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Incremental Offline/Online PIR

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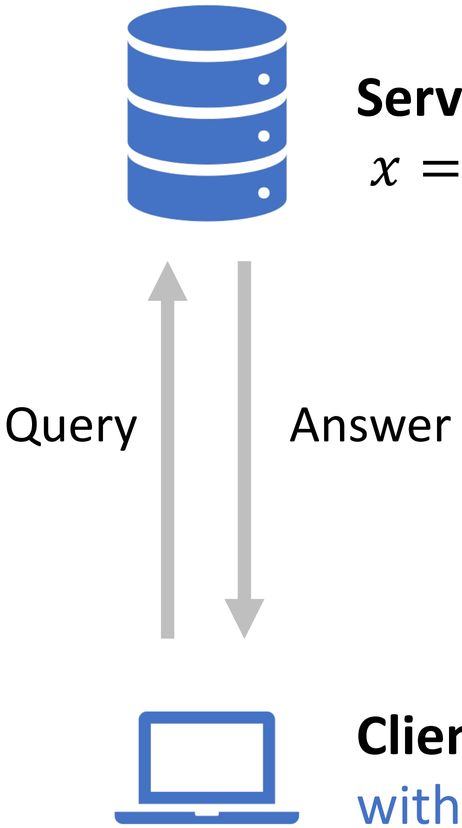
Private Information Retrieval (PIR): Basics

[CGKS95, KO97]



Server has a database
$x = x_1 x_2 \ldots x_n \in \{0,1\}^n$

Client wants to read an item x_i
without revealing the index i

Correctness

"Client gets the bits it wants"
(with overwhelming probability)

Security

"Server learns nothing from client's queries"
(information-theoretic or computational)

We will later discuss privacy against clients

Private Information Retrieval (PIR): Why interesting?

Server has a database
$x = x_1 x_2 \ldots x_n \in \{0,1\}^n$

Query

Answer

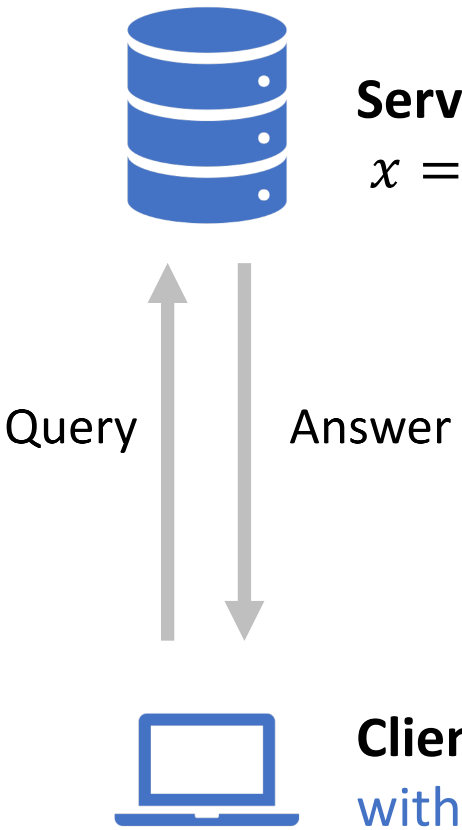
Client wants to read an item x_i
without revealing the index i

Black-box extensions
longer entries [CGN98],
keyword query [CGN98],
batched fetching [IKOS04], etc.

Real-world Applications

Private Information Retrieval (PIR): Applications

Server has a database
$x = x_1 x_2 \dots x_n \in \{0,1\}^n$



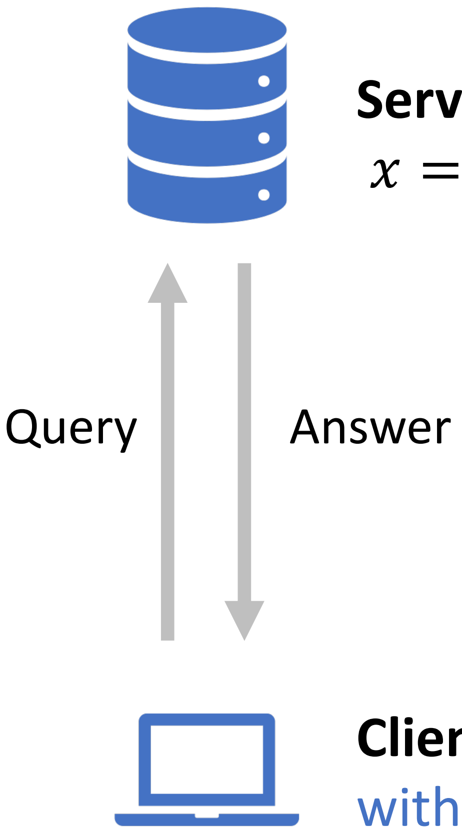
Client wants to read an item x_i
without revealing the index i

Systems built from PIR

Private contact discovery [DP5, PETS15]
Private stream service [Popcorn, NSDI16]
Metadata-private messaging [Pung, OSDI16]
Private search [DORY, OSDI20; Coeus, SOSP21]
Safe browsing [Checklist, Sec21]
Private key-value store [Pantheon, VLDB23]

