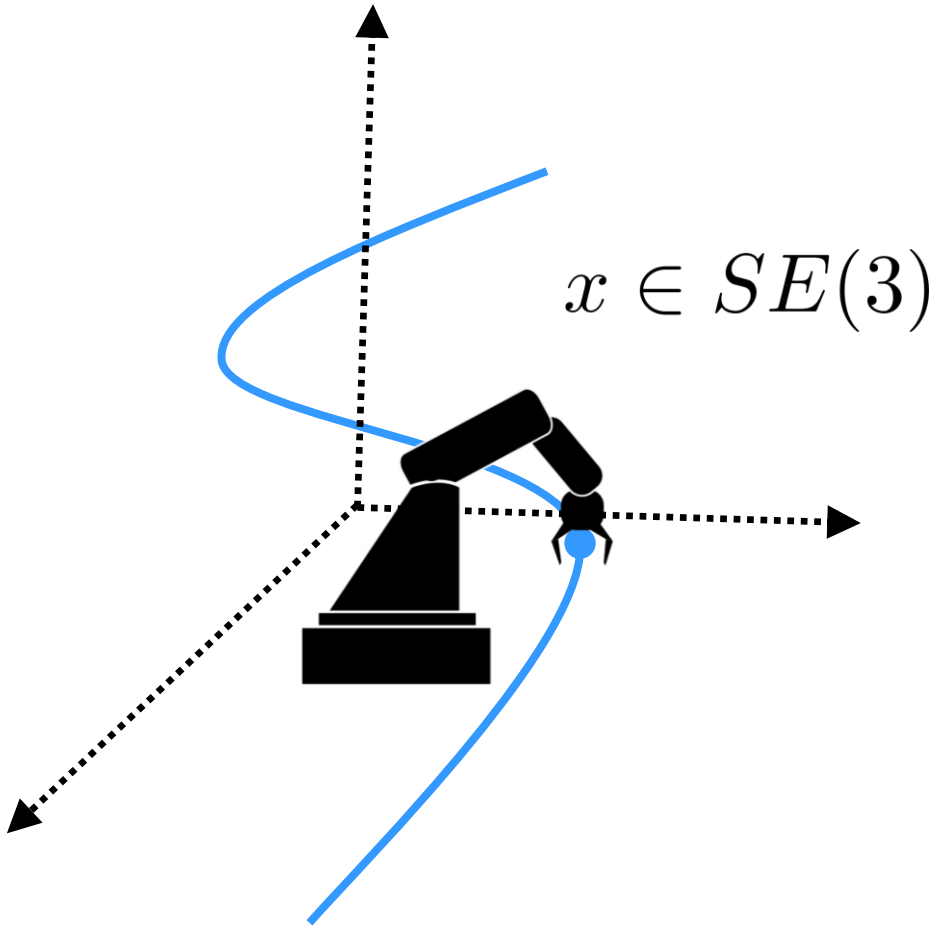
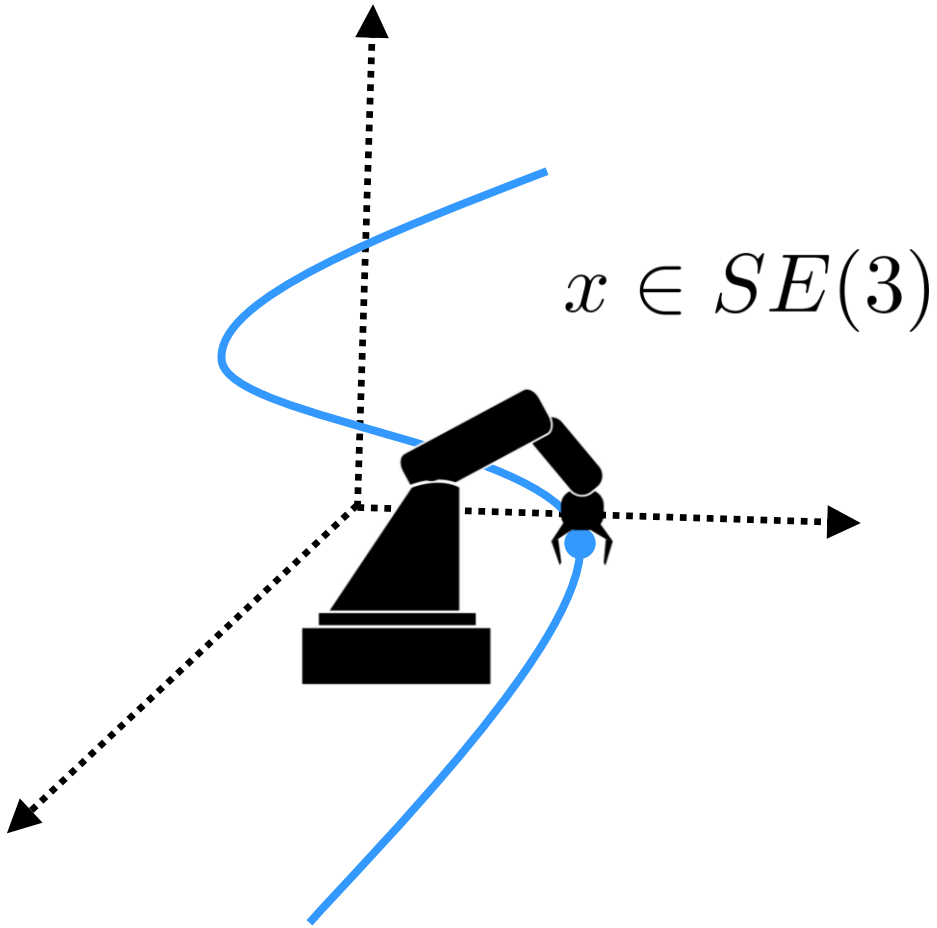
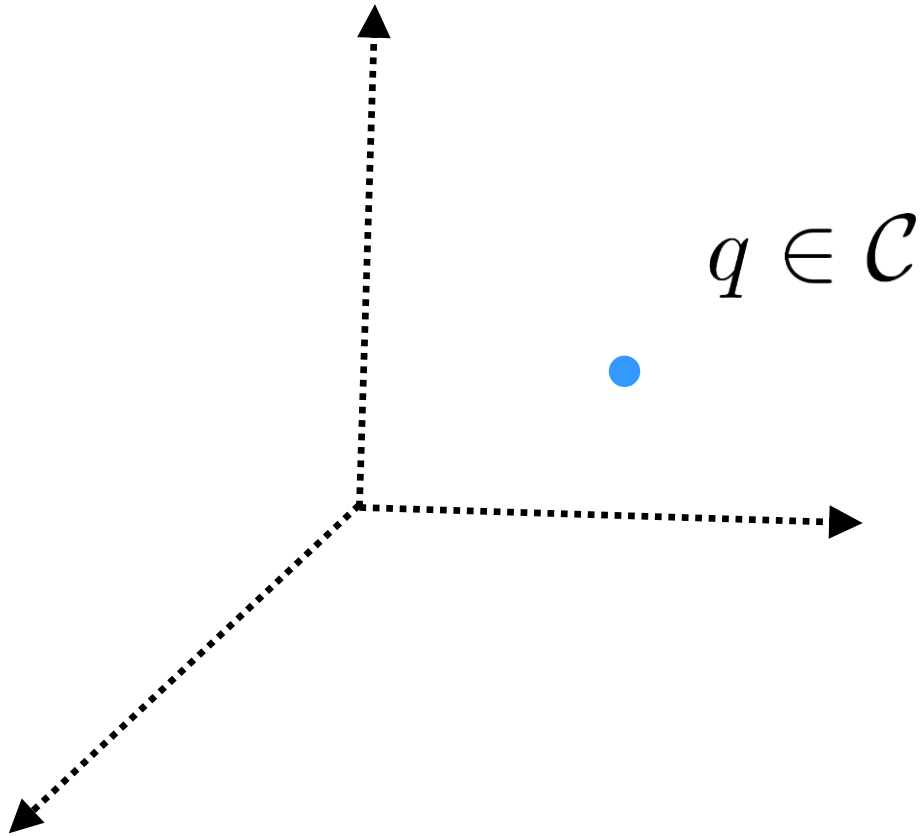
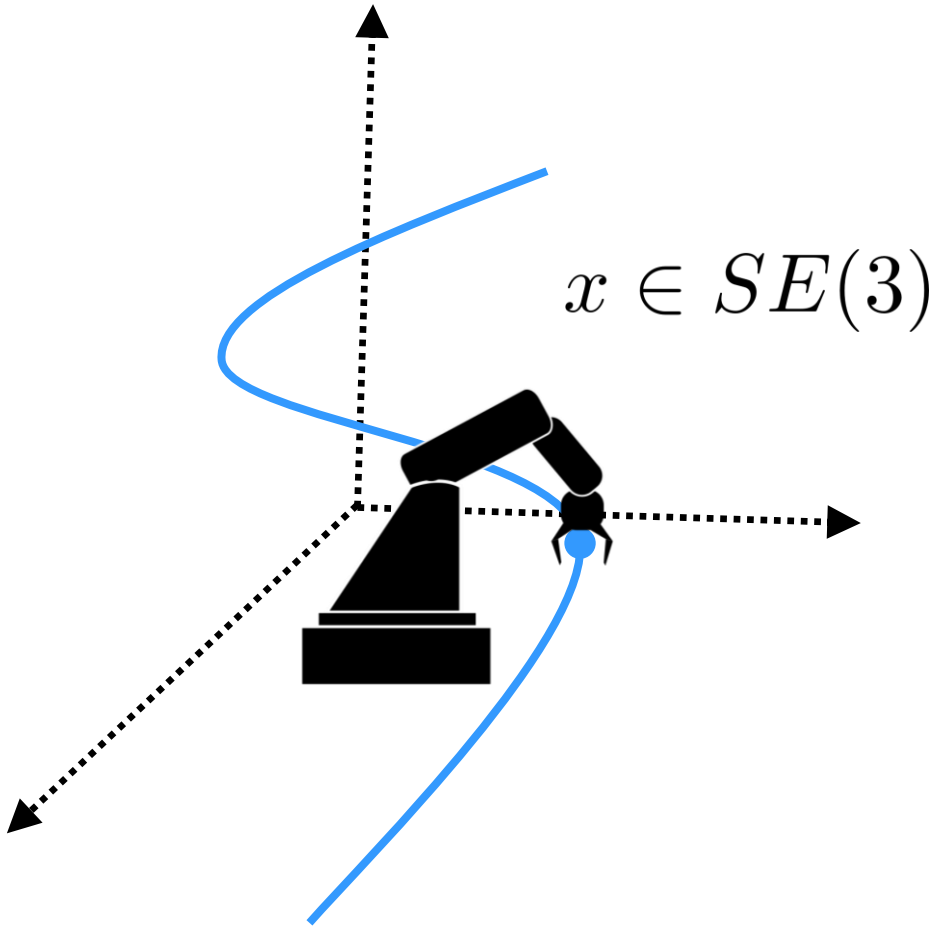


Task Space

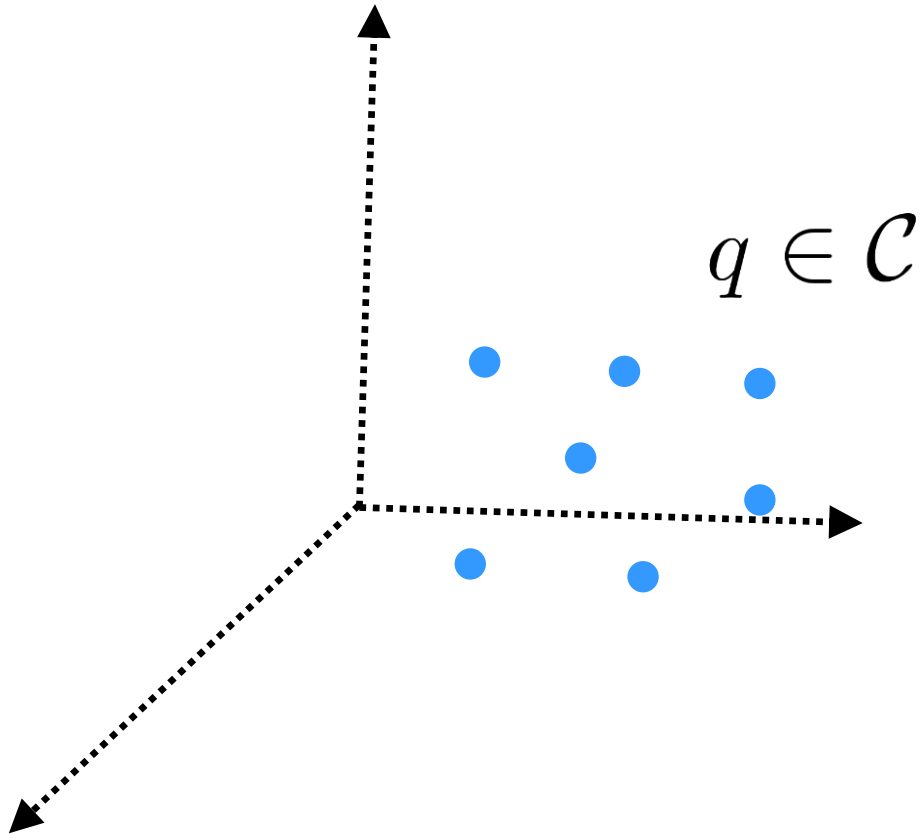




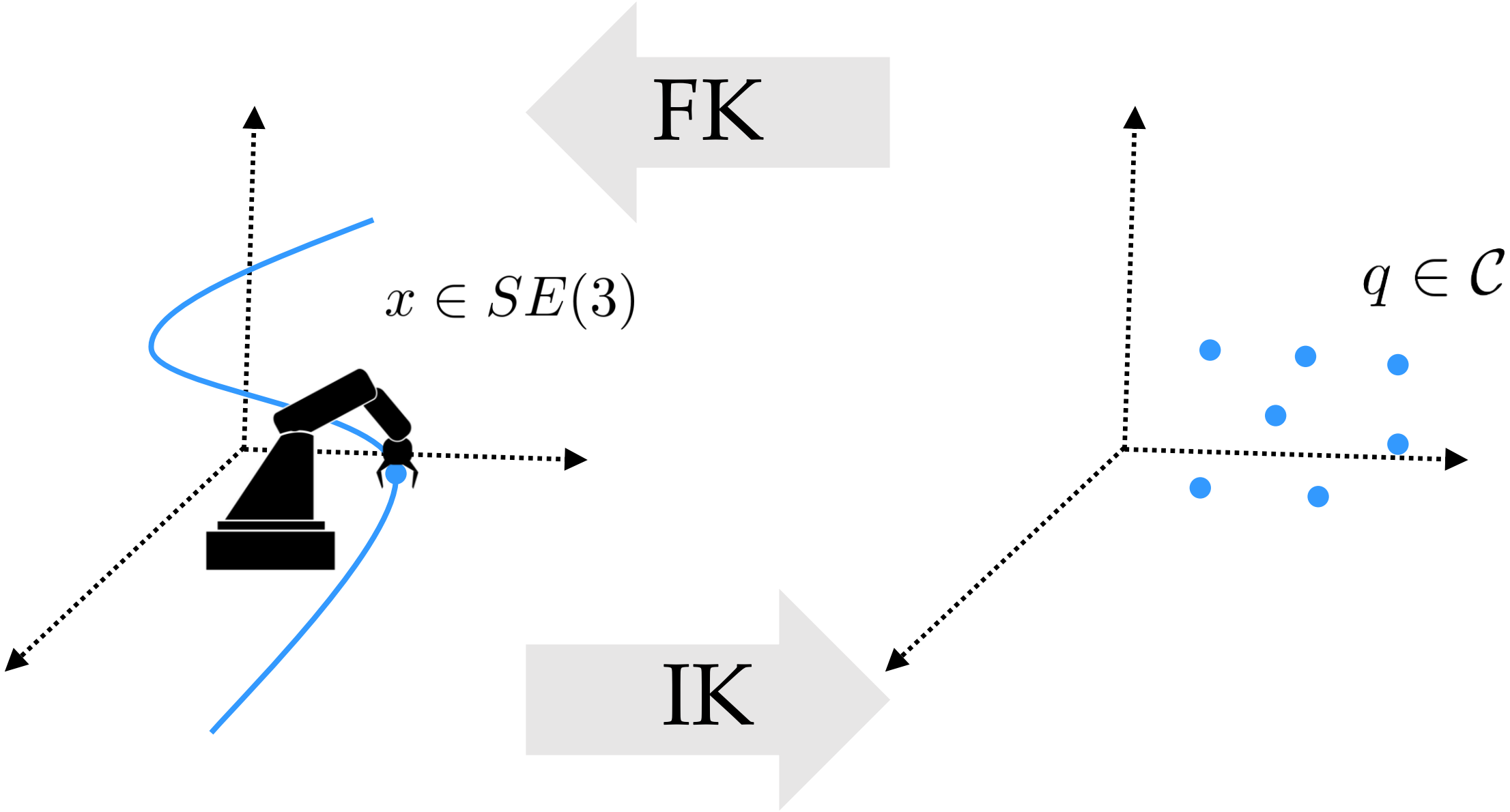
Task Space

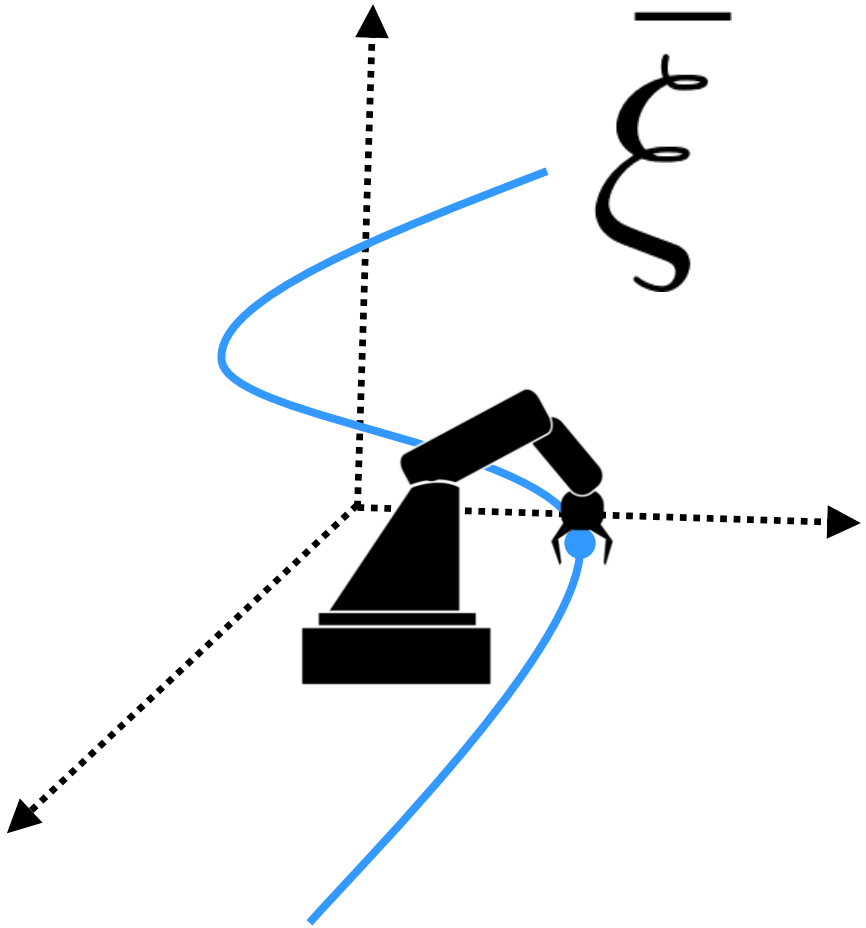


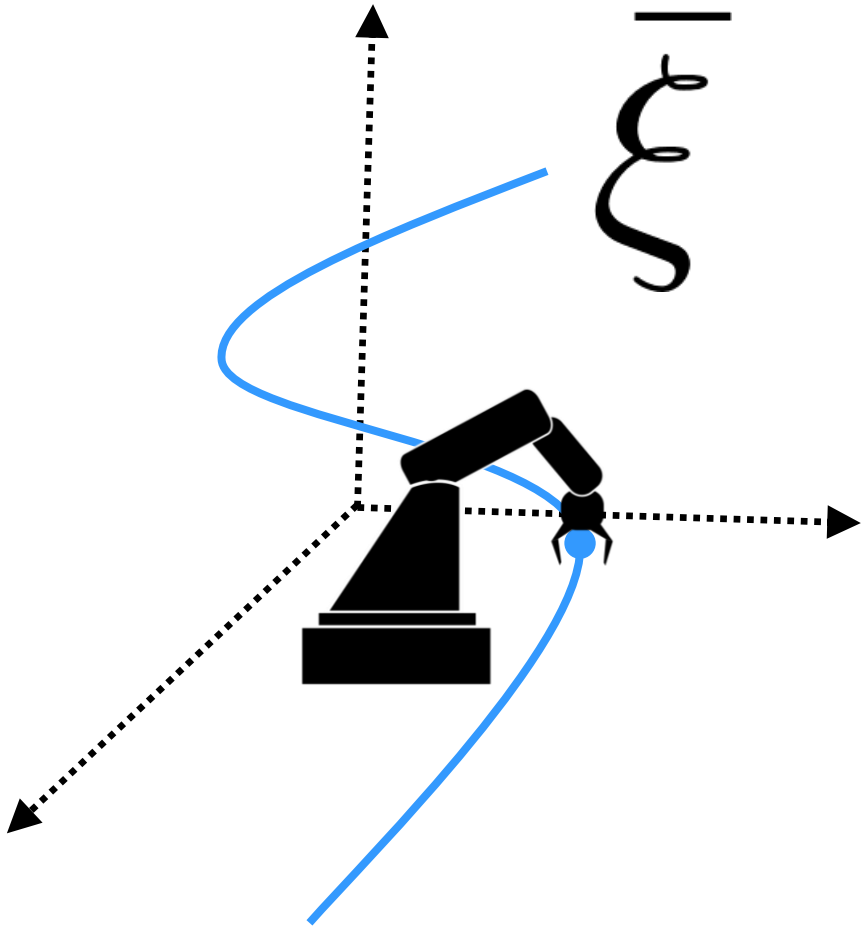
Task Space



Configuration Space


FK
$x \in SE(3)$
$q \in \mathcal{C}$
IK
Task Space
Configuration Space


$\bar{\xi}$



Goal: Generate a Configuration Space Path that Follows a Reference Path in Task Space.

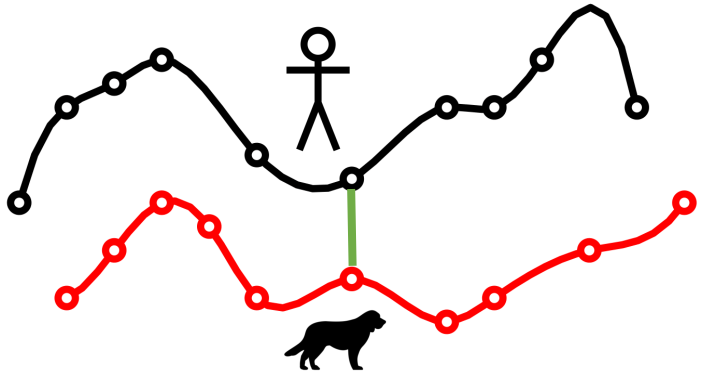
$$\xi^* = \arg\min_{\xi \in \Xi} ||FK(\xi) - \bar{\xi}||$$

$$\xi^* = \arg\min_{\xi \in \Xi} \underbrace{||FK(\xi) - \bar{\xi}||}$$

How do we capture distance?

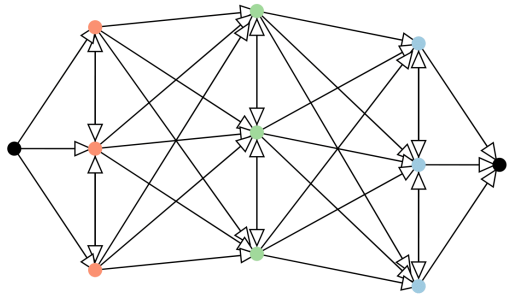
$$\xi^* = \underbrace{\arg\min_{\xi \in \Xi}}_{\text{How do we plan?}} \underbrace{||FK(\xi) - \bar{\xi}||}_{\text{How do we capture distance?}}$$

Distance Metrics

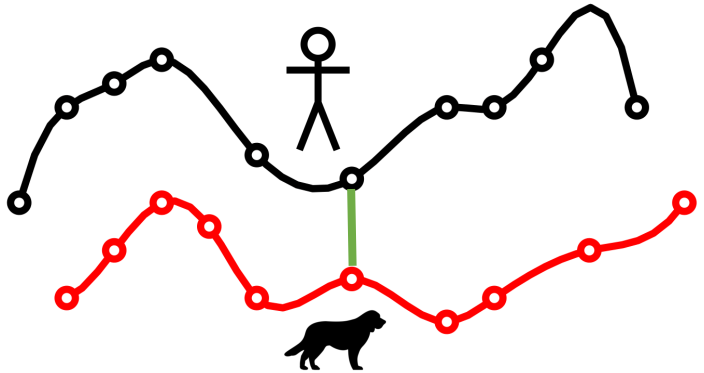


Trajectory Optimization

Cross Product Search

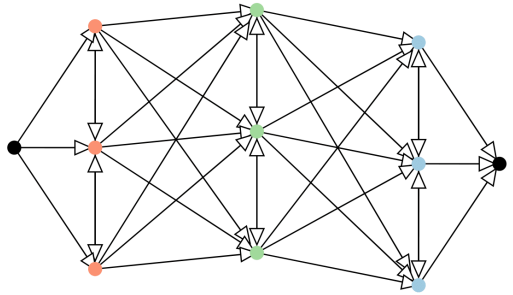


Distance Metrics



Trajectory Optimization

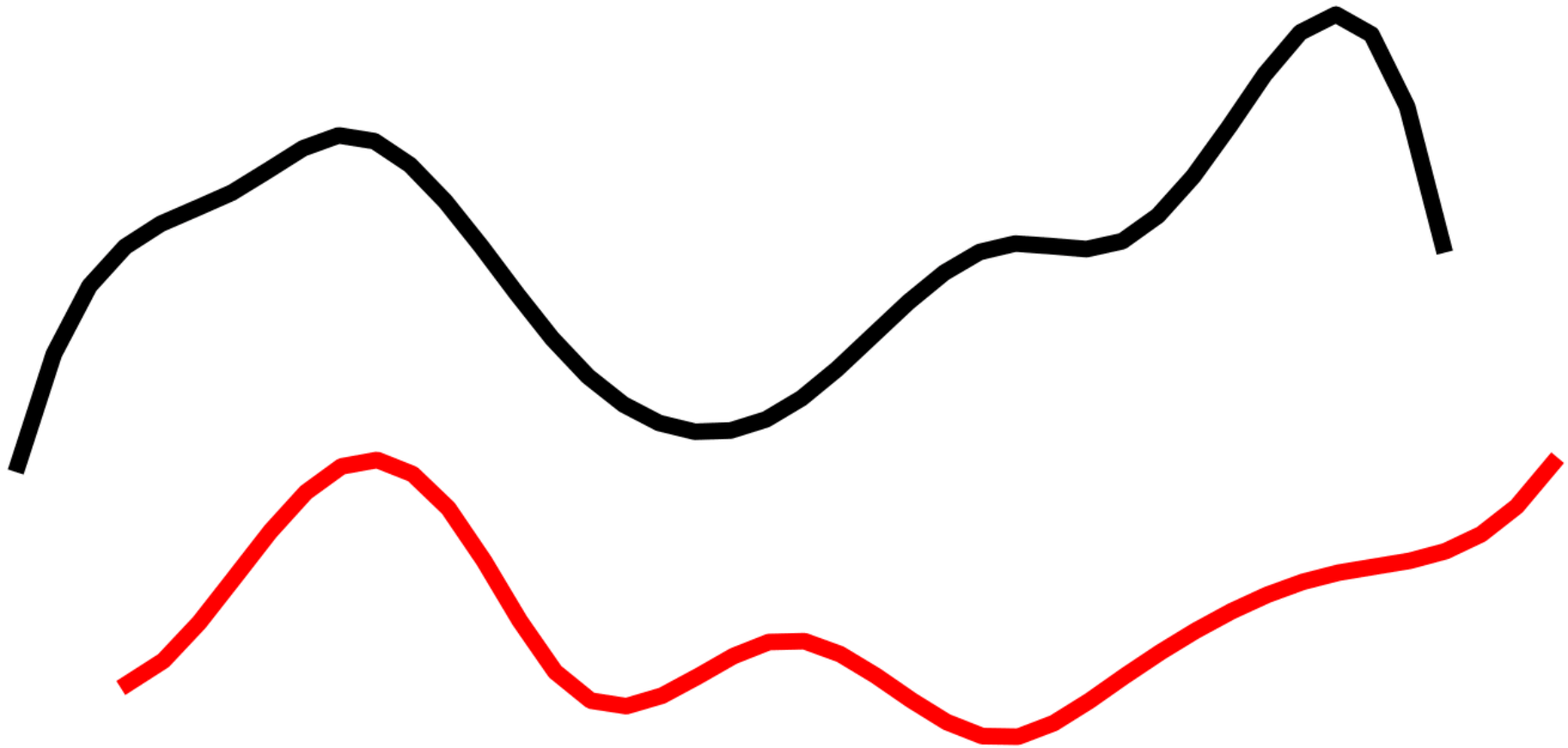
Cross Product Search

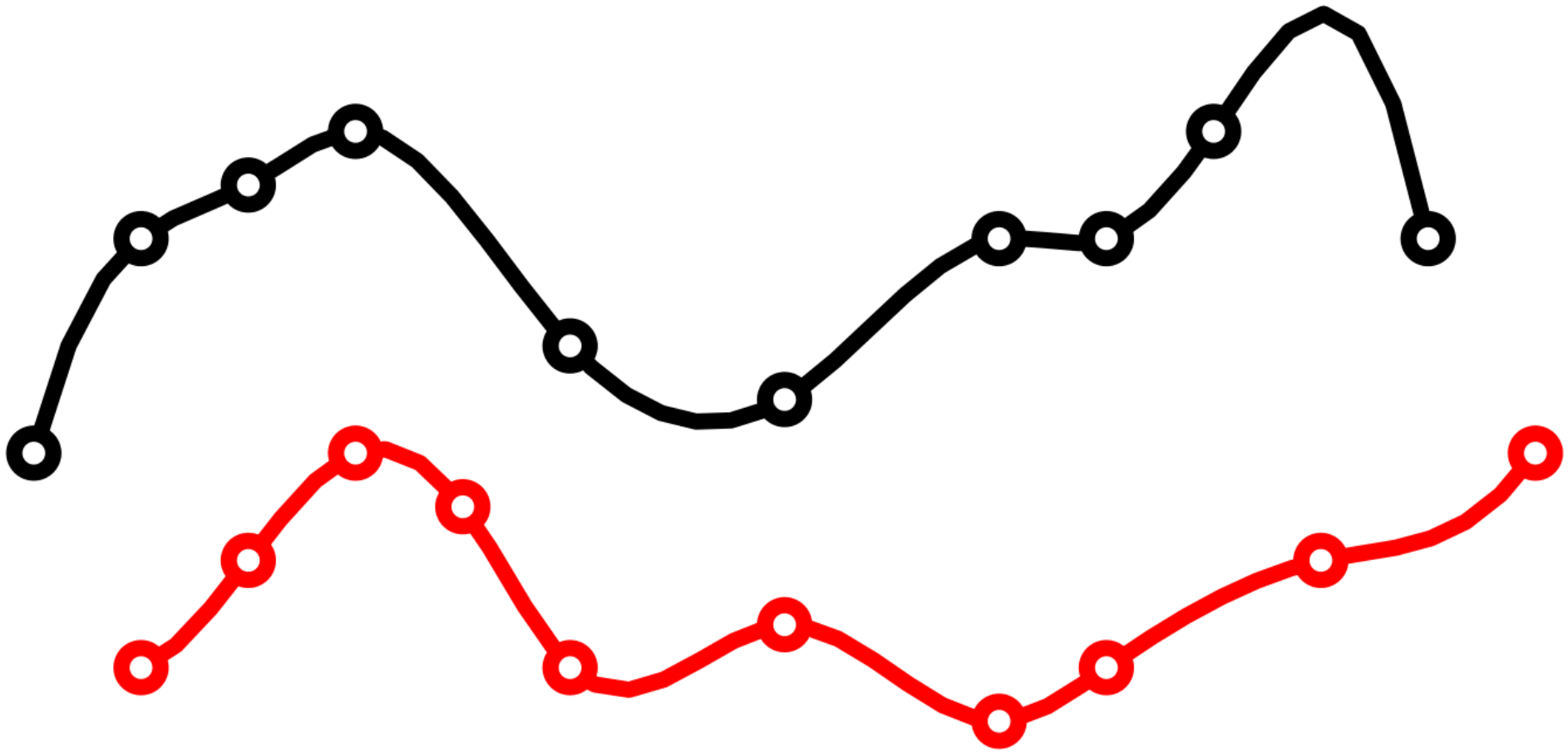


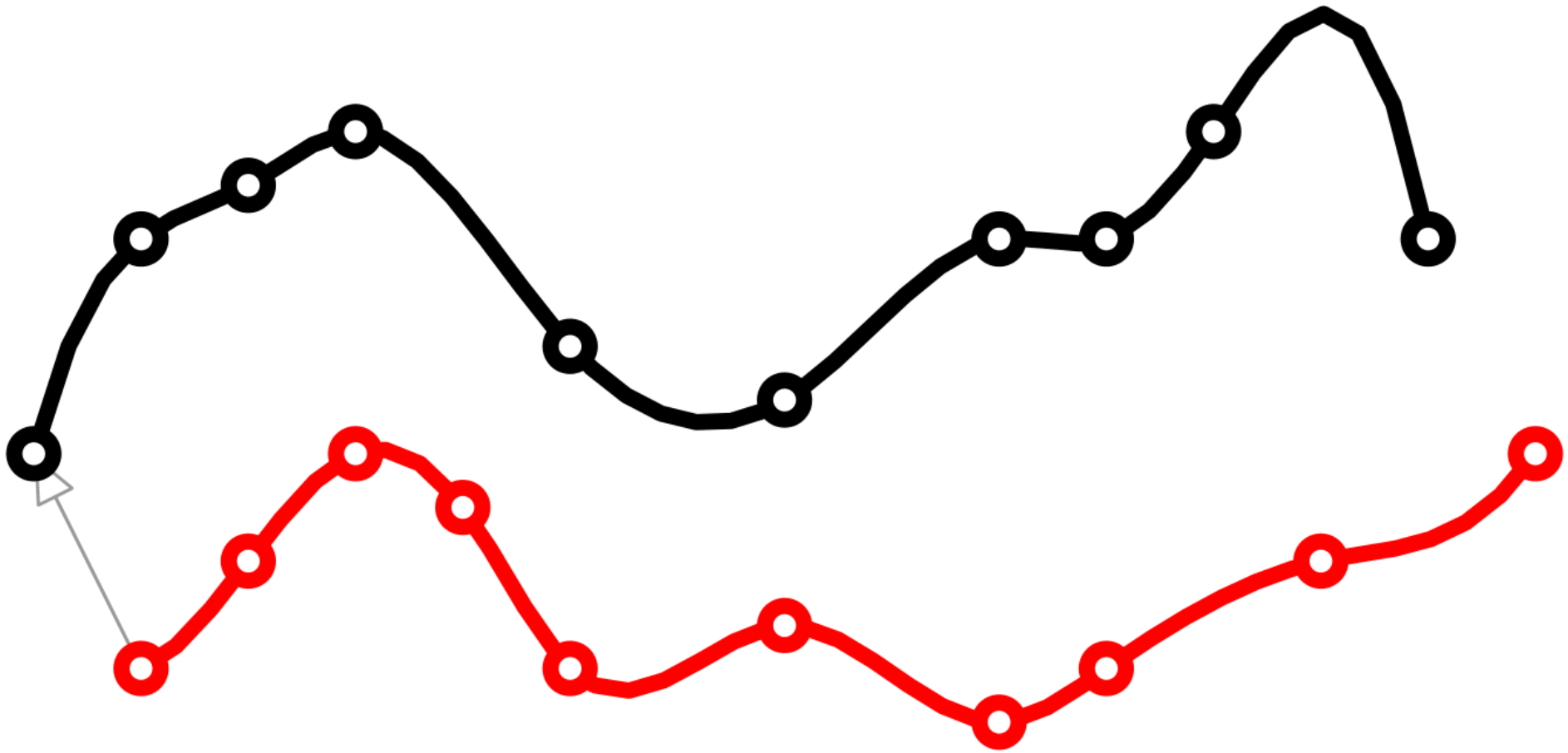
How to compare the distance between task space paths?

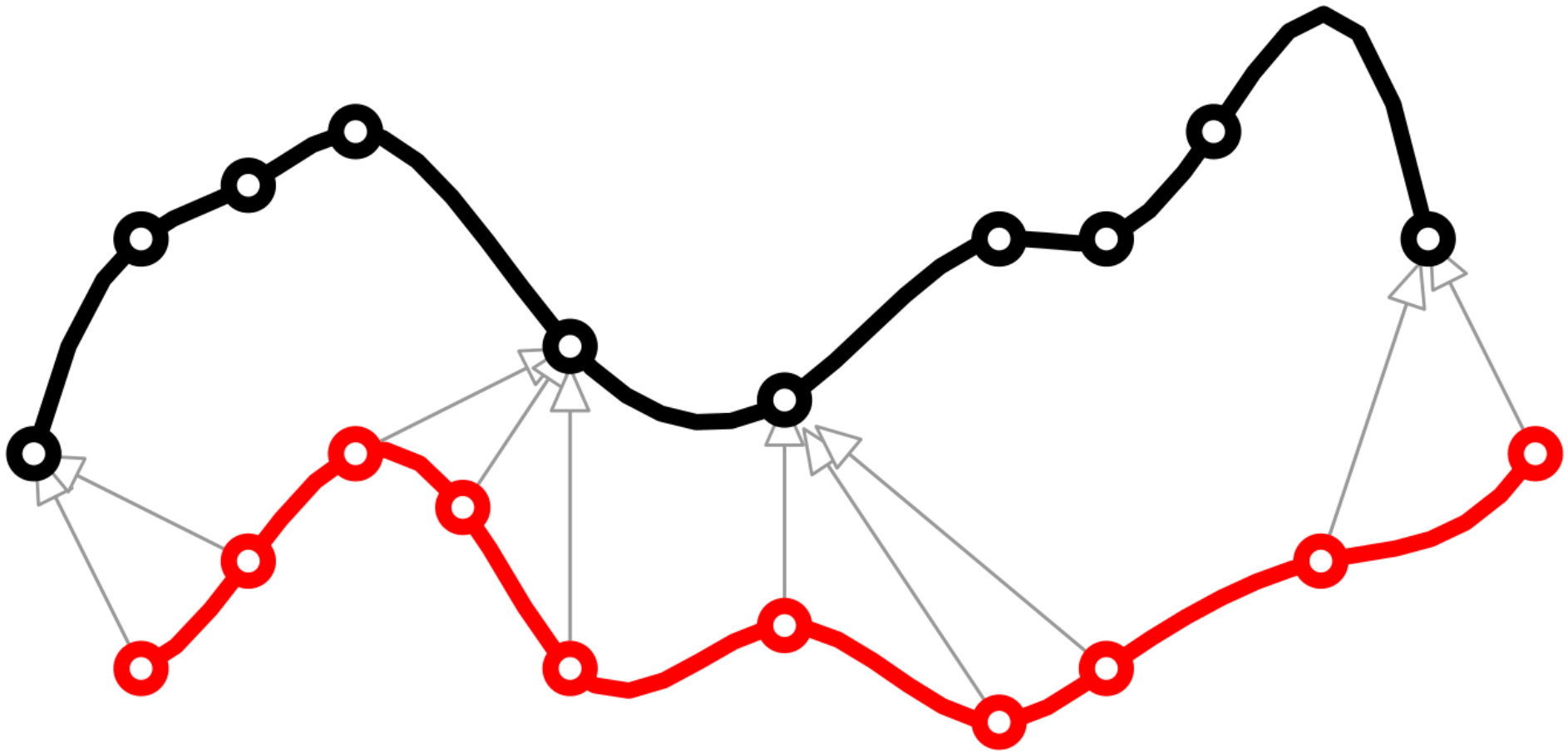
How to compare the distance between task space paths?

Borrow from computational geometry.