… and many more

Private Information Retrieval (PIR): Applications



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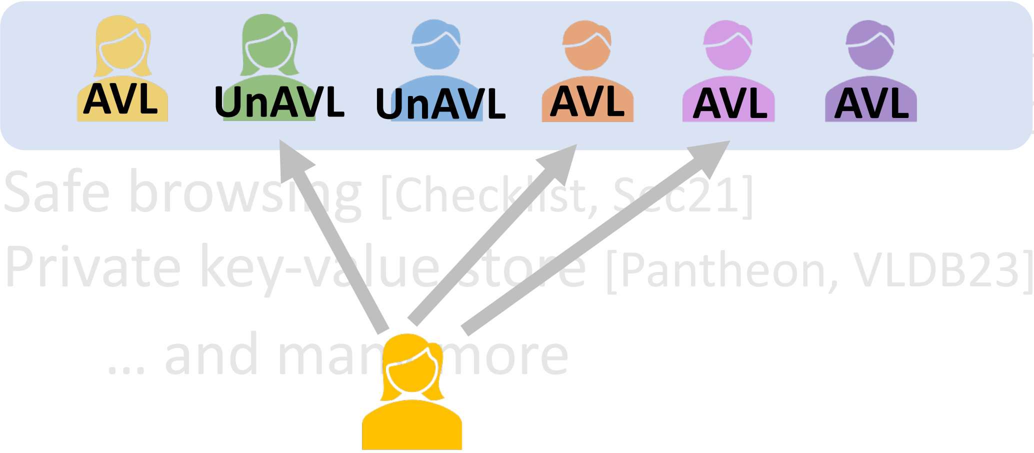
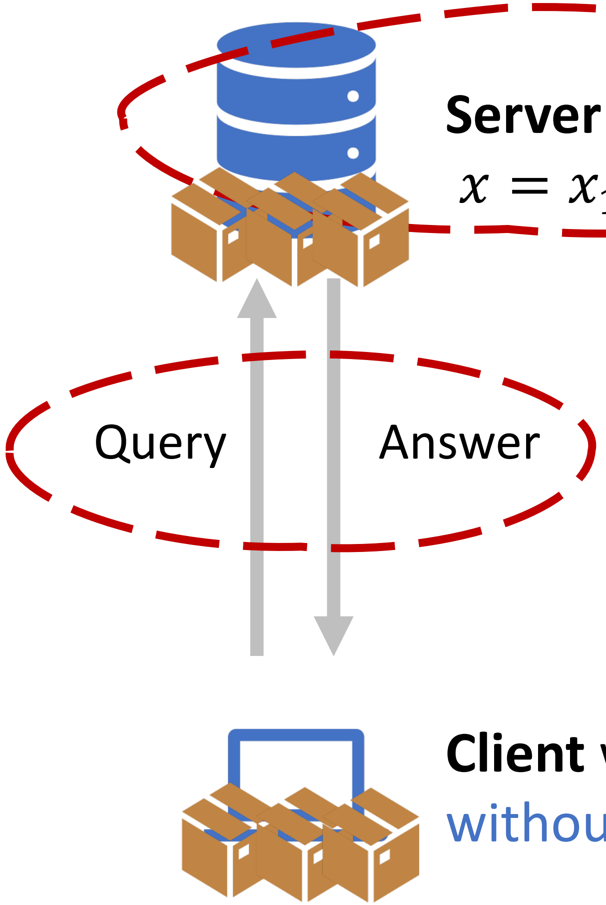


Safe browsing [Checklist, Sec21]
Private key-value store [Pantheon, VLDB23]
… and many more

Private Information Retrieval (PIR): Efficiency



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$x = x_1 x_2 \ldots x_n \in \{0,1\}^n$

Client wants to read an item x_i
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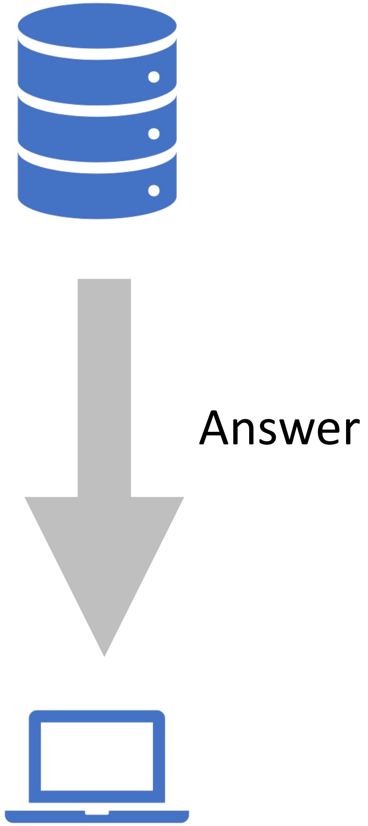
Cost is critical for applications:

- Communication
- Computation
  "work" [BIM04]:
  #bits the server reads
- Storage

Measure in terms of database size n

Reducing Communication Cost for PIR

- Trivial PIR has linear communication

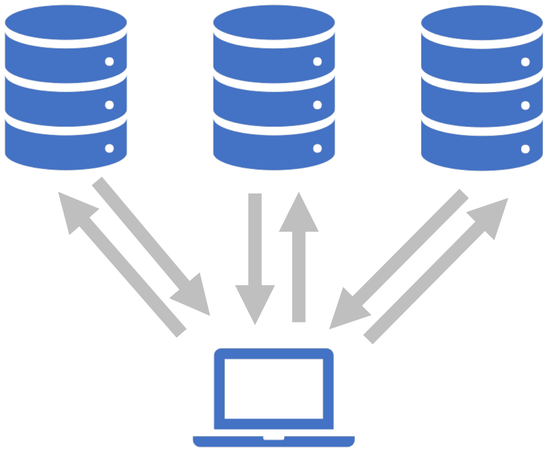


Reducing Communication Cost for PIR

- Trivial PIR has linear communication
- PIR with sublinear communication

Many schemes,
Some almost optimal

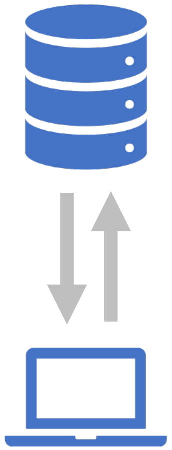
Non-colluding servers, database replicated