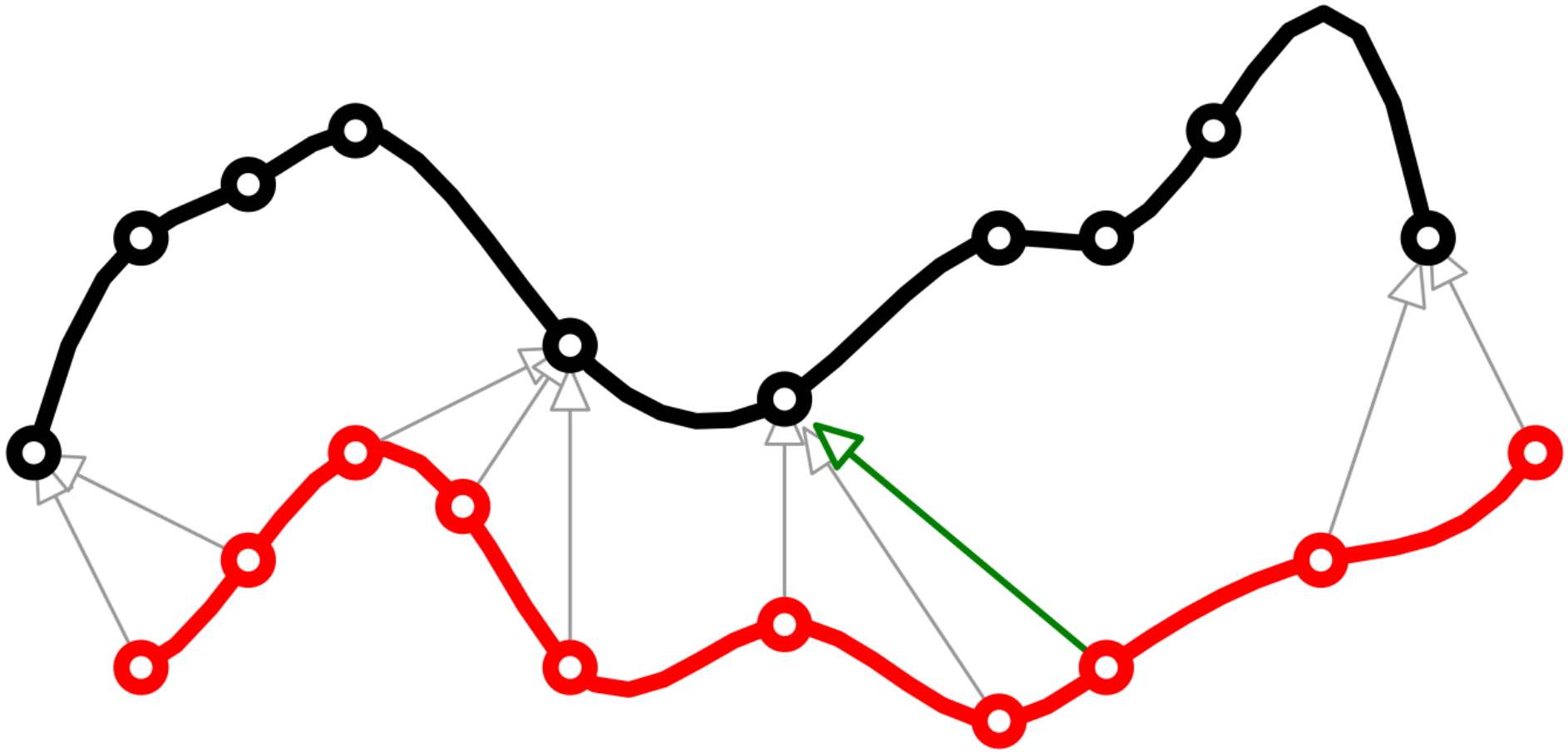


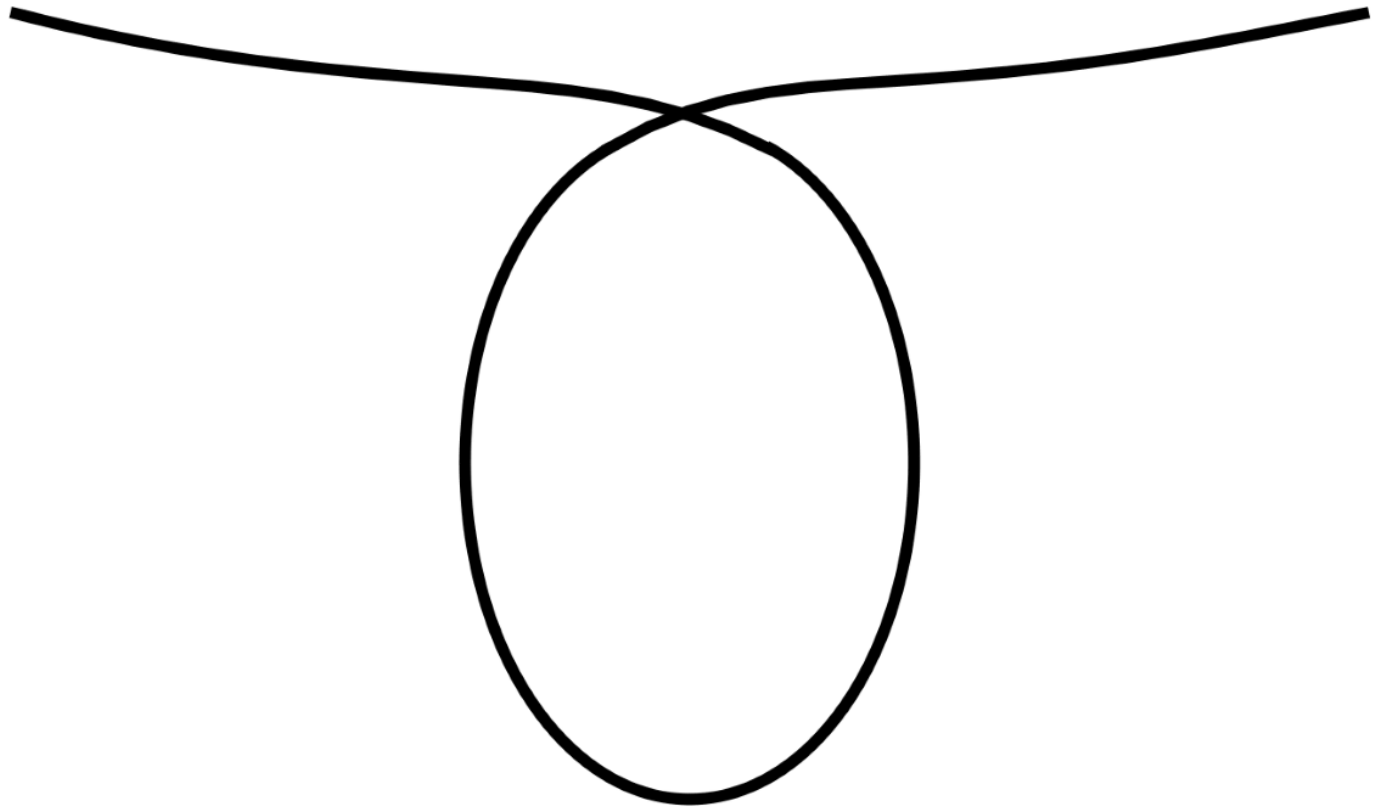


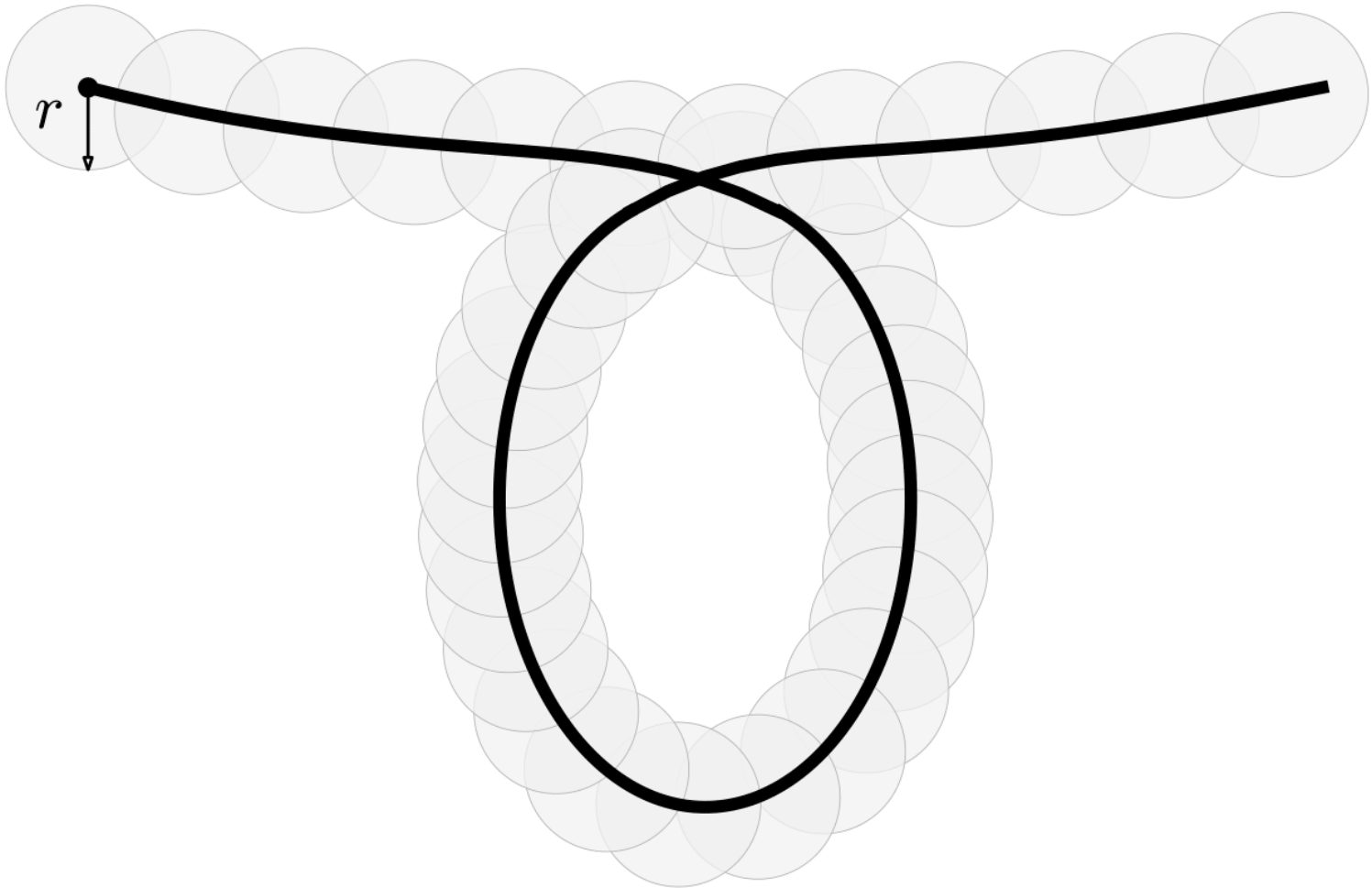




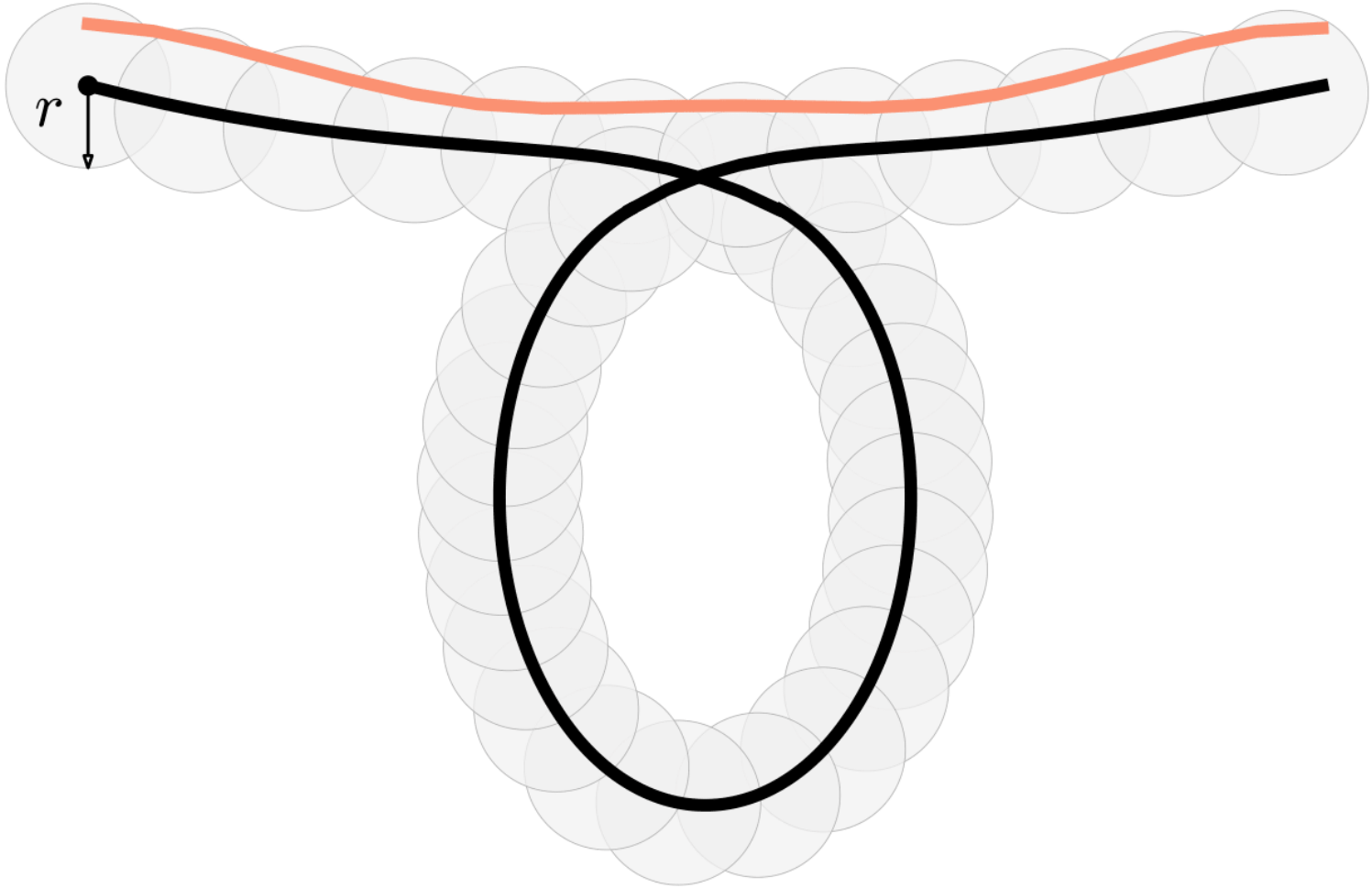
One-way Hausdorff Distance



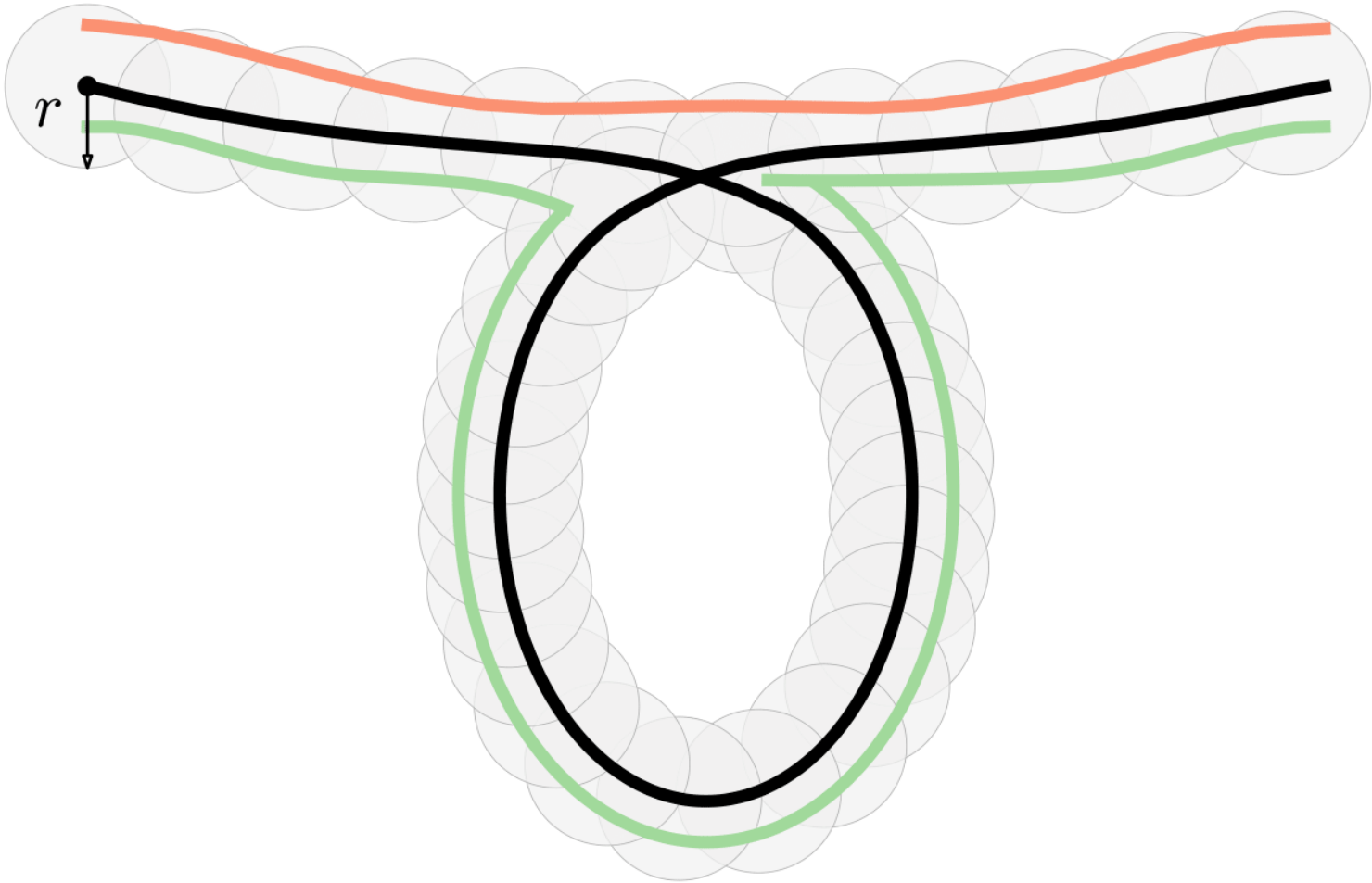



r

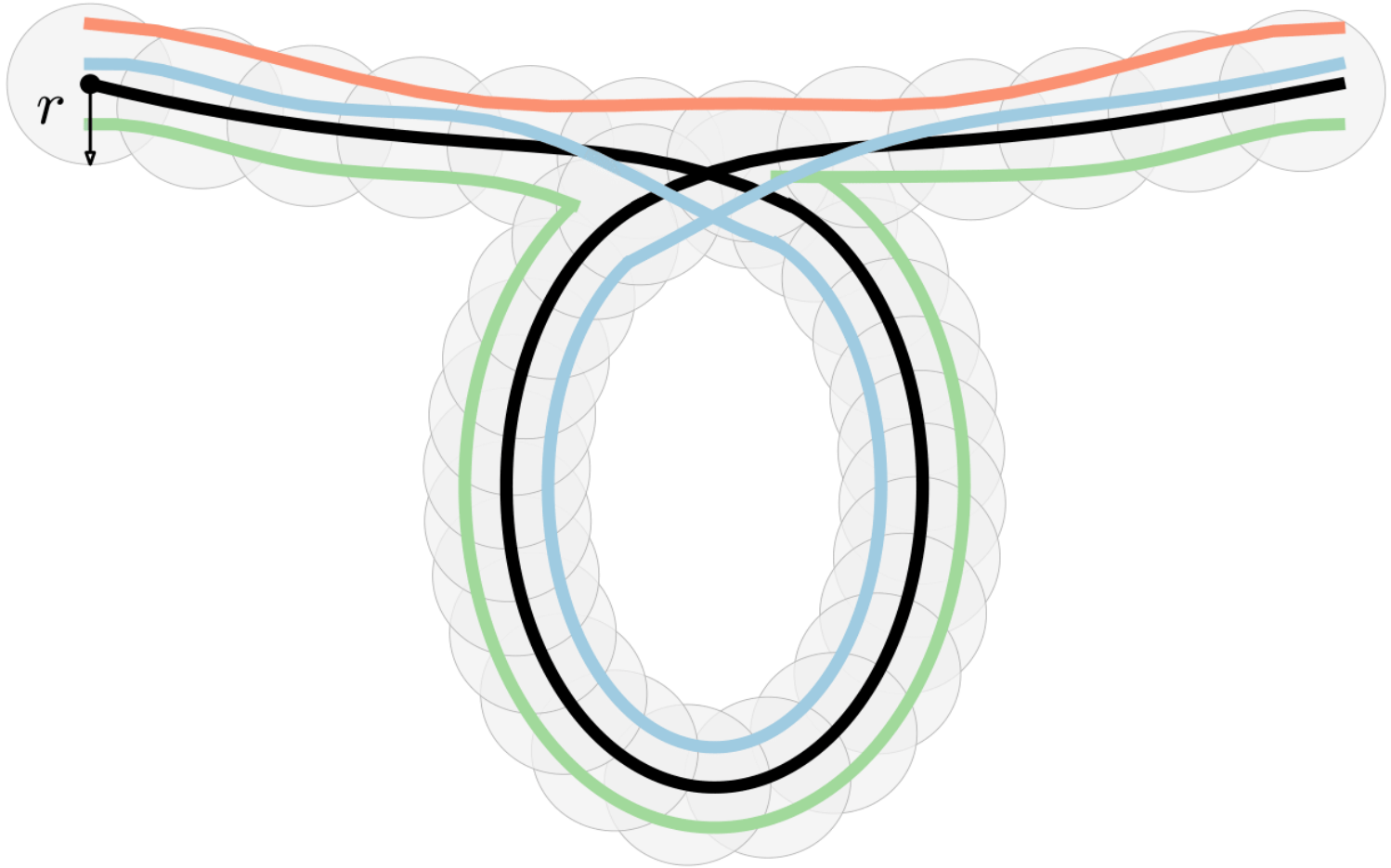
One-way Hausdorff Distance

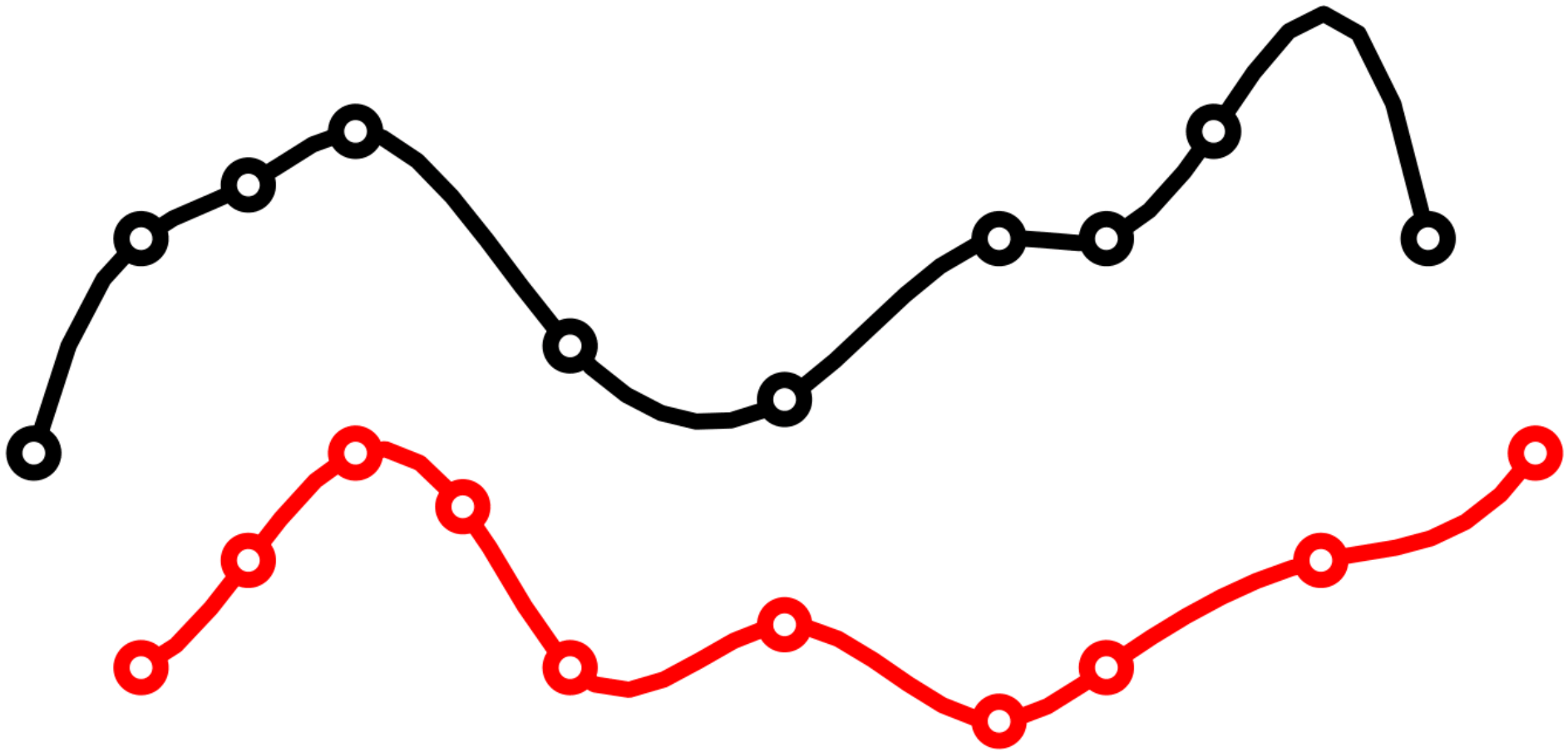


Two-way Hausdorff Distance



Follow Balls *in order*.

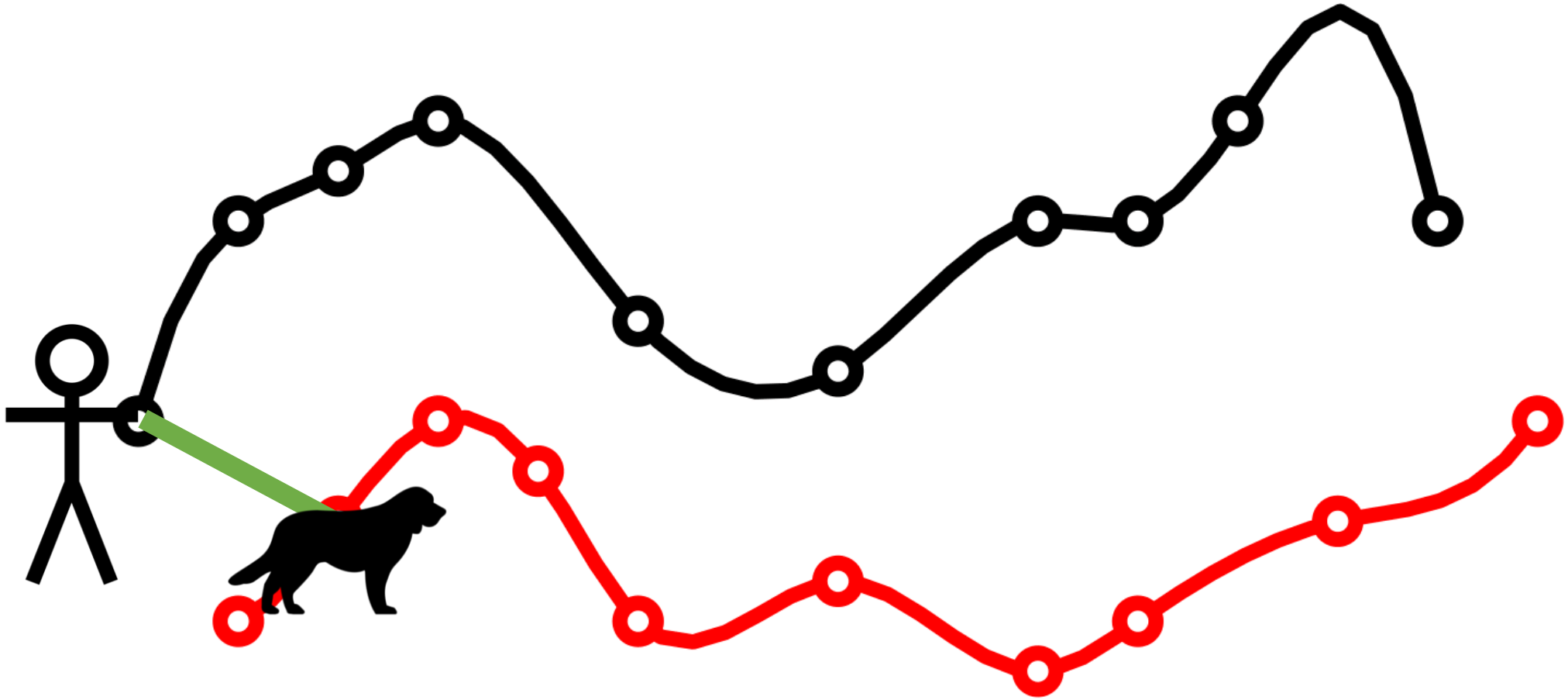




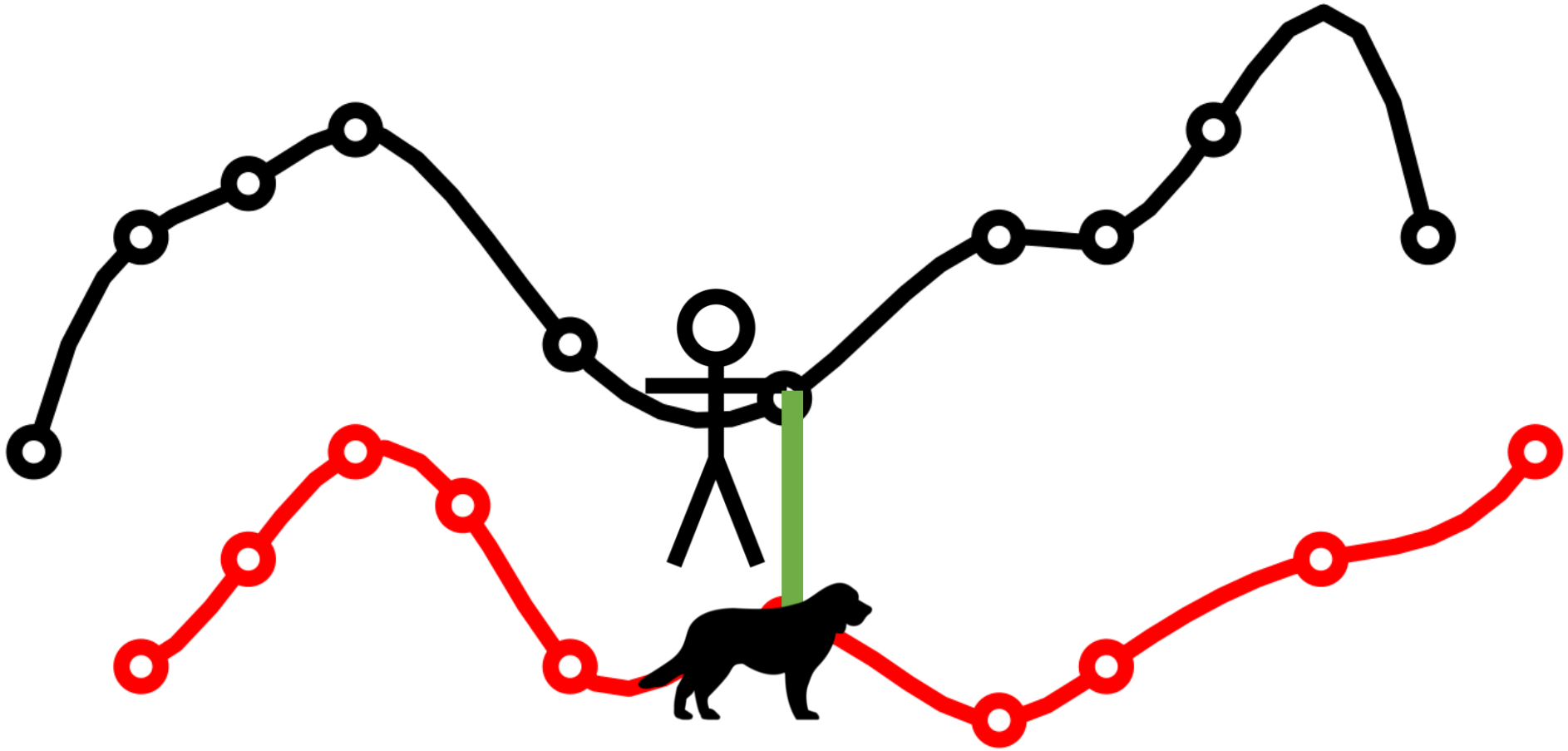
Fréchet Distance

Fréchet Distance

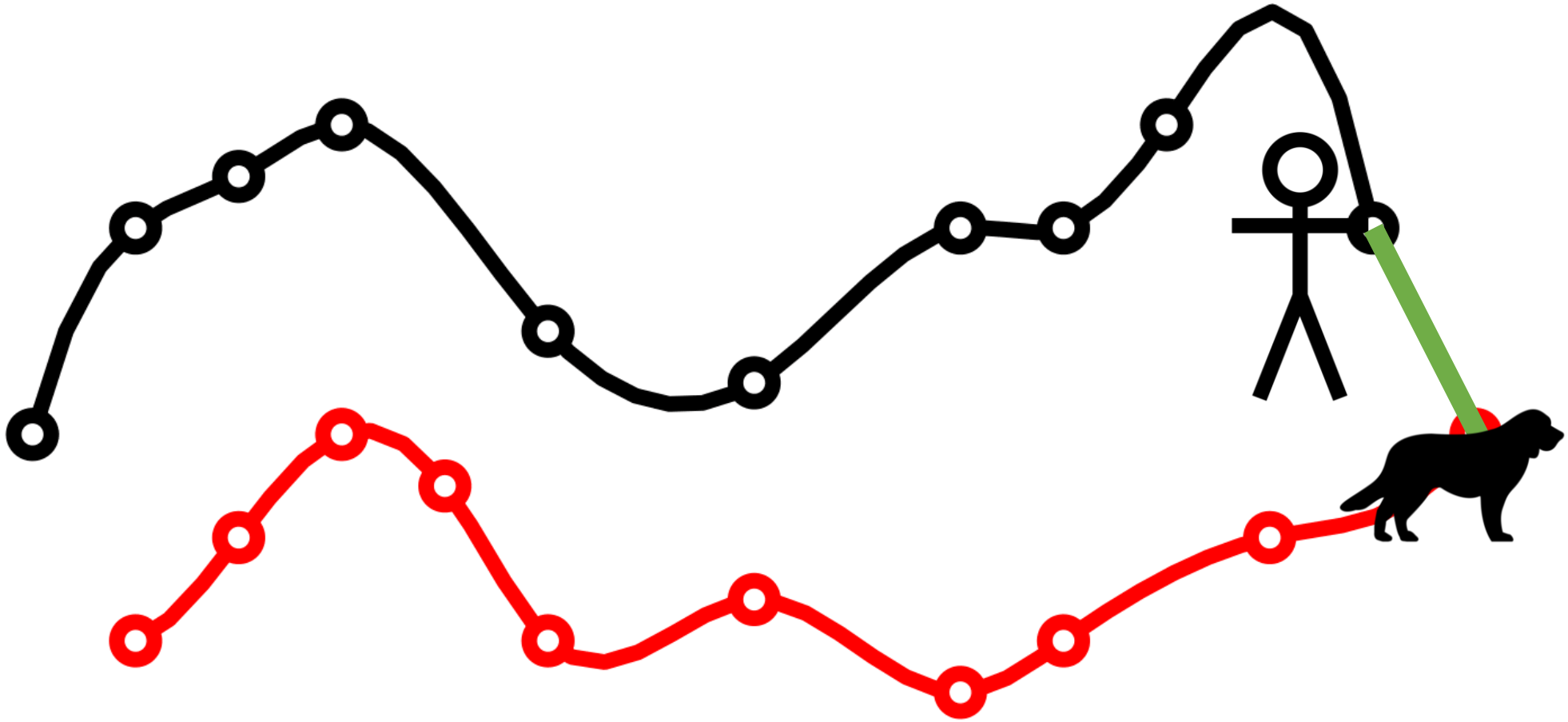
Fréchet Distance



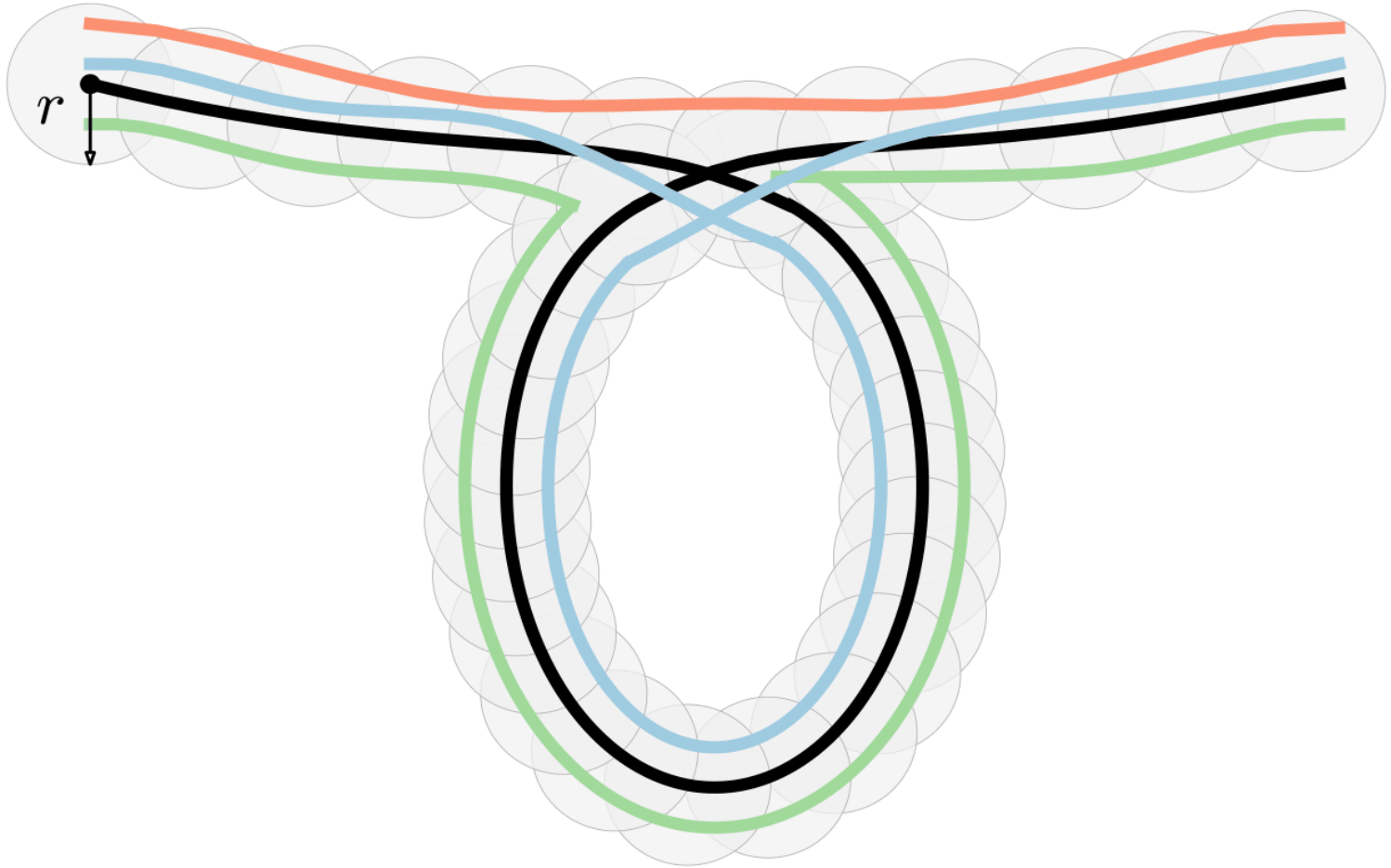
Fréchet Distance



Fréchet Distance



Fréchet Distance



$$\xi^* = \arg\min_{\xi \in \Xi} \underbrace{||\xi - \bar{\xi}||}_{} \quad \text{s.t. constraints}$$

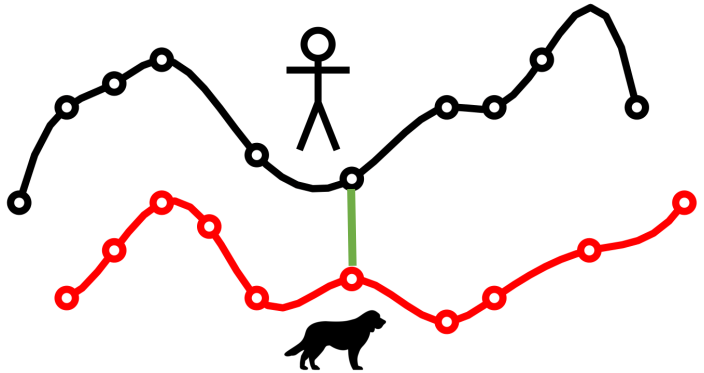
How do we capture distance?

$$\xi^* = \arg\min_{\xi \in \Xi} \underbrace{||\xi - \bar{\xi}||}_{} \quad \text{s.t. constraints}$$

How do we capture distance?