Information-theoretic
[CGKS95, BIK04, Yek08, Efr12, DG16, …]

Computational
[BGI16, …]

Single server



Information-theoretic
[KO97]
Not possible unless linear communication

Computational
[KO97, CMS99, KO00, GR05, OS07, …]

Reducing Computation Cost for PIR?

- Trivial PIR has linear computation or "work" [BIM00]
- PIR with sublinear computation?

A lower bound [BIM00]**:**

If the database has no redundancy (i.e., no extra storage at the server or the client), answering a single query requires in expectation $\Omega(n)$ total server "work", where n is database size.

Reducing Computation Cost for PIR?

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☹ Linear computation makes PIR hard to scale to large databases

Reducing Computation Cost for PIR?

- Trivial PIR has linear computation or "work" [BIM00]
- PIR with sublinear computation: hope?

A lower bound [BIM00]**:**

If the database has no redundancy (i.e., no extra storage at the server or the client), answering a single query requires in expectation $\Omega(n)$ total server "work", where n is database size.

Reducing Computation Cost for PIR: An important idea

Push (necessary) linear work to an offline stage and generate hints along the way…

and get sublinear computation for each query with the hints!

[BIM00, CHR17, BIPW17, HOWW18, PPY18, CK20, SACM21, CHK22, …]

PIR with preprocessing

Reducing Computation Cost for PIR: An important idea

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PIR with preprocessing

Superlinear-sized hints at the server

Reducing Computation Cost for PIR: An important idea

Push (necessary) linear work to an offline stage and generate hints along the way…

and get sublinear computation for each query with the hints!

Our work is based on this flavor

[BIM00, CHR17, BIPW17, HOWW18, PPY18, CK20, SACM21, CHK22, …]

Offline/Online model

Sublinear-sized hints at the client

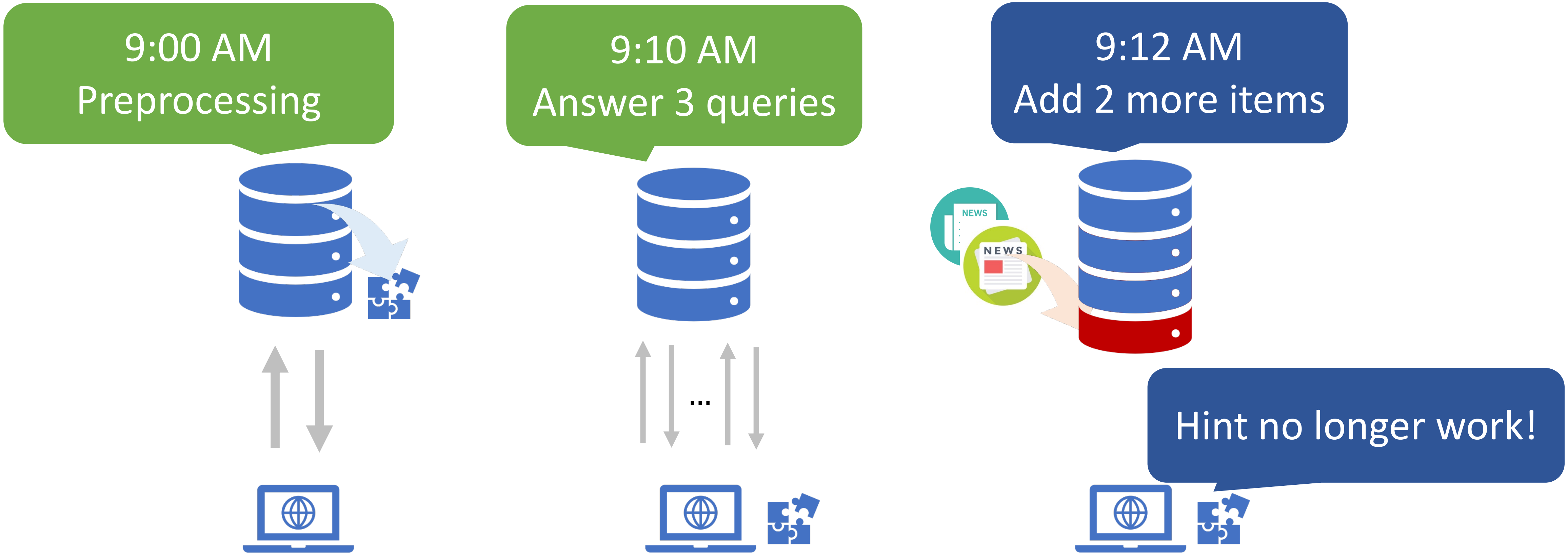
PIR with Preprocessing : Are we done?