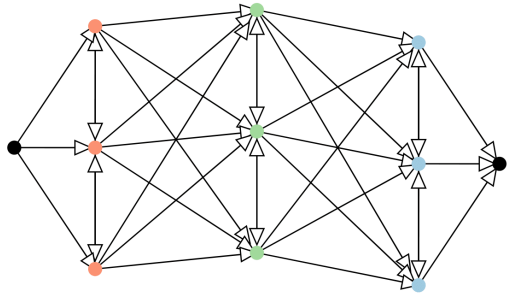
Discrete Fréchet Distance

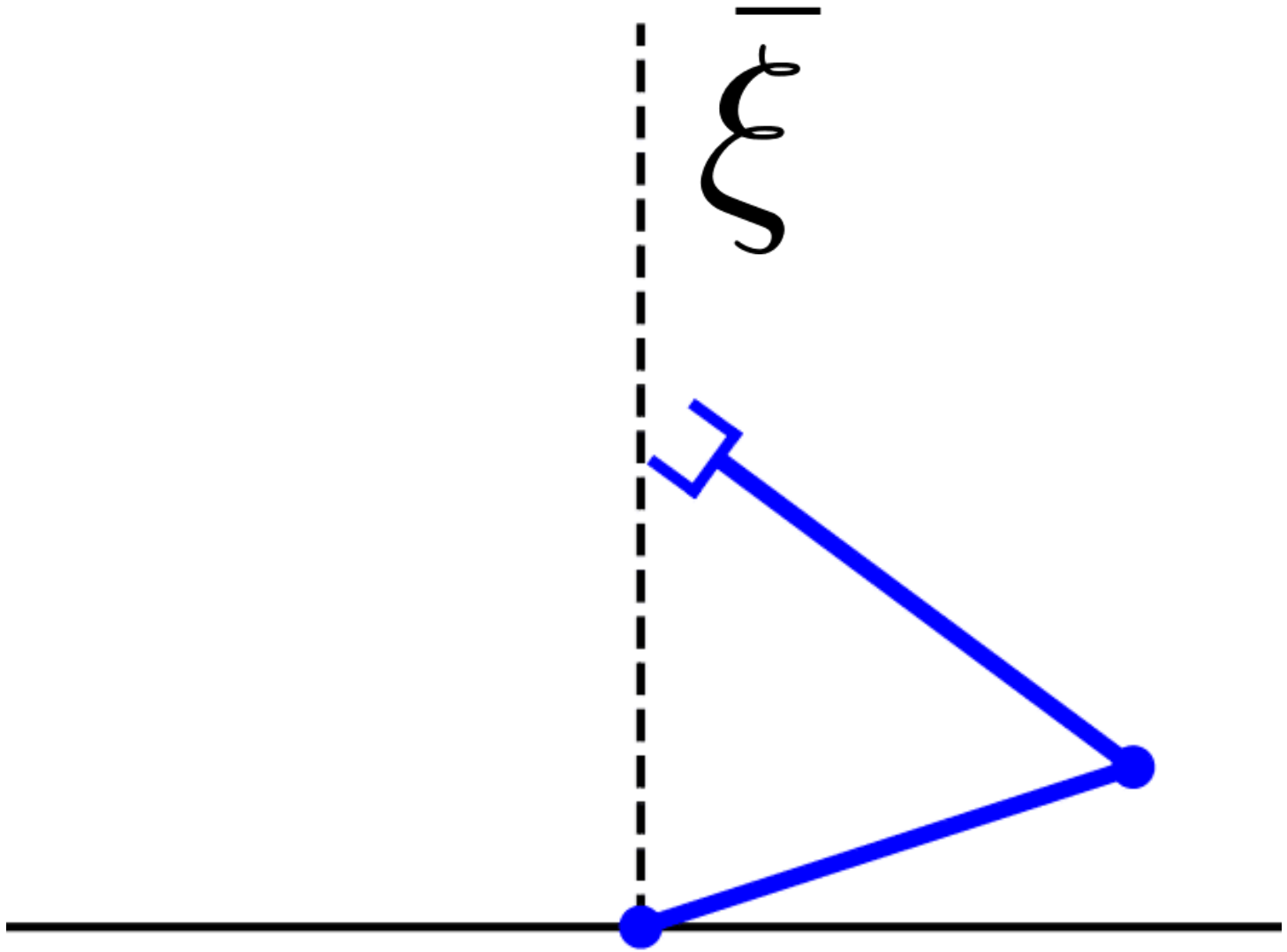
Distance Metrics



Trajectory Optimization

Cross Product Search

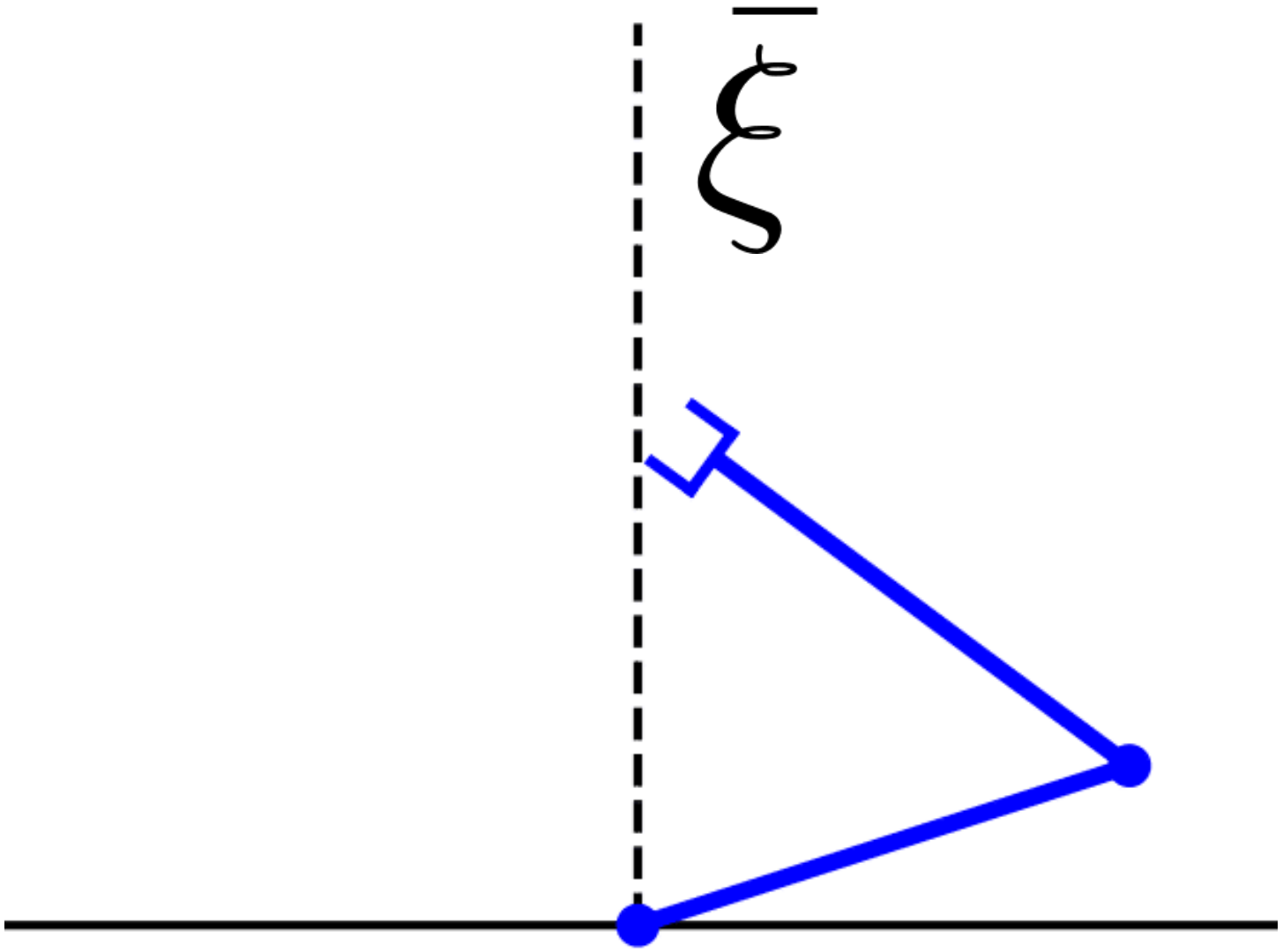



$\bar{\xi}$

Initialize

Start: q_0

End: q_n

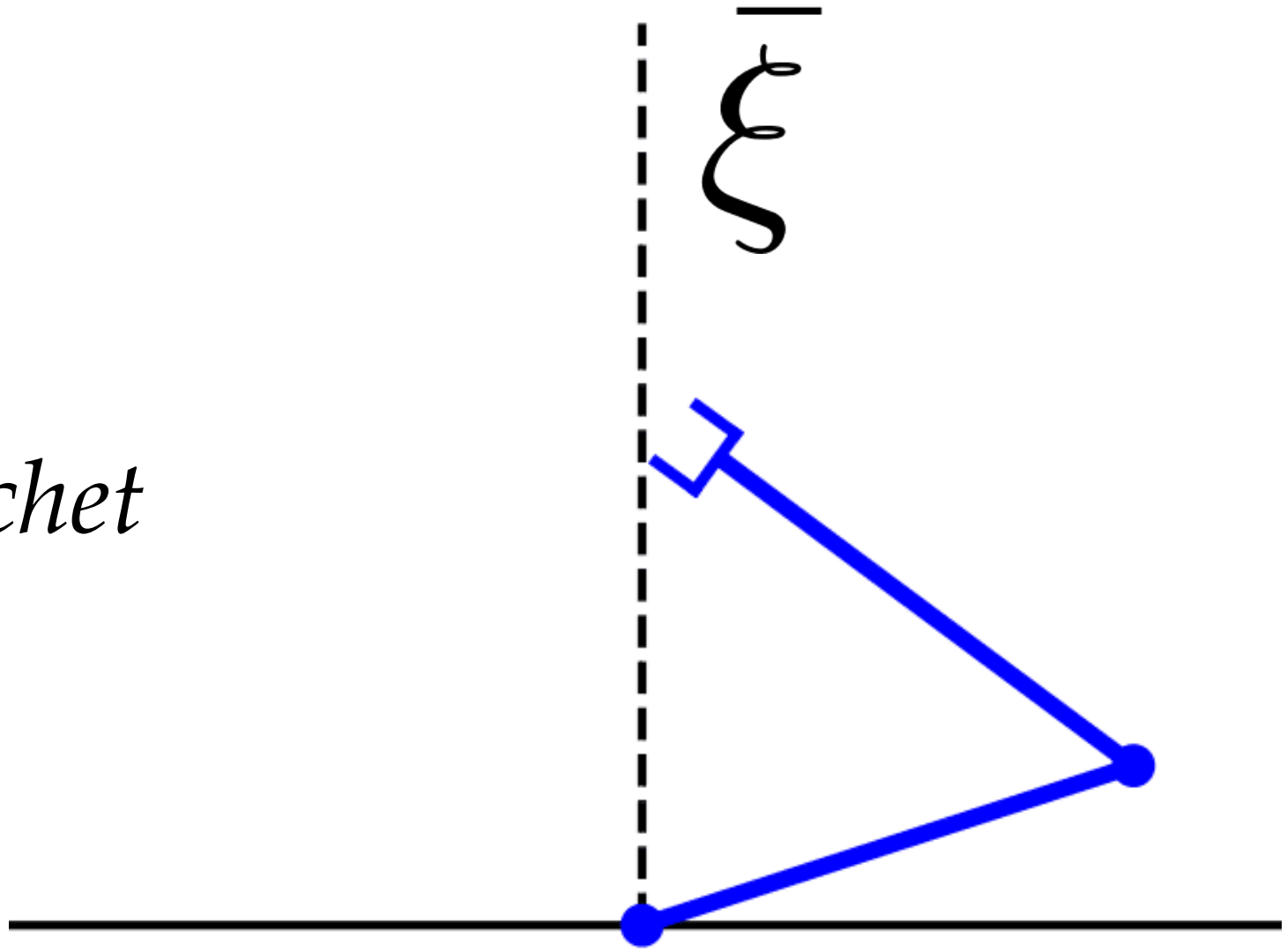


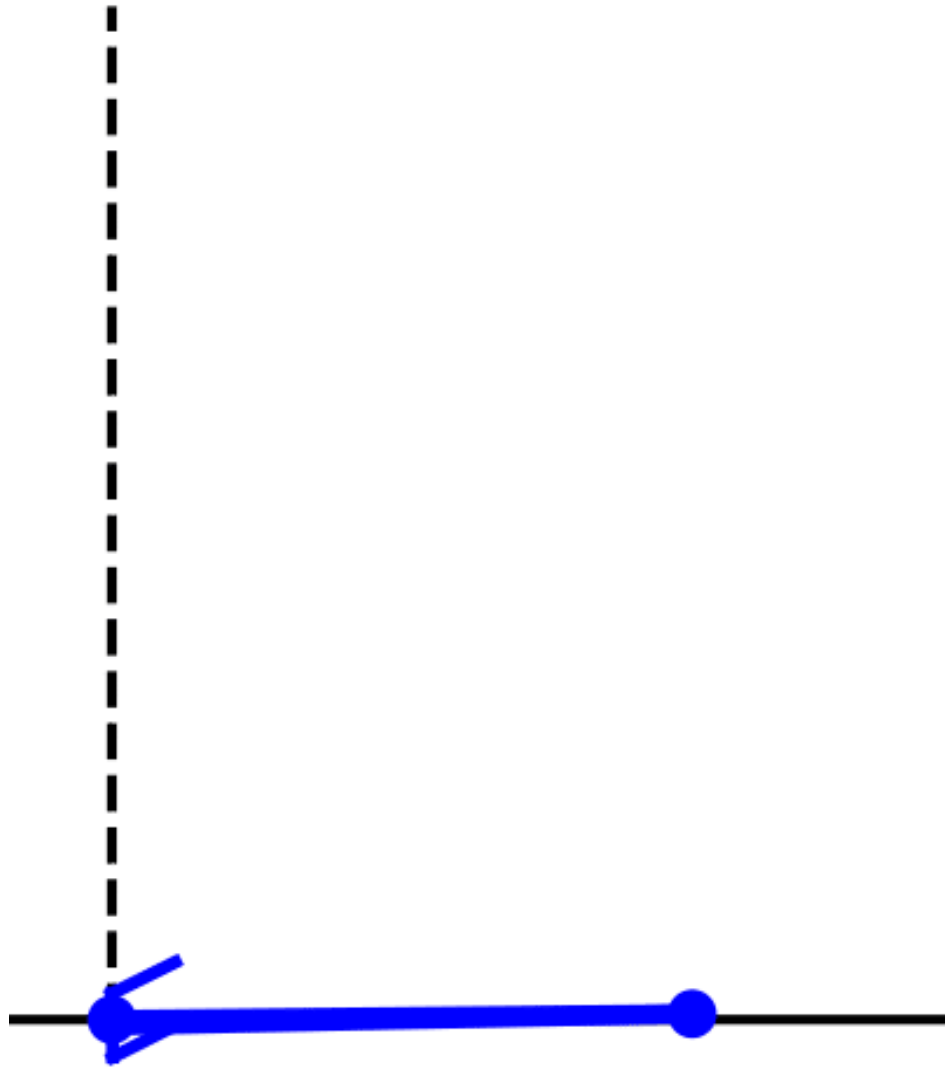
Initialize

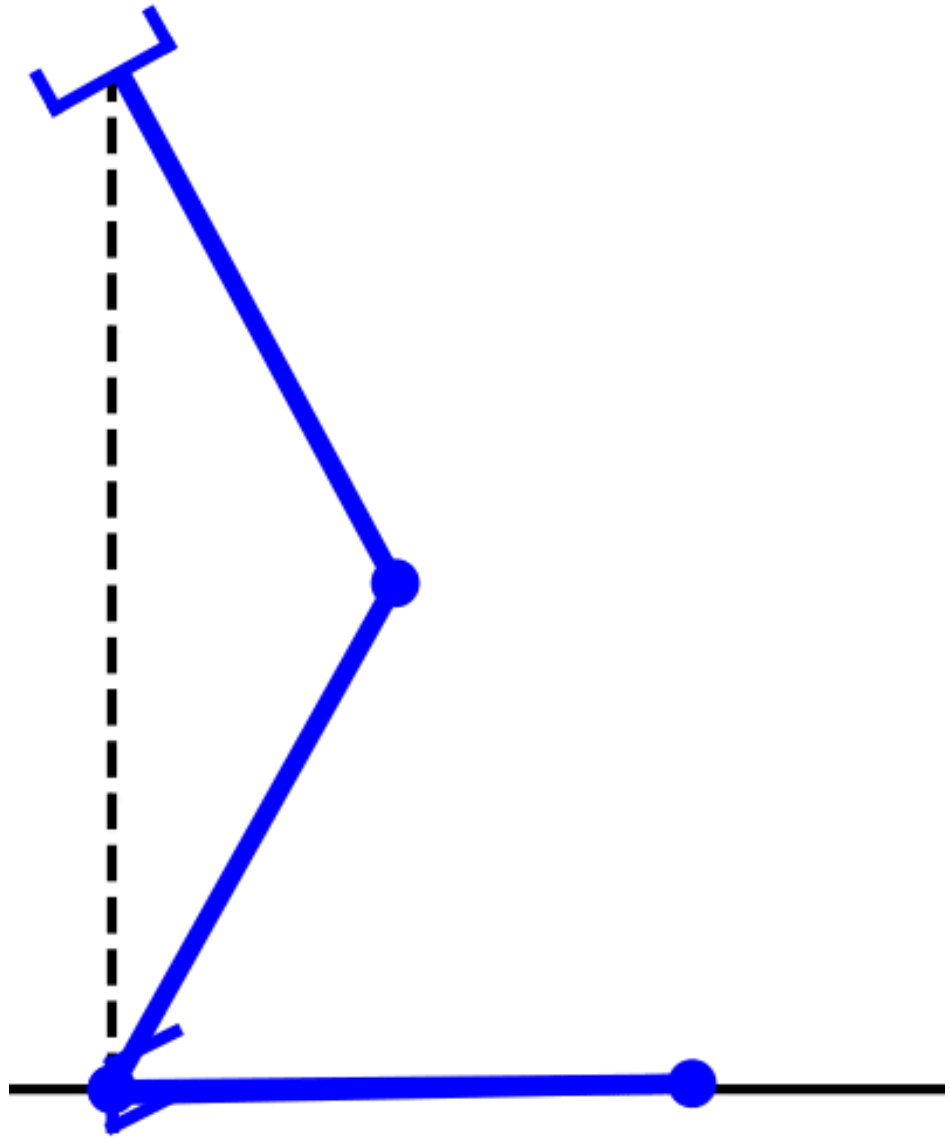
Start: q_0

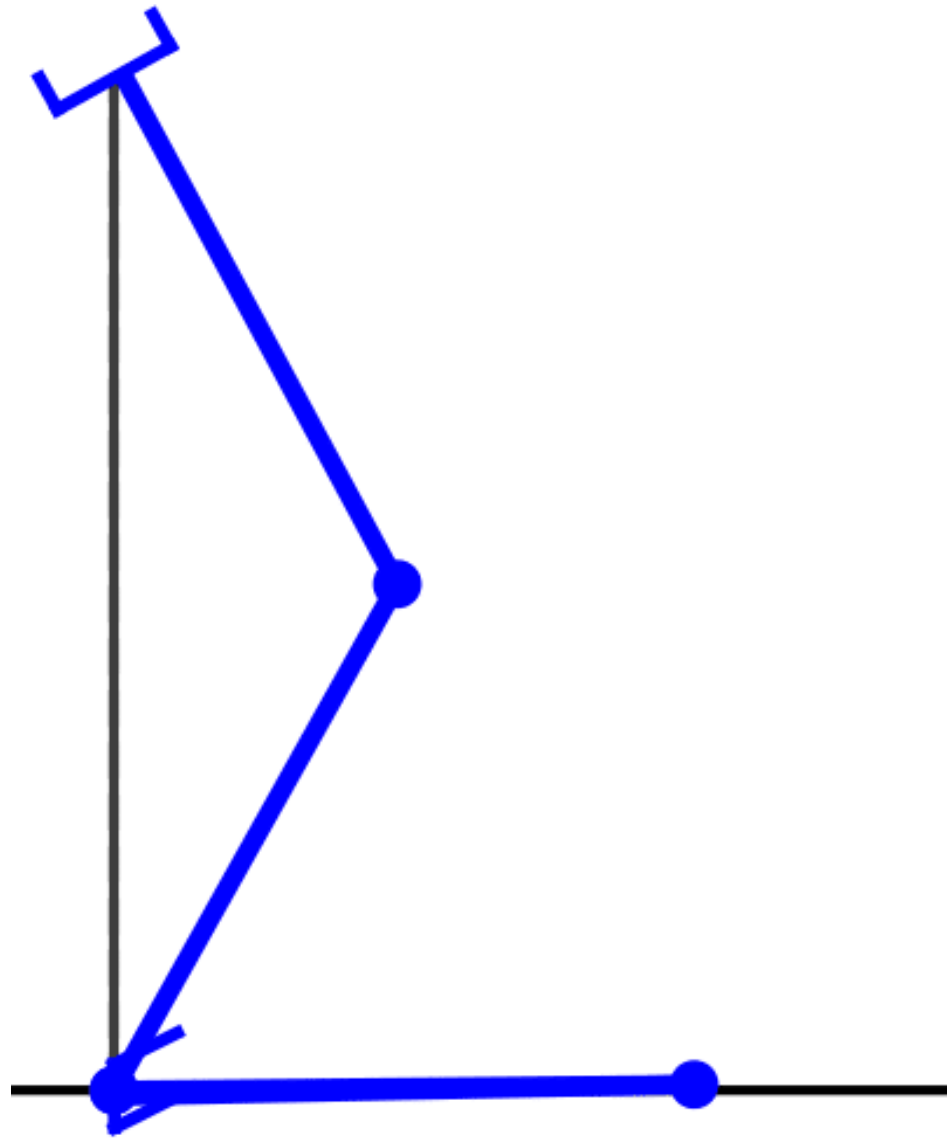
End: q_n