- Preprocessing phase: (super) linear computation, generate *hints*
  - Preprocessing can be done only by the servers [BIM00, BIPW17, CHR17] or interactive with clients (per-client) [PPY18, CK20, KC21, SACM21, CHK22]
  - Hints can be stored at the server or the clients
- Query phase: sublinear computation for each query utilizing the hints
  - #Queries can be unbounded or polynomially many

Sublinear computation and sublinear communication

PIR with Preprocessing in Applications

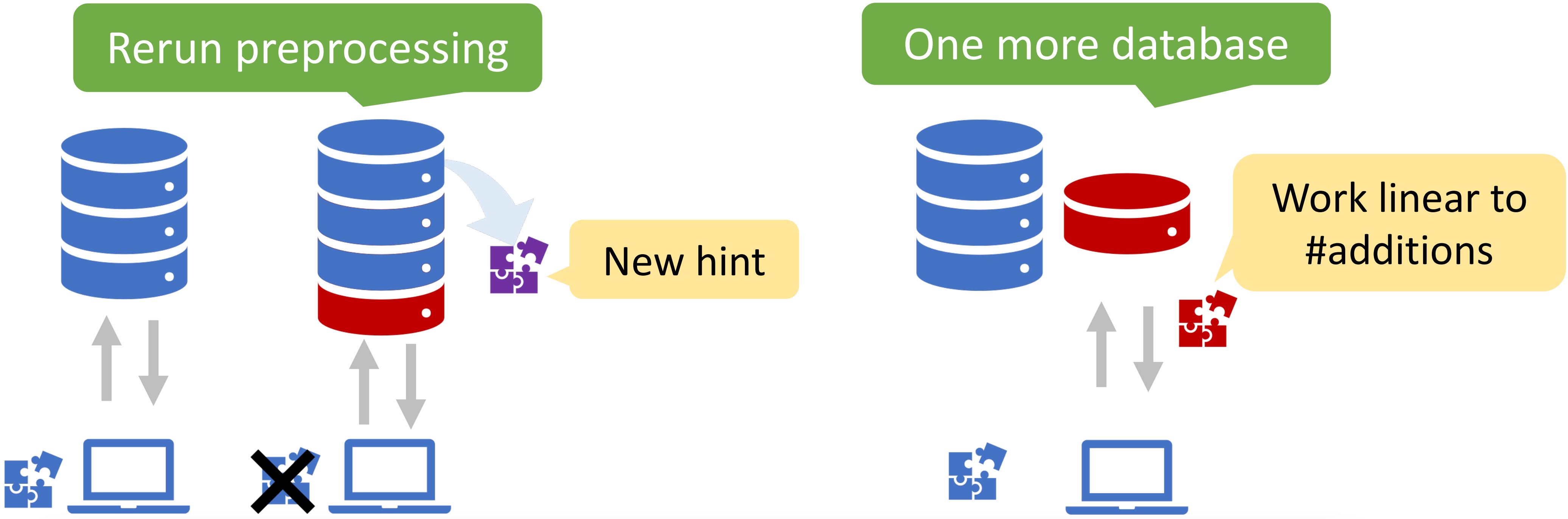
PIR with Preprocessing in Applications



PIR with Preprocessing: More things to do

Databases are not static,
but old hints no longer work!

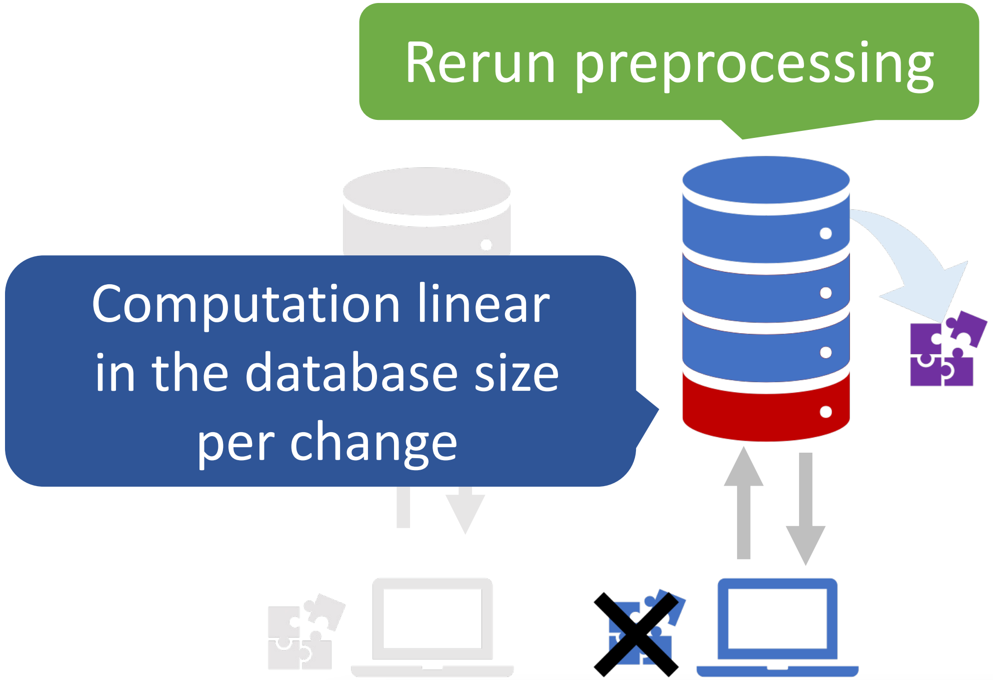
PIR with Preprocessing: Patches



Features of common applications

- A handful of changes (small compared to the database size)
- Changes periodically happen