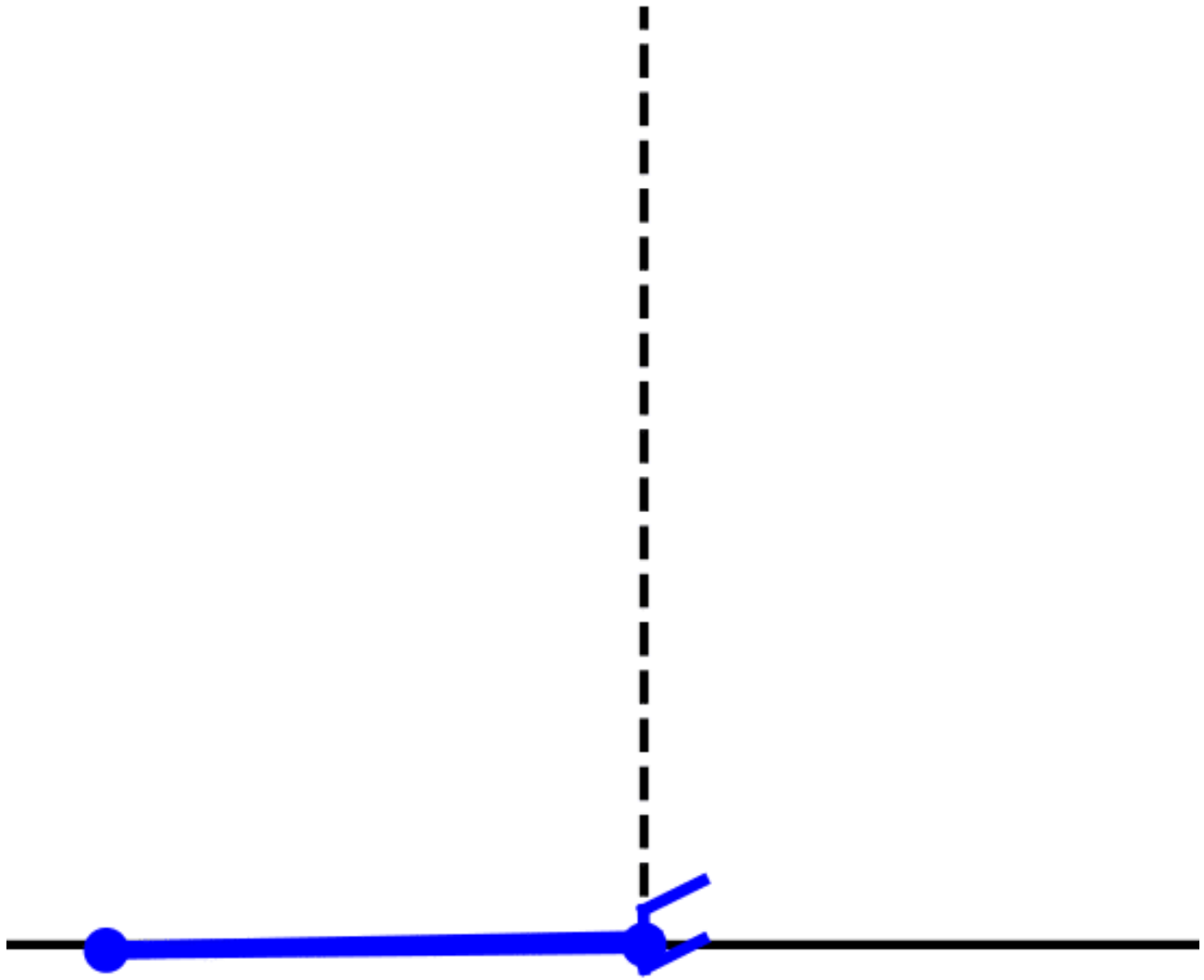
Cost: *Frechet*

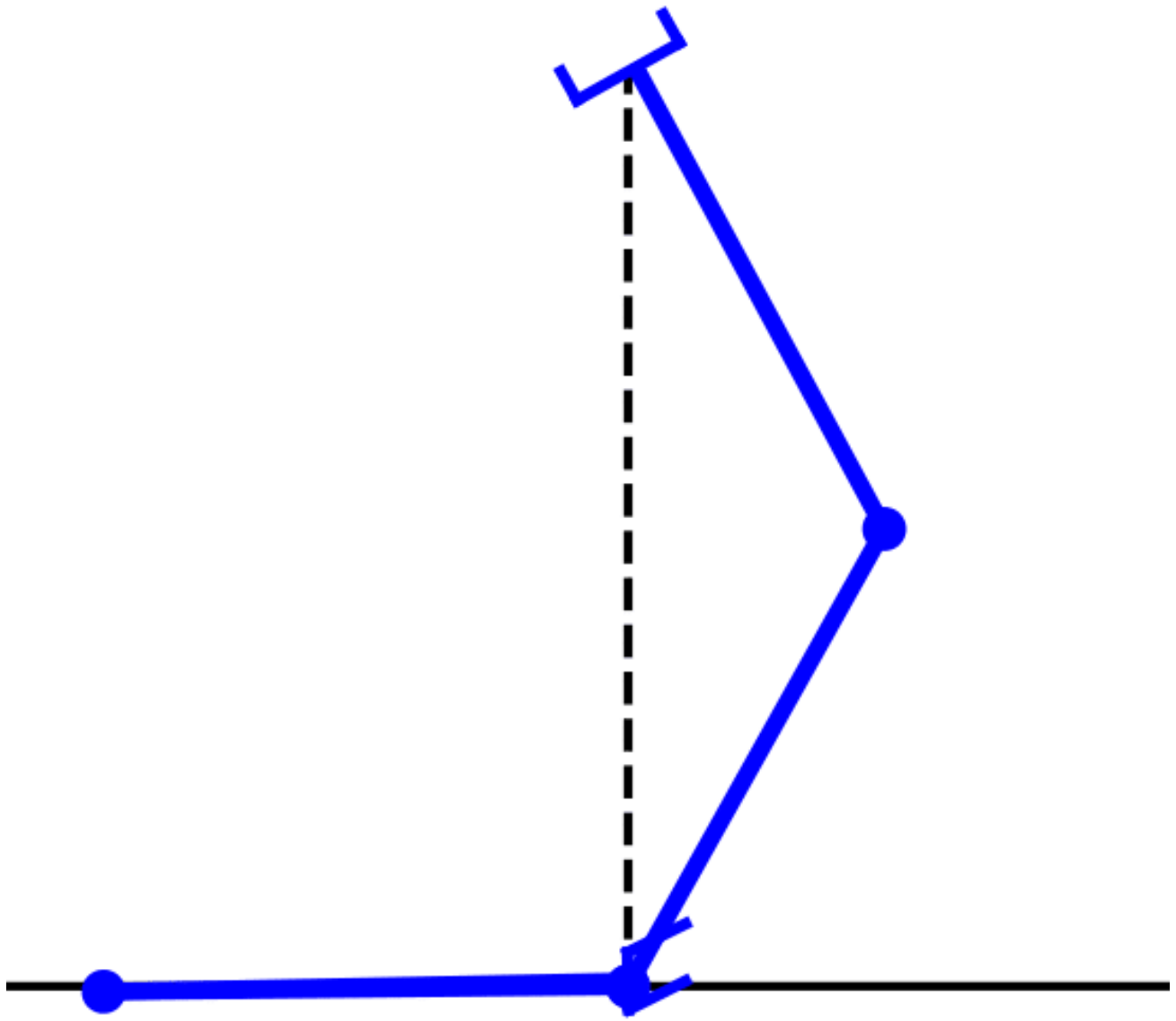


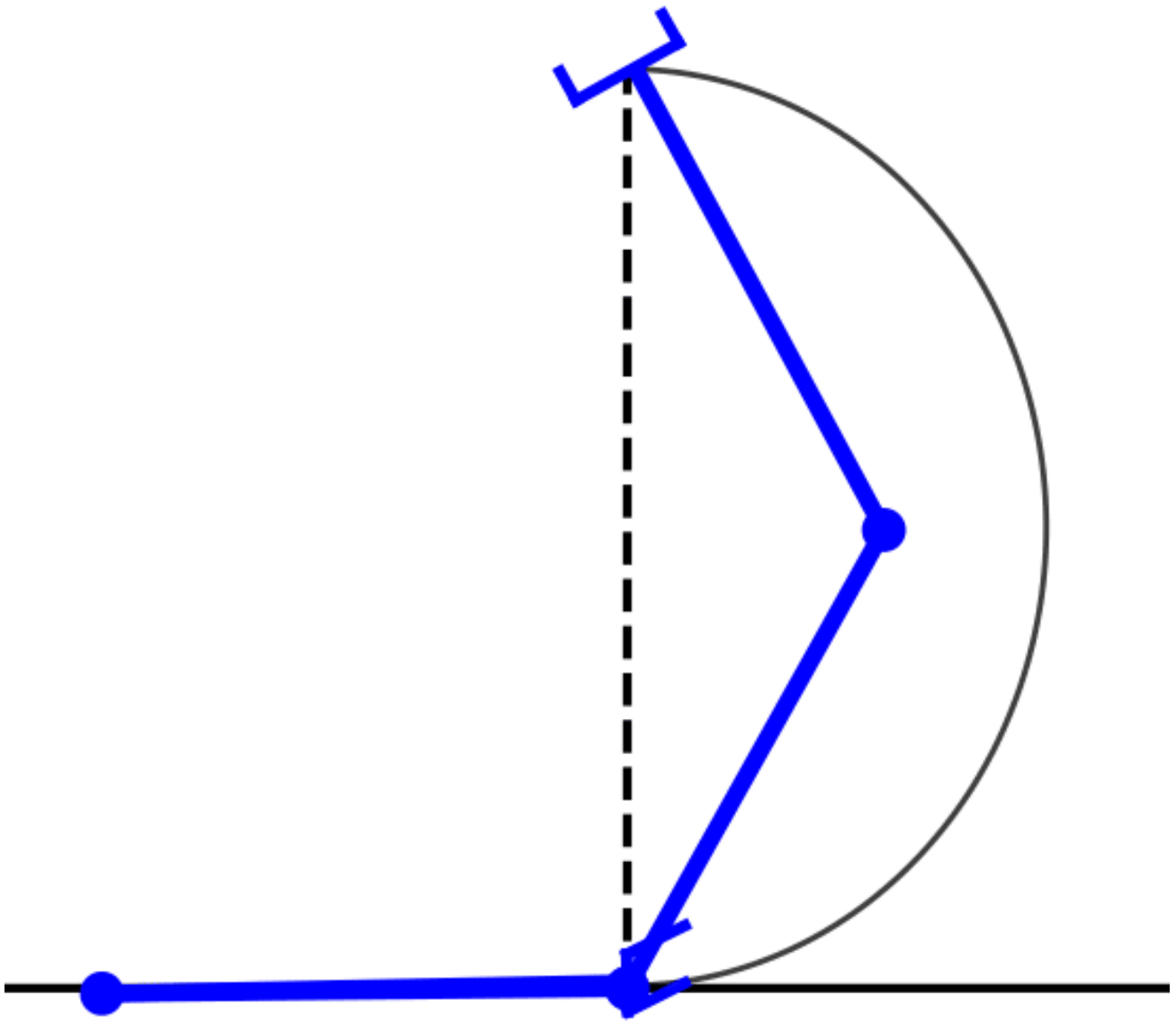


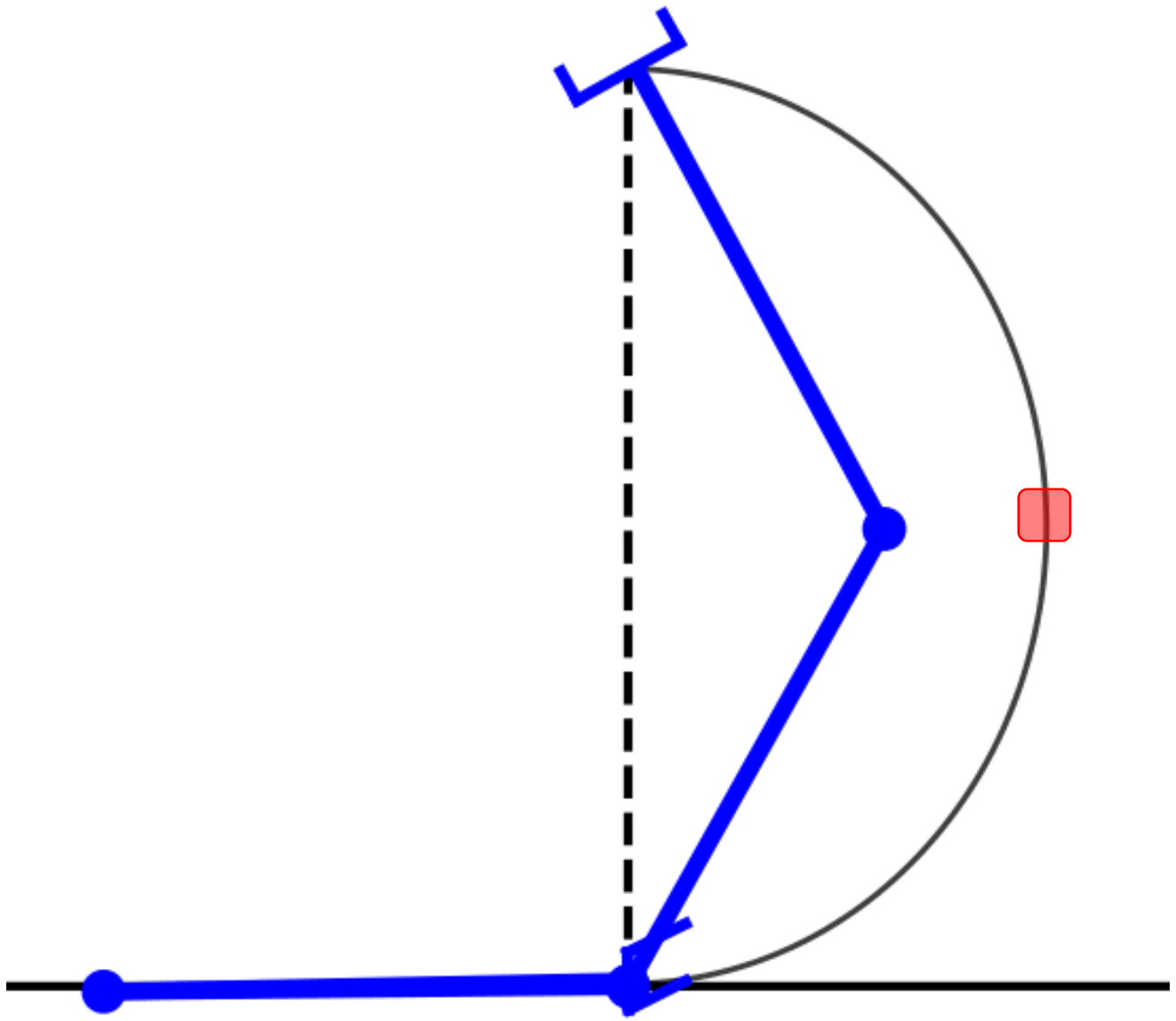


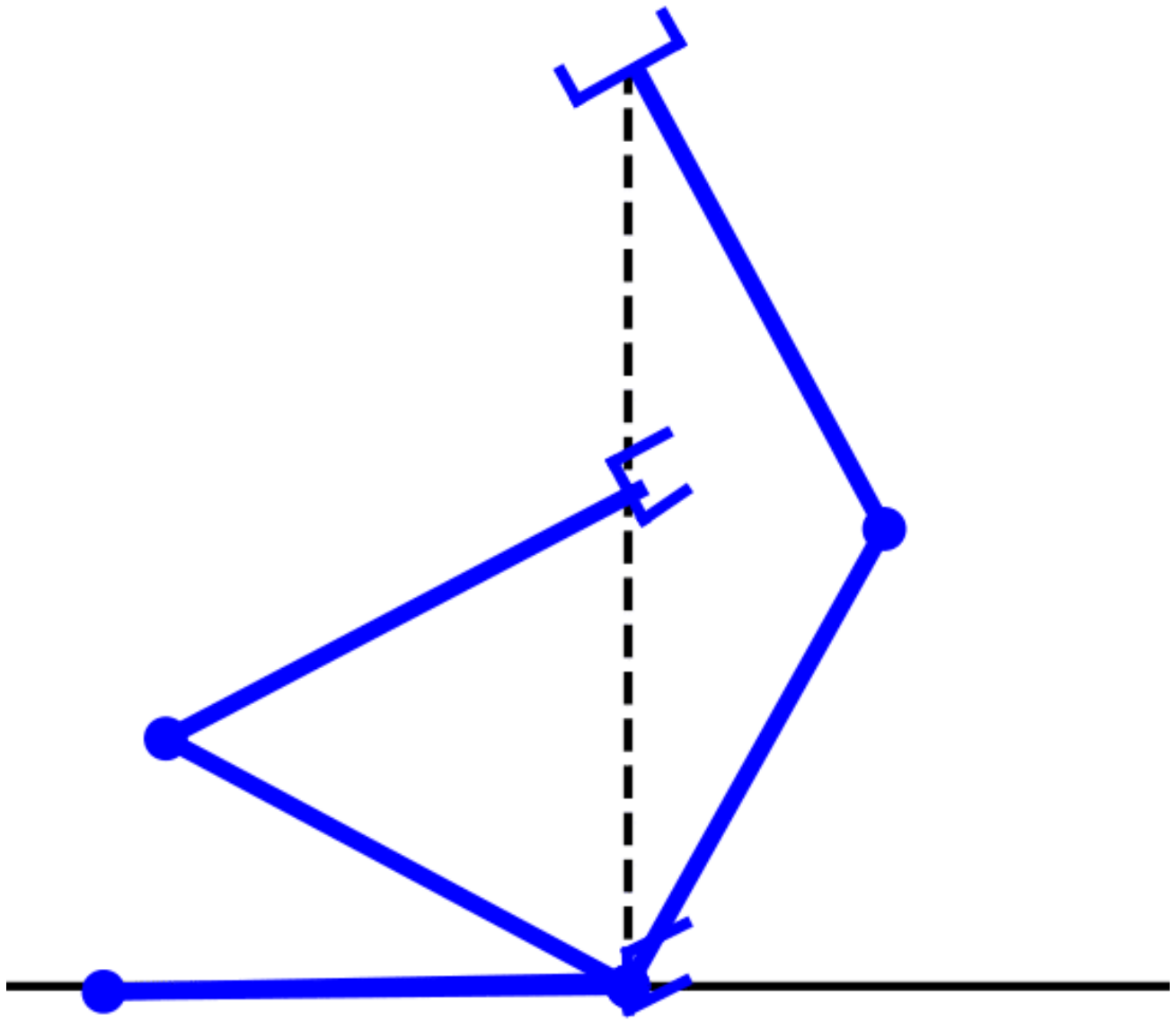


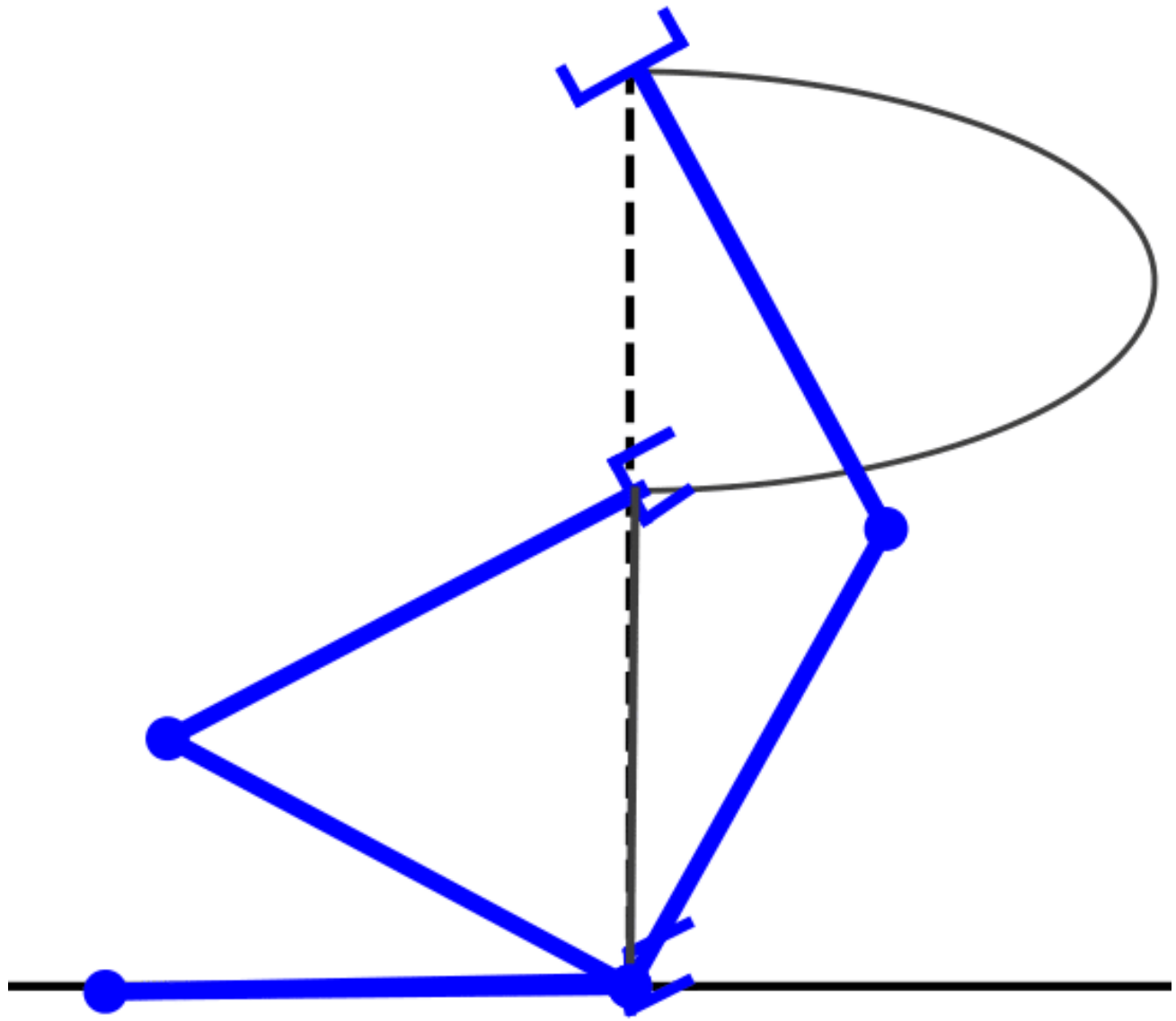


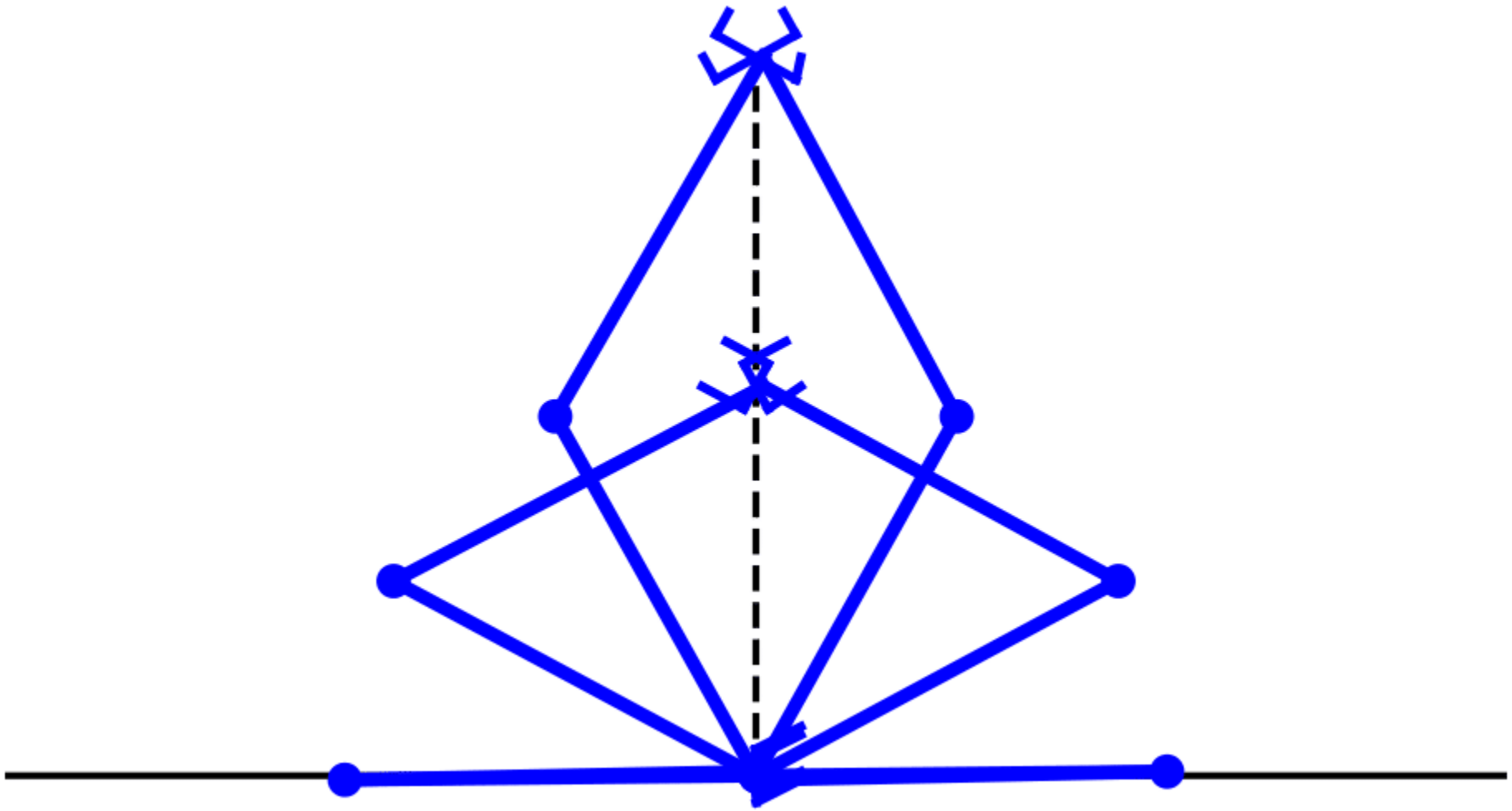






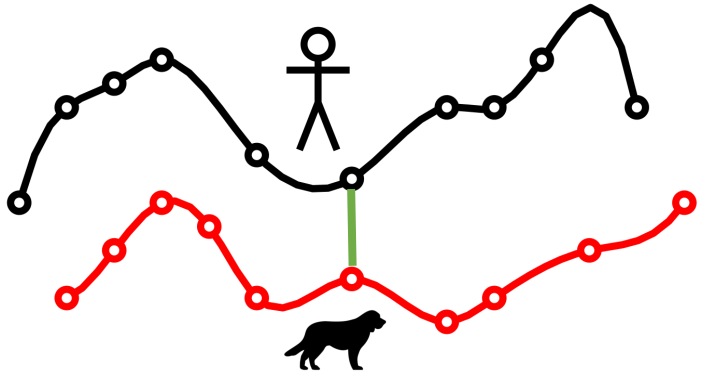






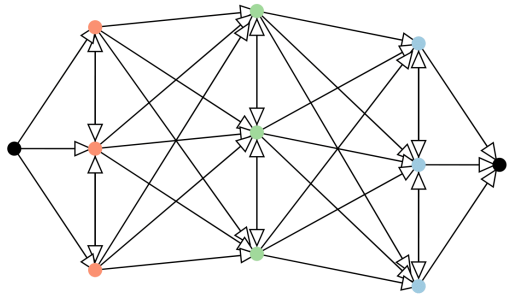
Key Insight
Search along the reference path
And the Space of IK Solutions