PIR with Preprocessing: Patches



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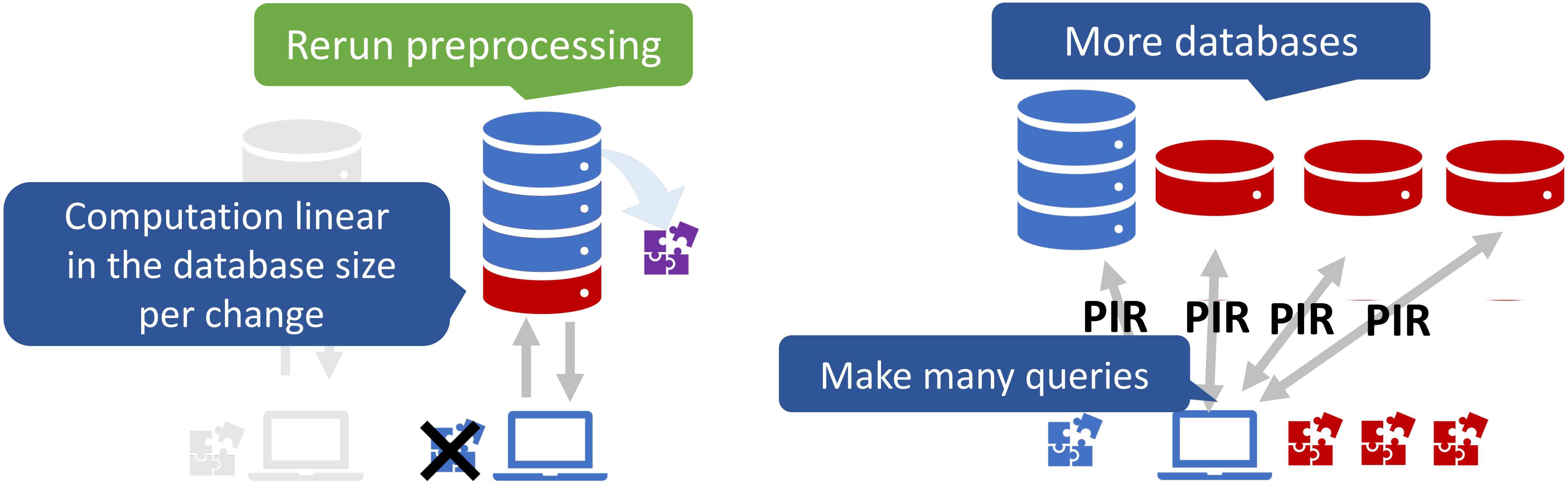

Features of common applications

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- Changes periodically happen

PIR with Mutable Preprocessing

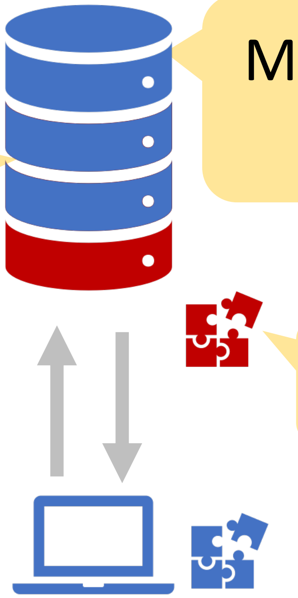
Our approach to handle dynamic database
preserves all the properties of the solutions for the static database

PIR with Mutable Preprocessing

Preprocessing for updates

Minimum necessary server work:
linear in #additions

A single database



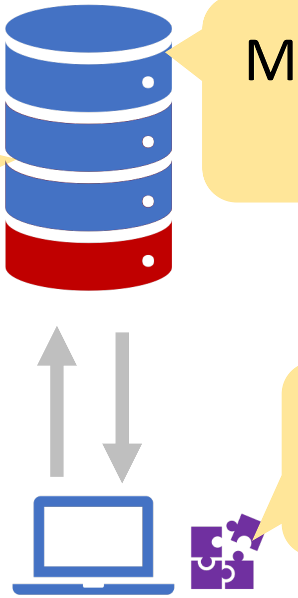
A small hint

PIR with Mutable Preprocessing

Preprocessing for updates

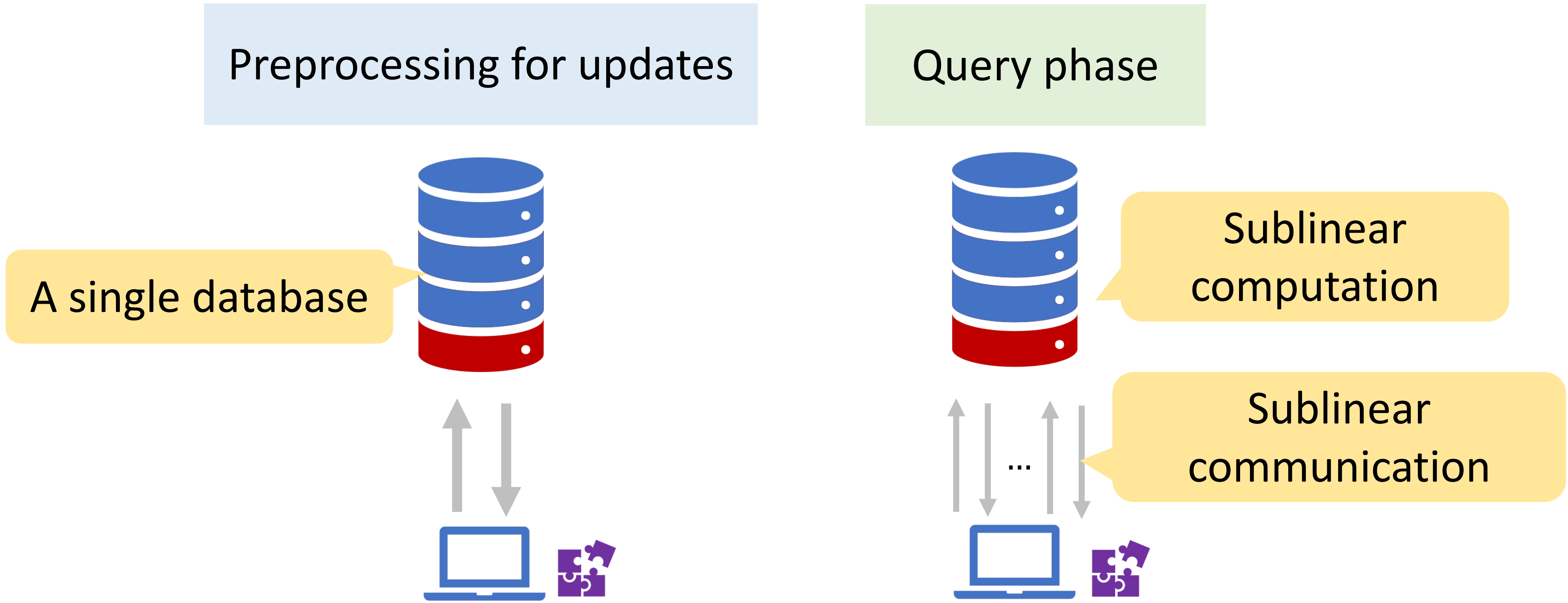
A single database

Minimum necessary server work:
linear in #additions



A new hint for the updated database

PIR with Mutable Preprocessing



Rest of This Talk

- Motivation
- Our solution: update existing hints at a cost proportional to the changes
  - Based on [CK20], [SACM21]
  - Experimental evaluation
- Open questions

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Protocol-specific manner

- Discussion

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Protocol-specific manner

A Two-Server Offline/Online PIR [CK20]

- Generate “hints” in offline phase for online queries

Non-colluding servers, database replicated

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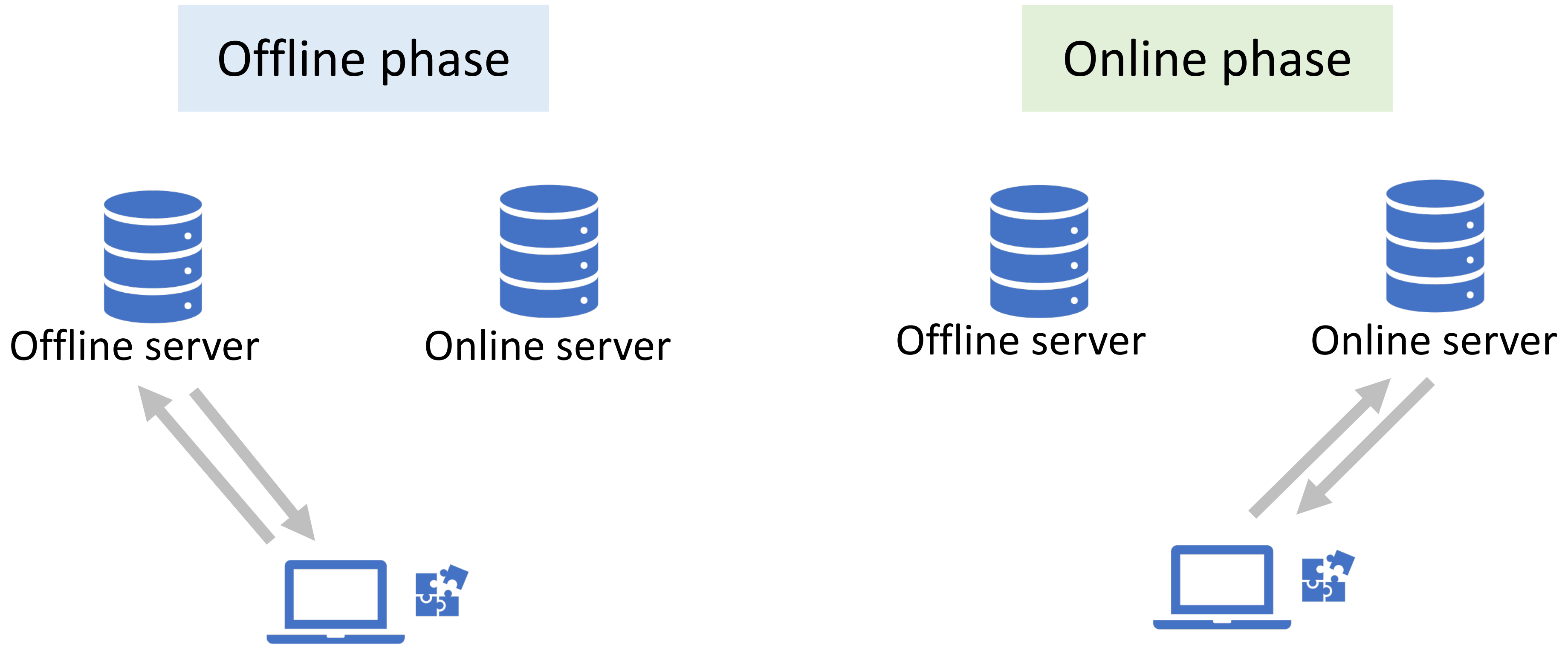

A Two-Server Offline/Online PIR [CK20]

Offline phase

Database $x = x_1 x_2 \ldots x_n$ replicated among two servers

Offline server

Online server

λ ensures the subsets cover $[n]$ with high probability.