Distance Metrics

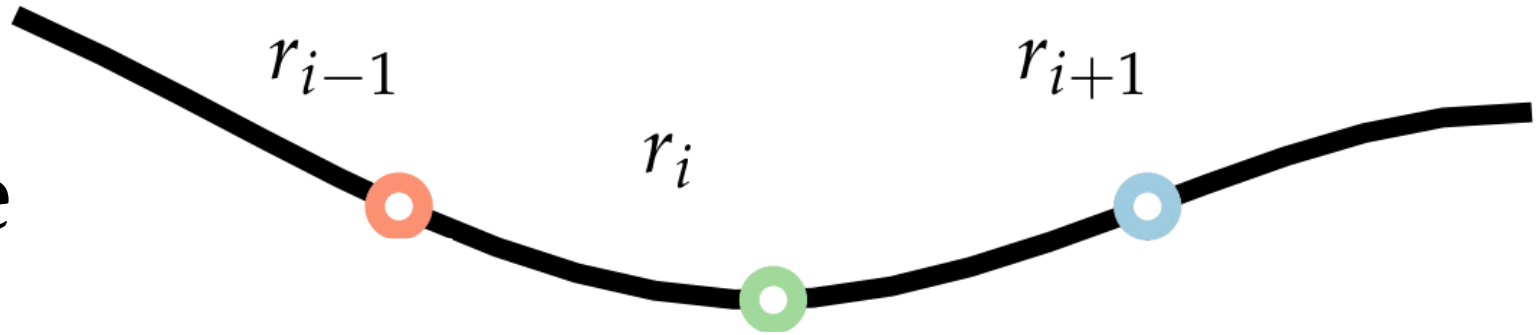


Trajectory Optimization

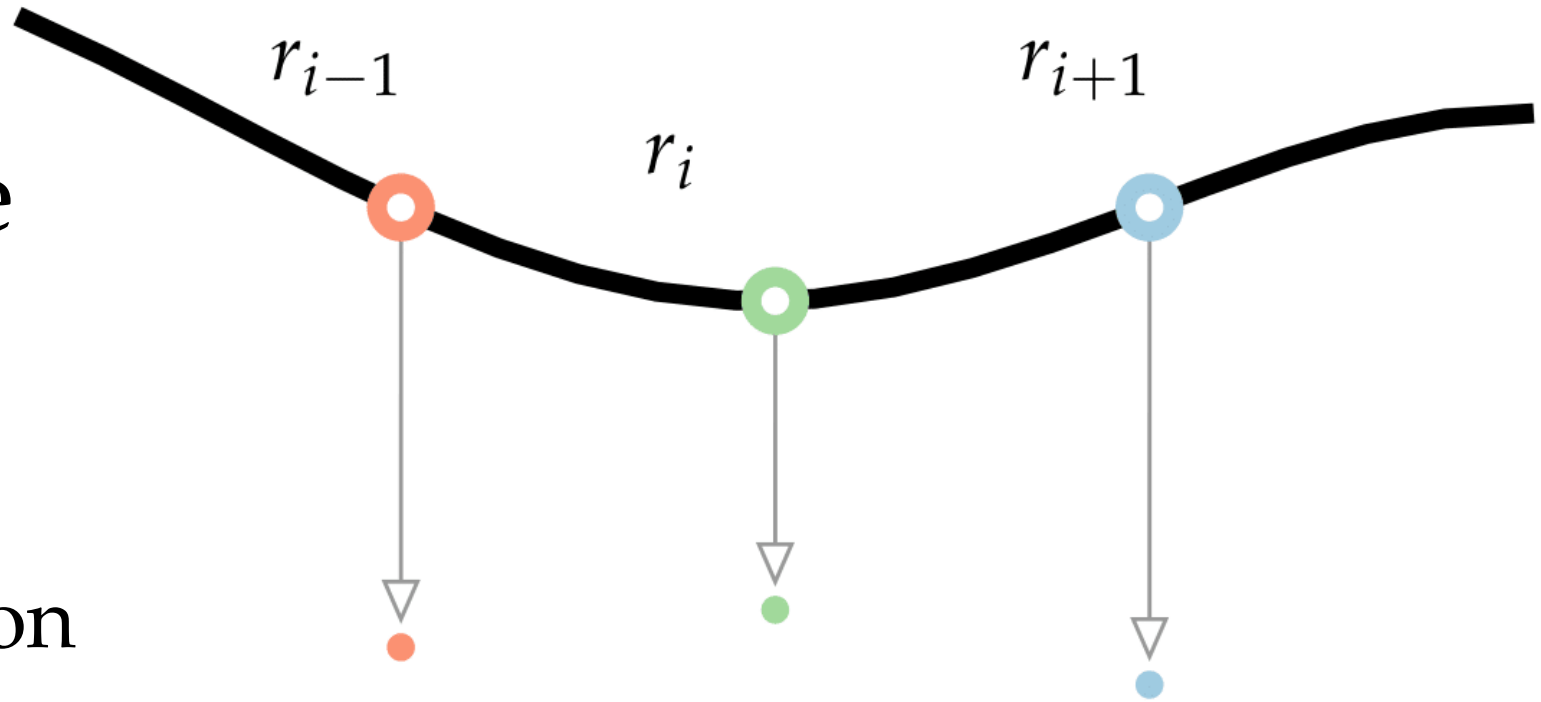
Cross Product Search



Task Space



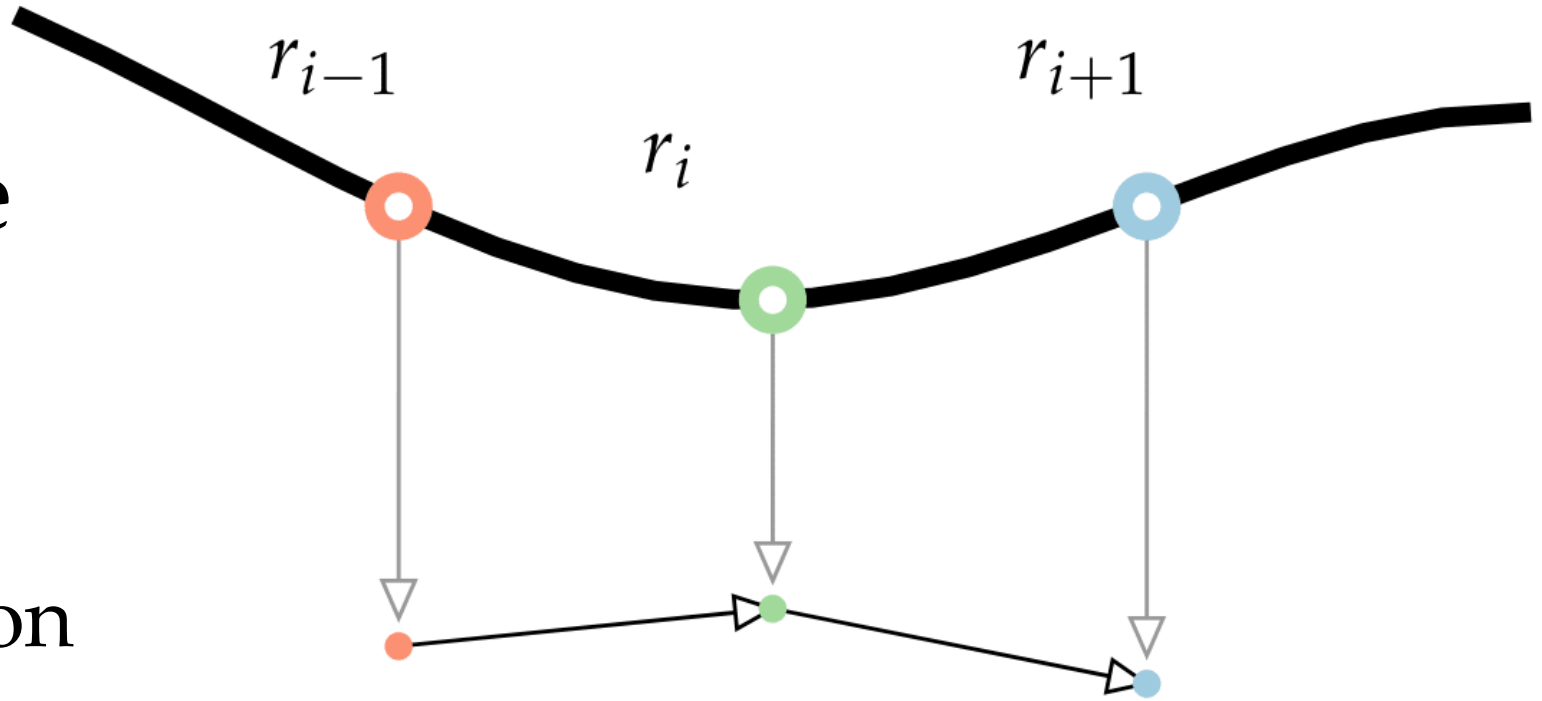
Task Space



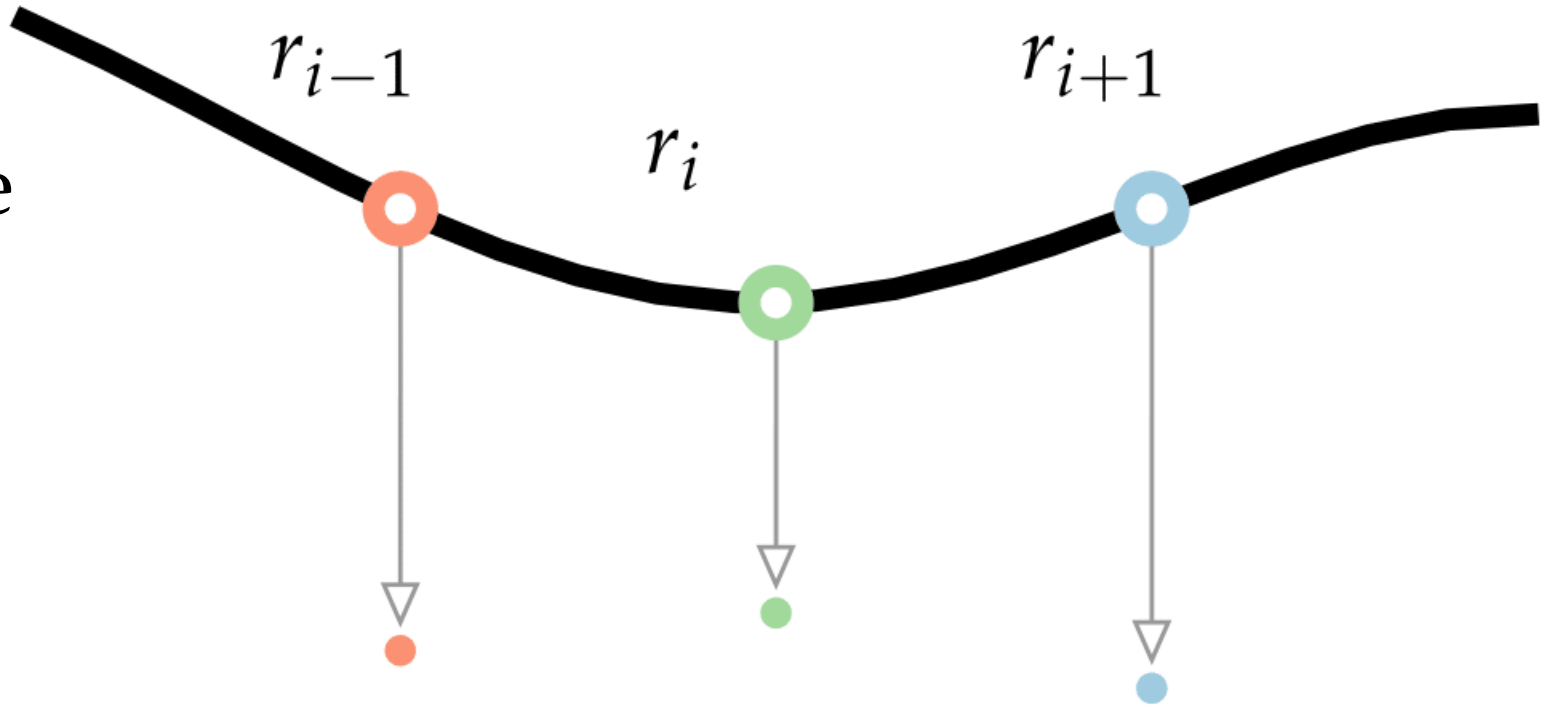
Configuration Space

Task Space

Configuration Space

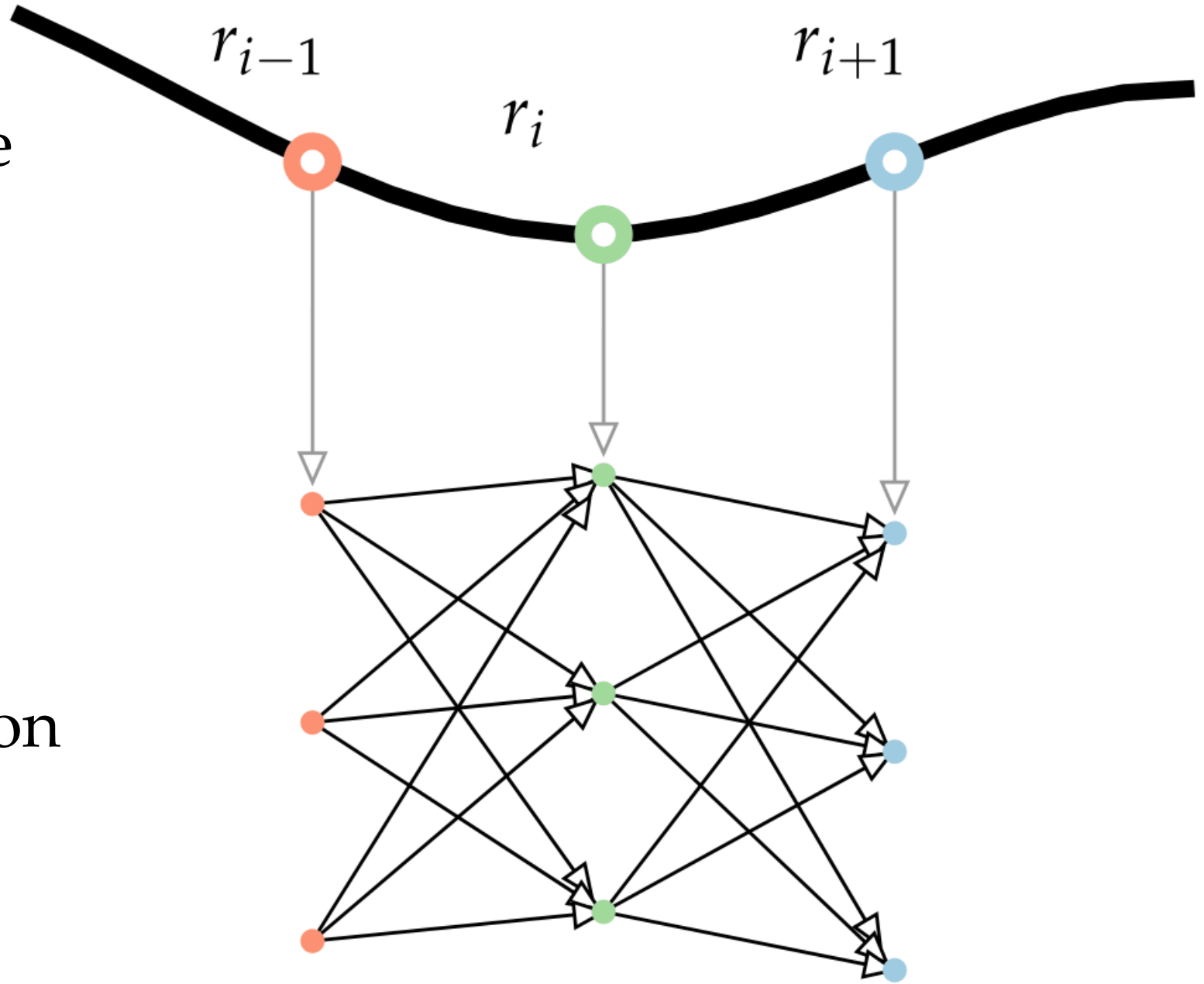


Task Space



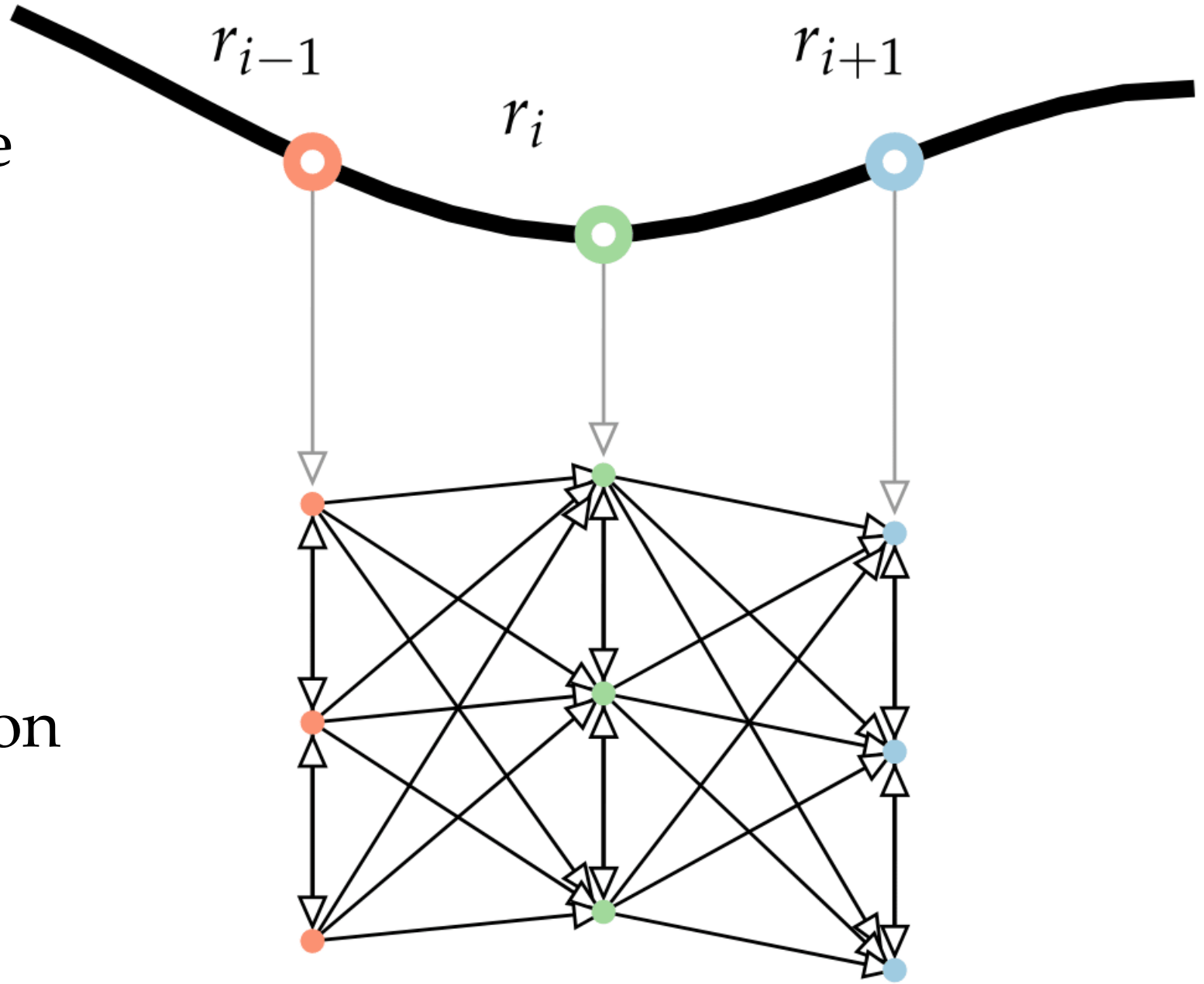
Configuration Space

Task Space

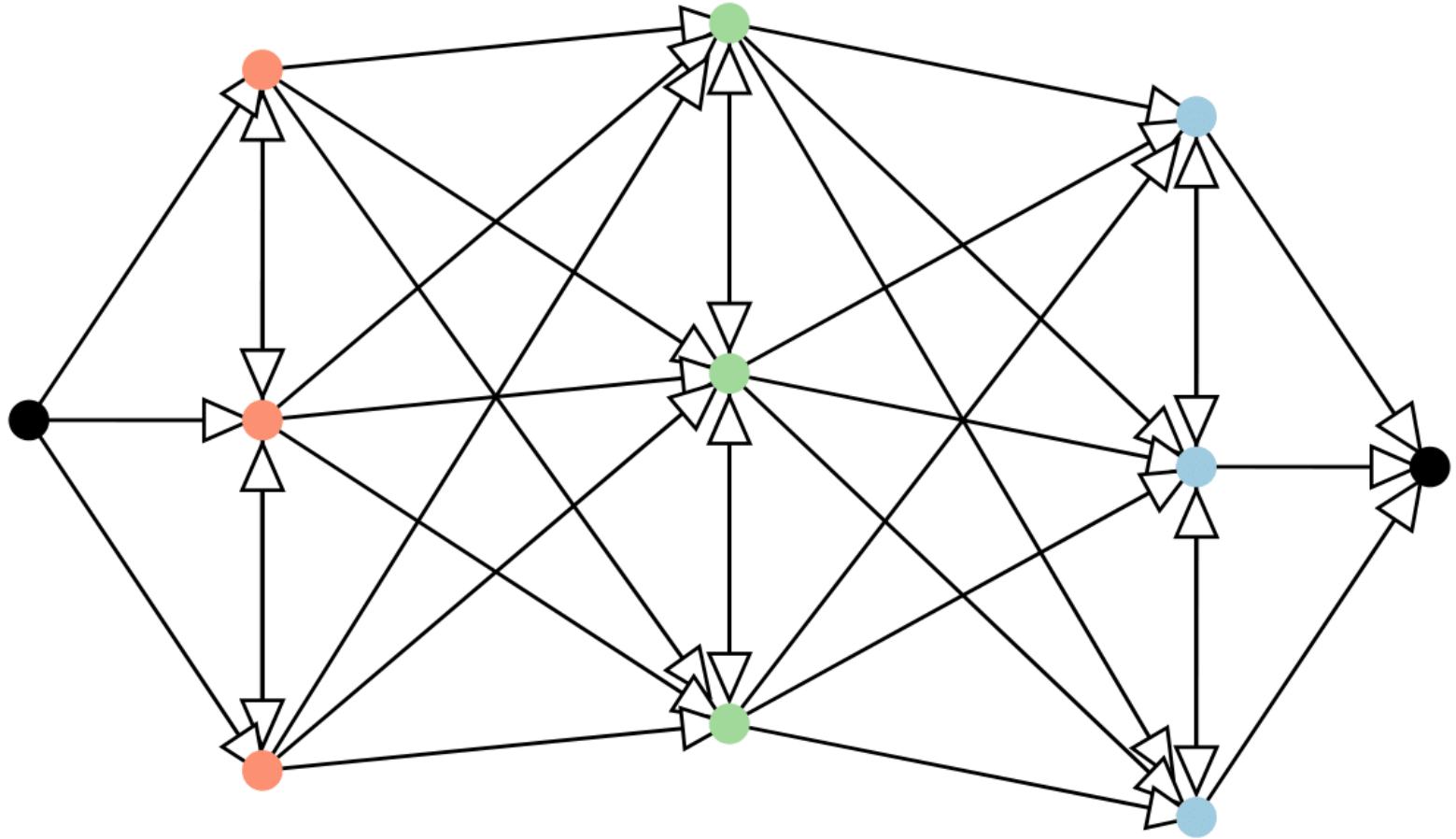


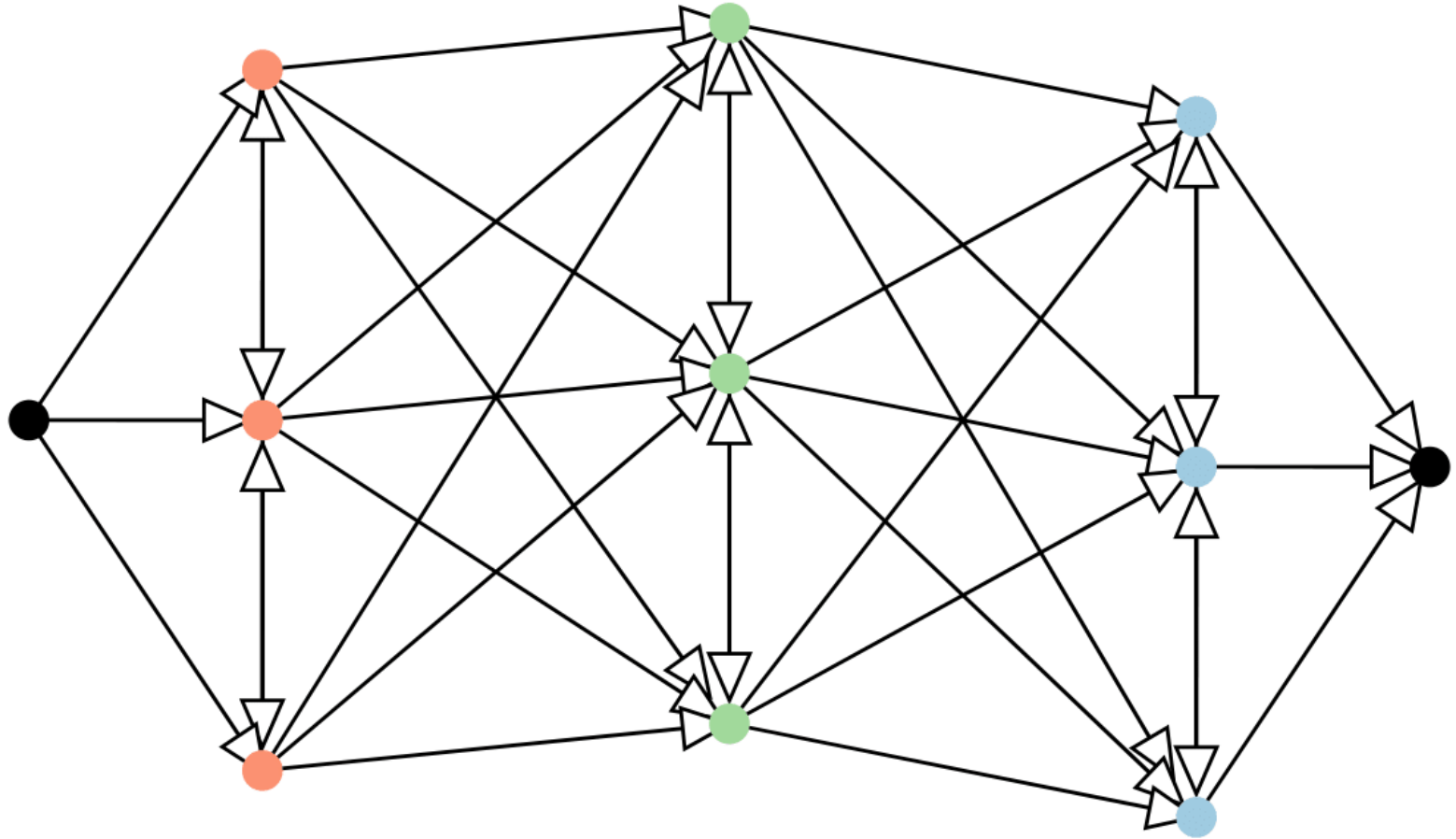
Configuration Space

Task Space

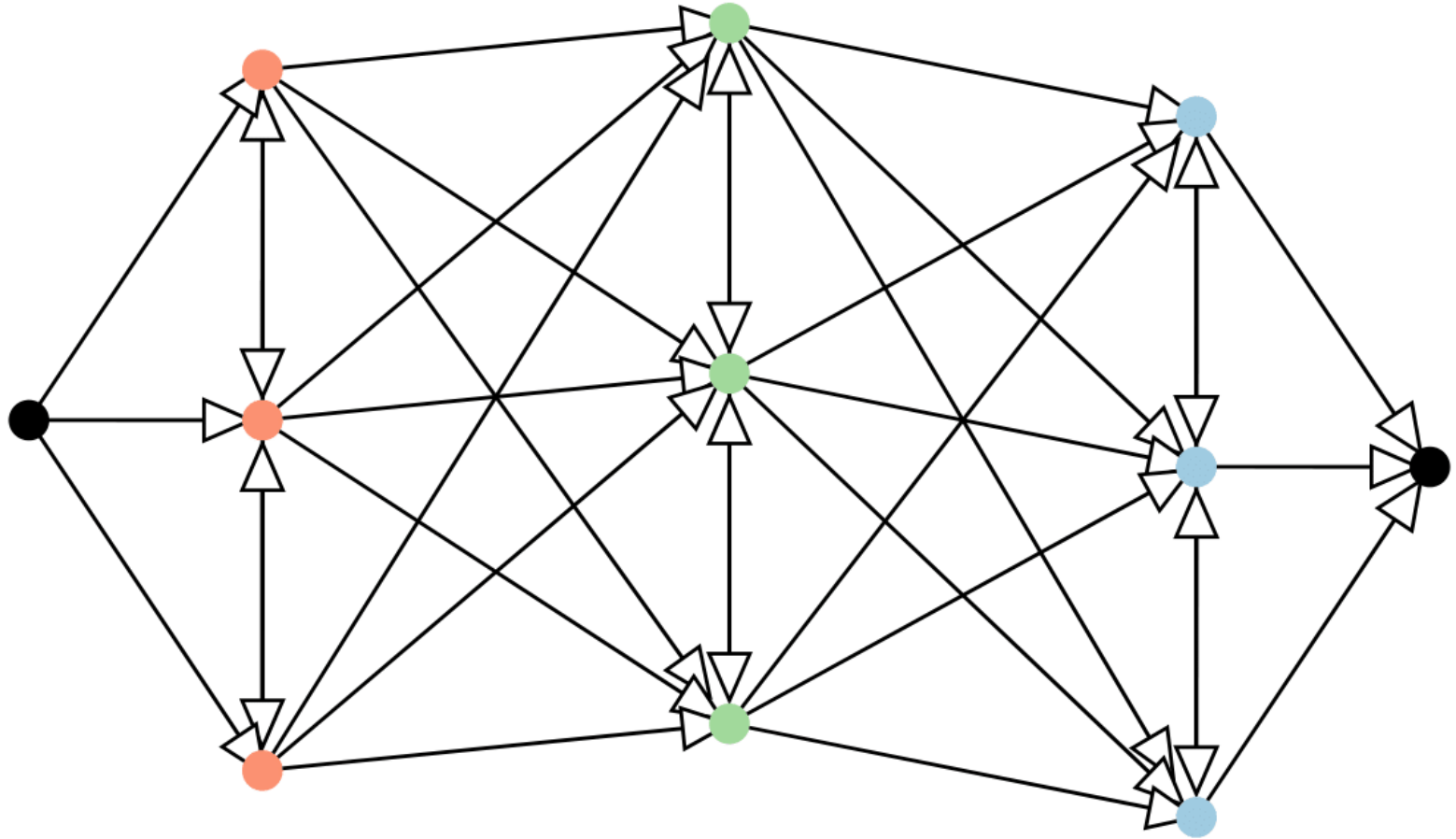


Configuration Space



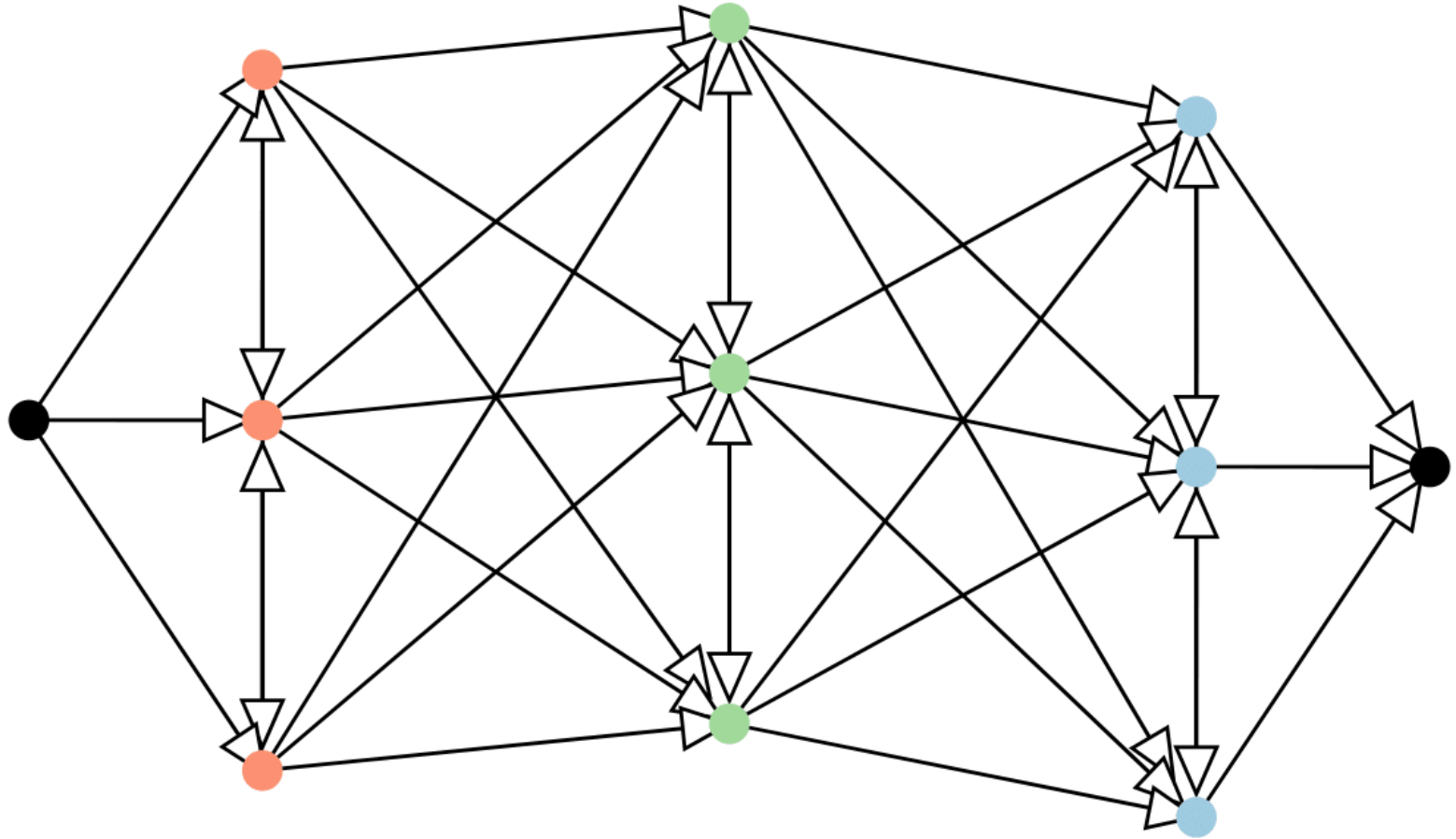


$$\xi^* = \arg\min_{p \in L} Frechet(FK(p), \bar{\xi})$$



$$\xi^* = \arg\min_{p \in L} Frechet(FK(p), \bar{\xi})$$

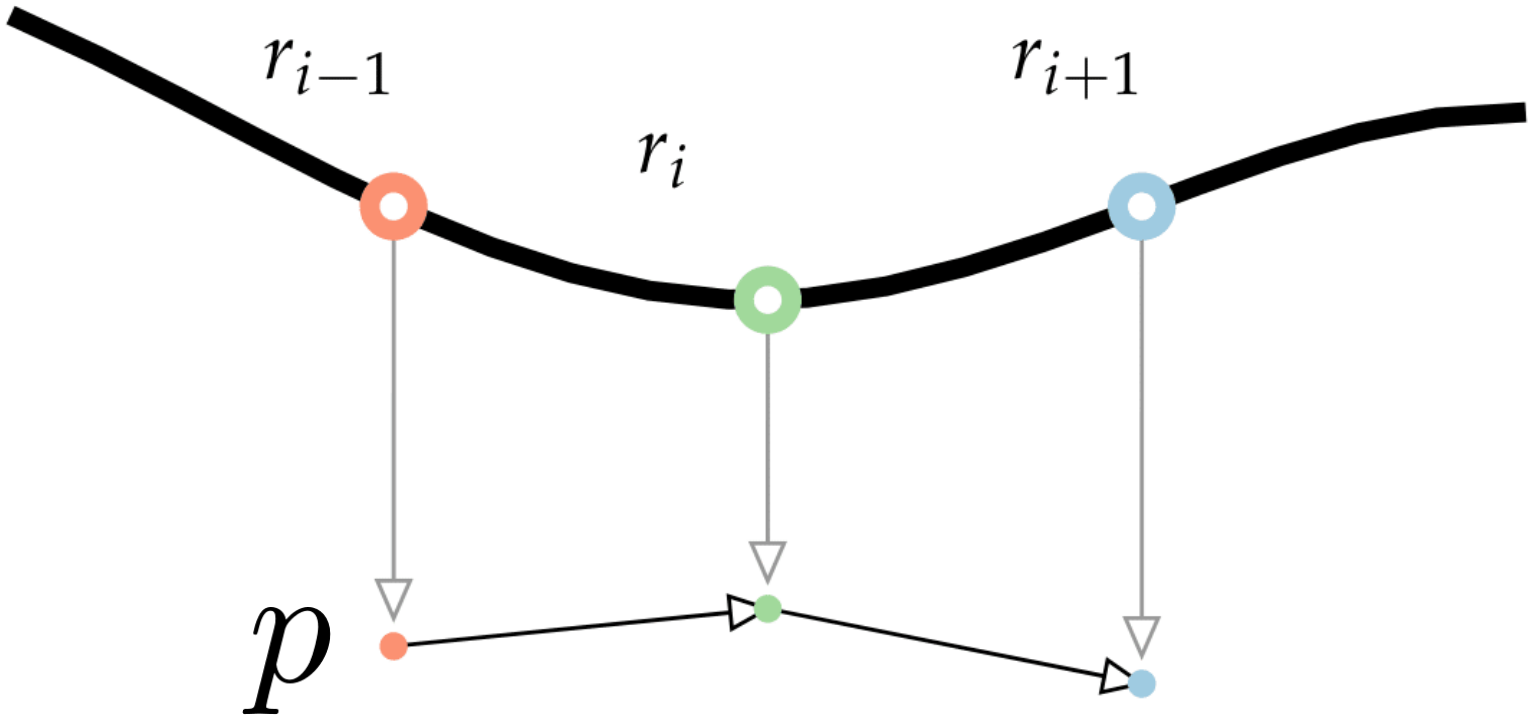
$$|p \in L|$$

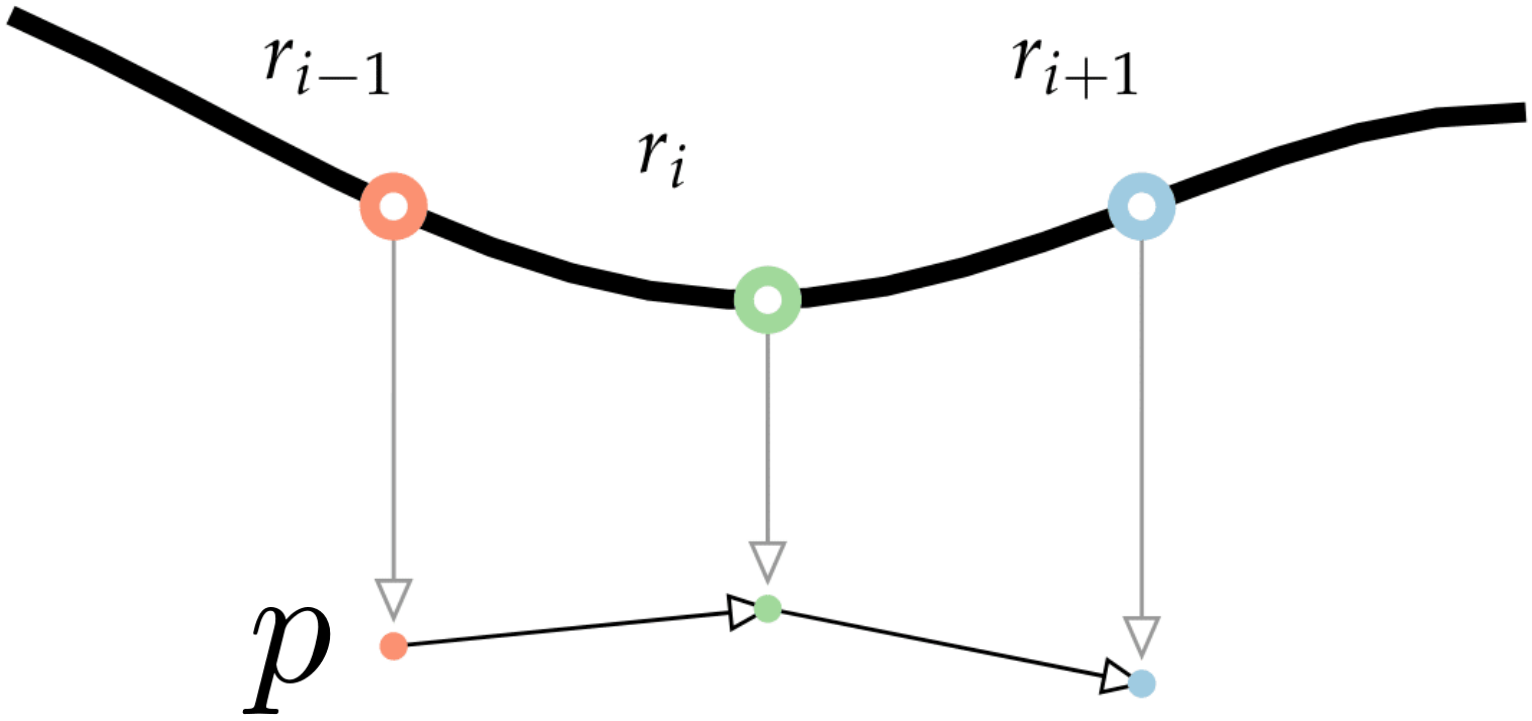


$$\xi^* = \arg\min_{p \in L} Frechet(FK(p), \bar{\xi})$$

$$|p \in L| \in \mathcal{O}(n^k)$$

Can we be more intelligent in our search?


r_{i-1}
r_i
r_{i+1}
p

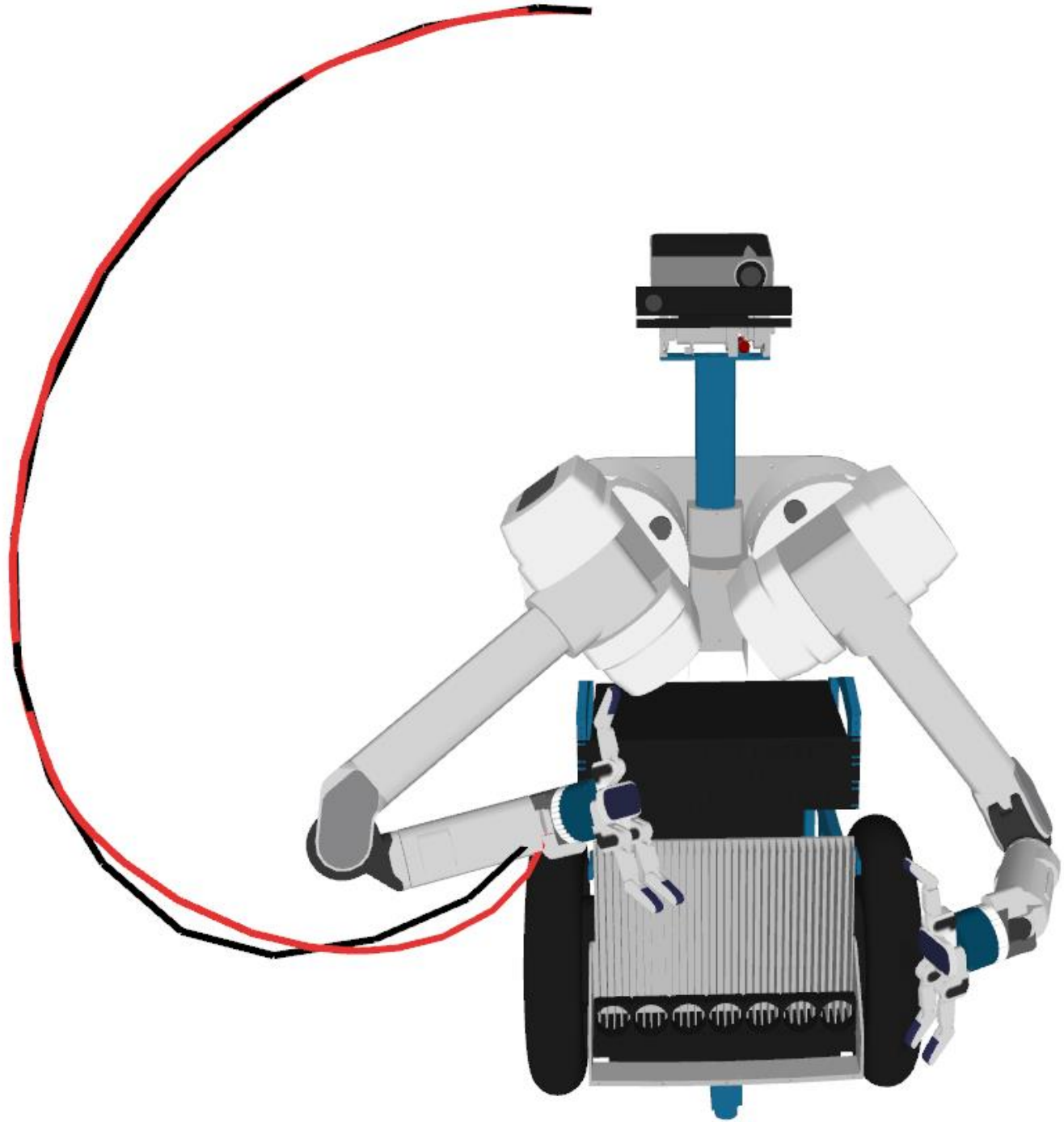


$$Frechet(FK(p), \bar{\xi})$$

Search the Cross Product Space of the two paths to find the Minimum Leash.

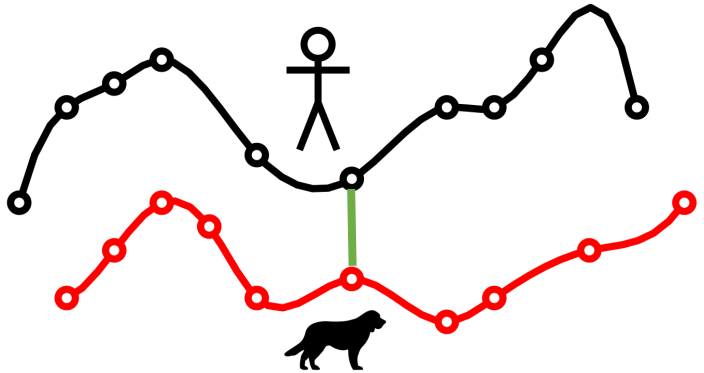
Search the Cross Product Space of
the reference path and the graph
to find the Minimum Leash.

Search the Cross Product Space of *the reference path and the graph* to find the *Bottleneck Shortest Path*.



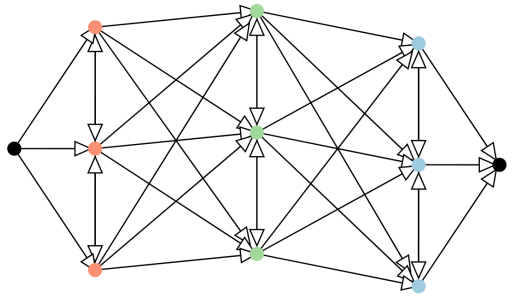
$$\xi^* = \underbrace{\arg\min_{\xi \in \Xi}}_{\text{How do we plan?}} \underbrace{||FK(\xi) - \bar{\xi}||}_{\text{How do we capture distance?}}$$

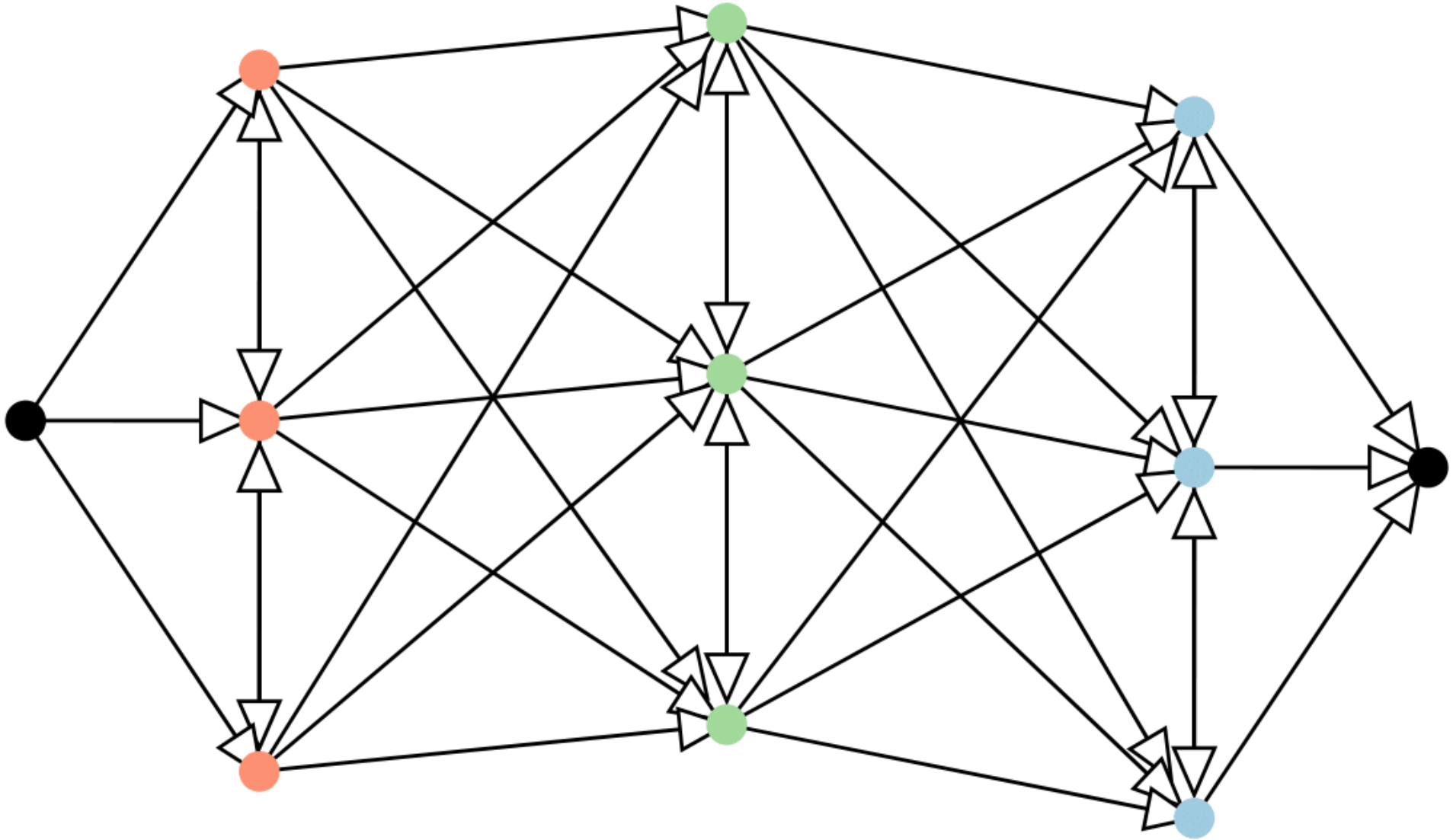
Distance Metrics

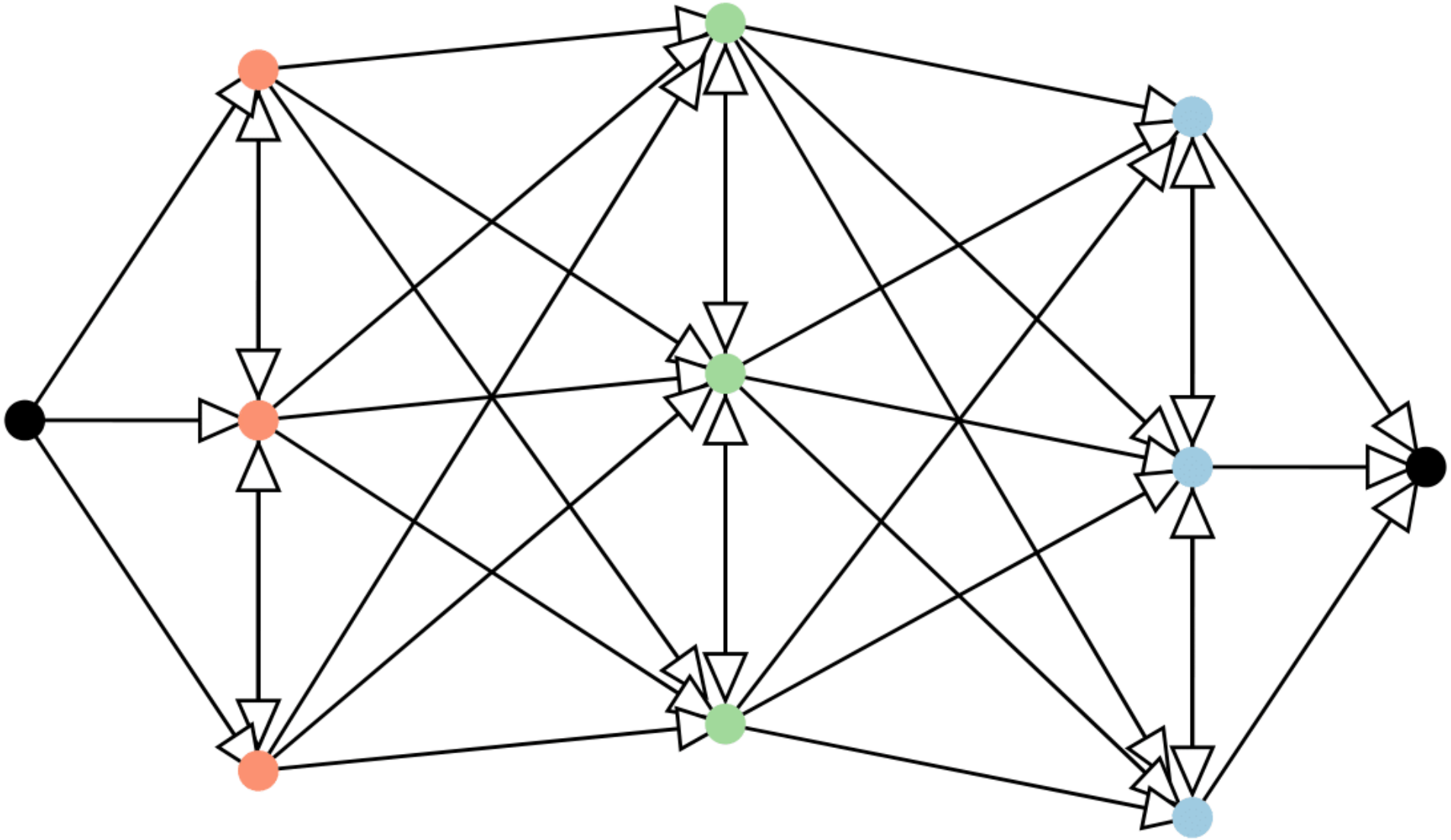


Trajectory Optimization

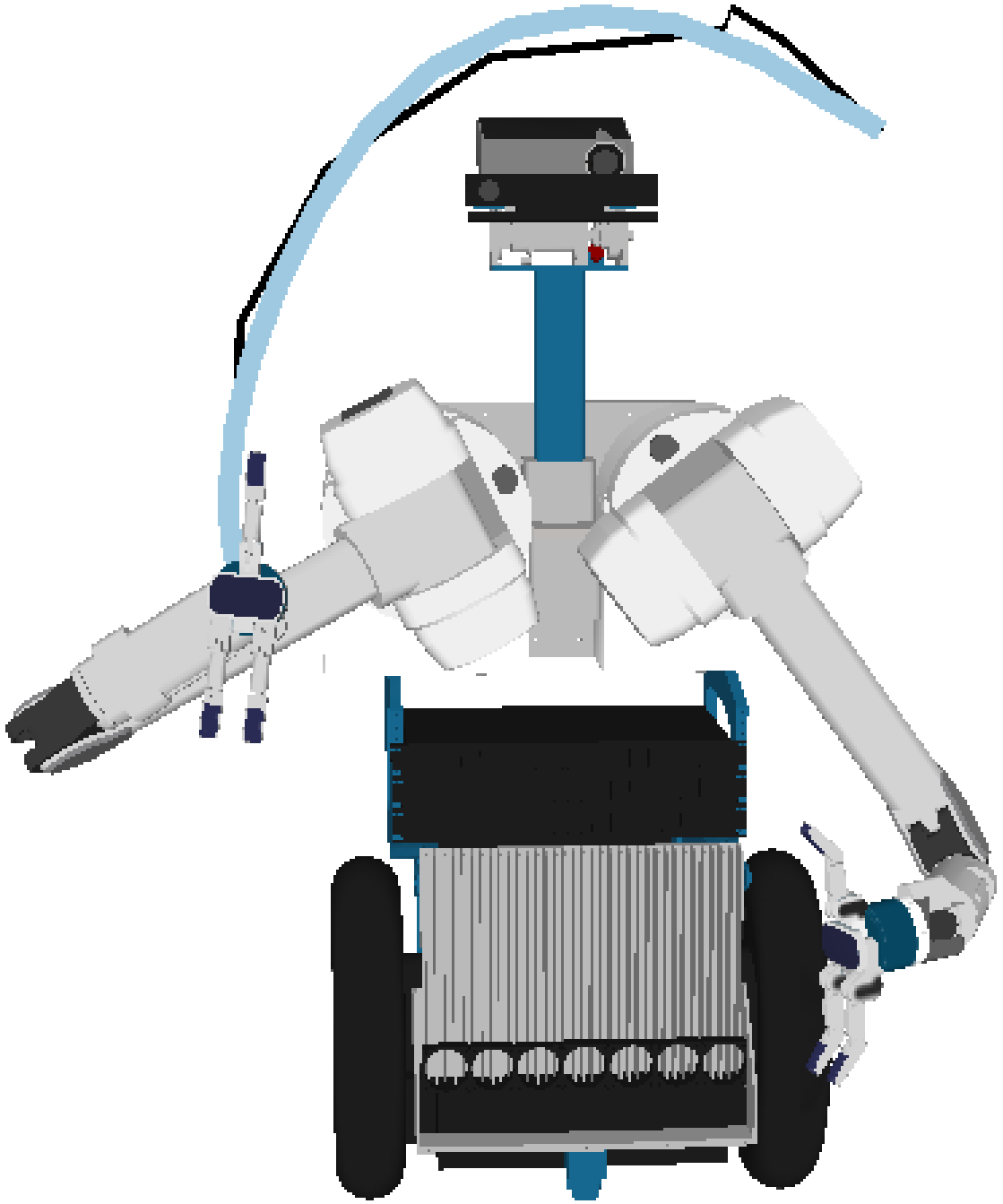
Cross Product Search







Graph Densification



Following Paths in Task Space: Distance Metrics and Planning Algorithms

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