Client generates $T = \lambda\sqrt{n}$ random subsets of $[n]$, each of size $s = \sqrt{n}$. Denote as $S_1, S_2, \ldots, S_T$.

Notation $[n] \coloneqq \{1,2,\ldots,n\}$

A Two-Server Offline/Online PIR [CK20]

Offline phase

Offline server

Online server

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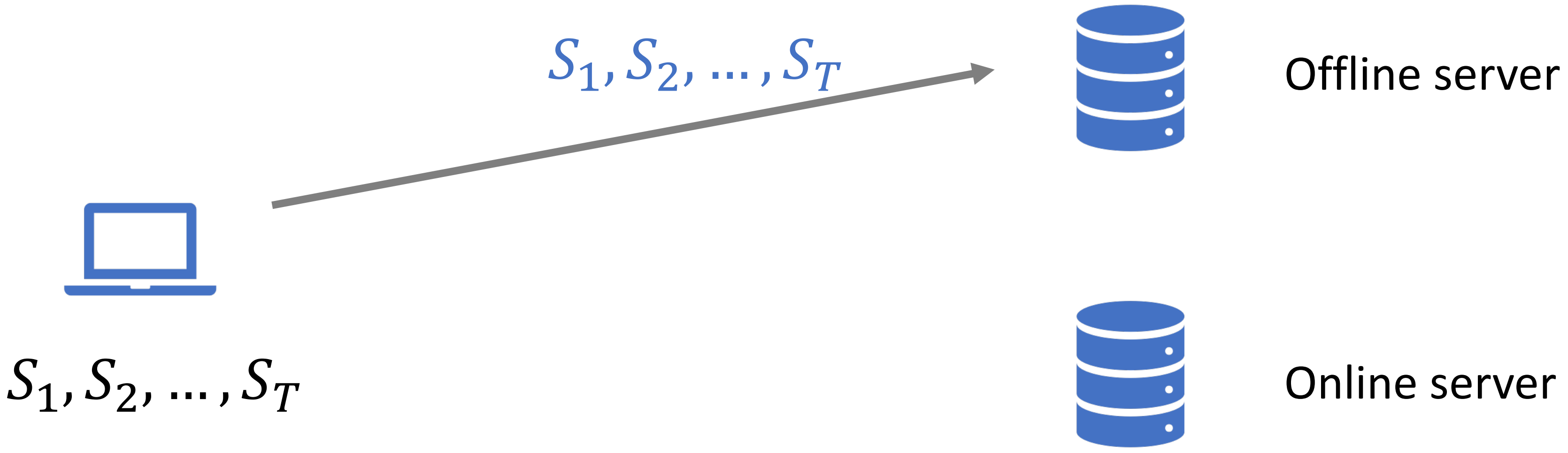
A Two-Server Offline/Online PIR [CK20]

Offline phase



A Two-Server Offline/Online PIR [CK20]

Offline phase

$k_1, k_2, \dots, k_T$

Offline server

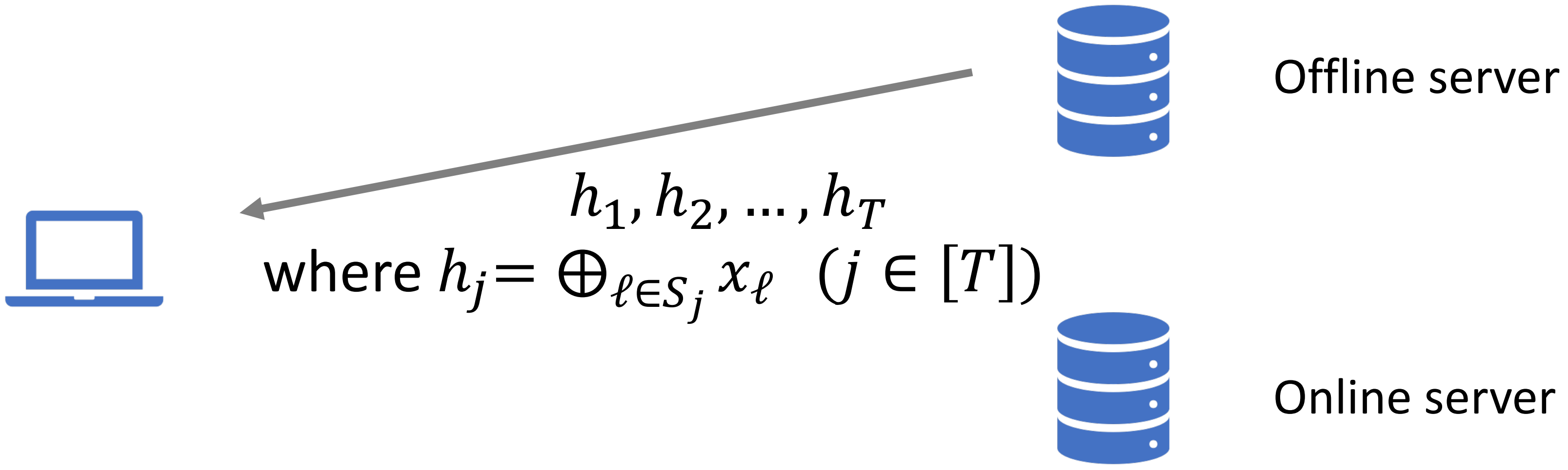
Online server

Client generates random subsets of $[n]$, $S_1, S_2, \dots, S_T$, all of size s.

$k_1, k_2, \dots, k_T$

A Two-Server Offline/Online PIR [CK20]

Offline phase



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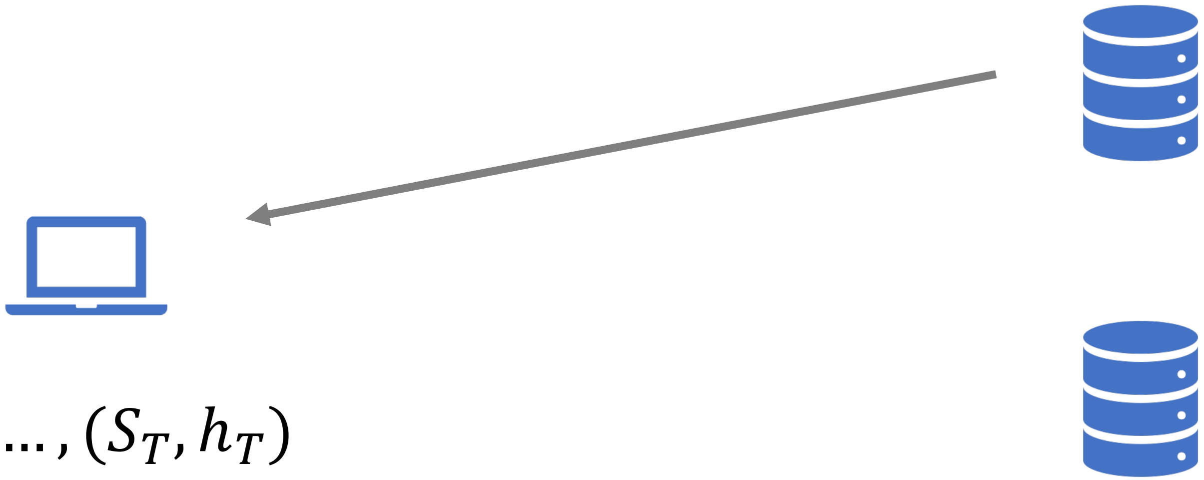
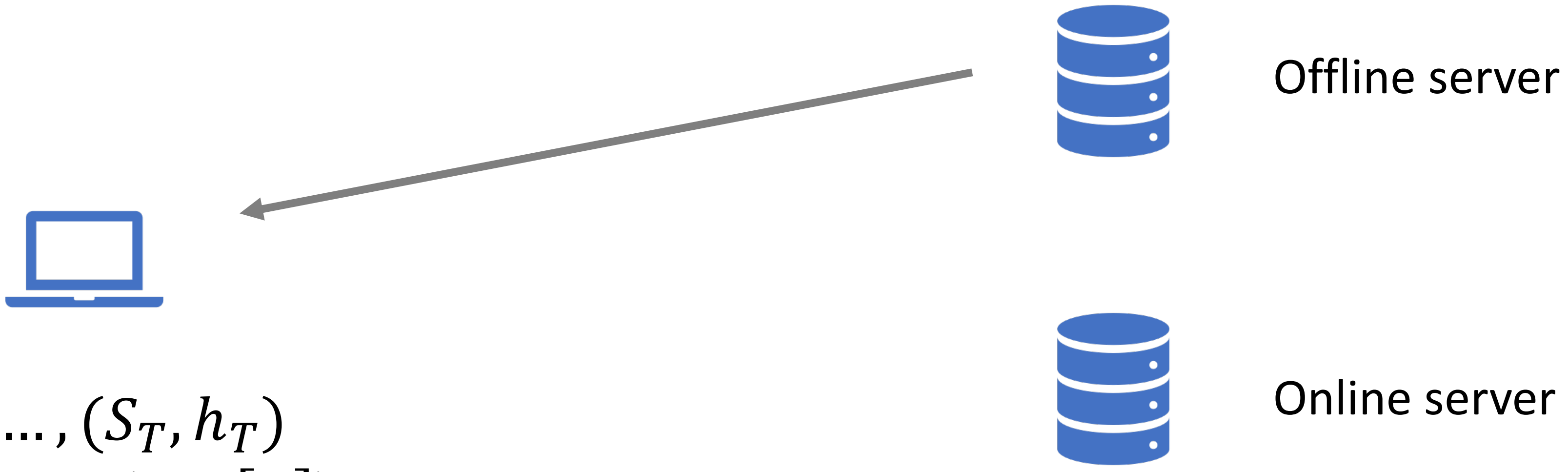


$(S_1, h_1), (S_2, h_2), \dots, (S_T, h_T)$
where $h_j = \bigoplus_{\ell \in S_j} x_\ell \quad (j \in [T])$

A Two-Server Offline/Online PIR [CK20]

Offline phase

$O(\lambda n)$ server computation
$O(\lambda\sqrt{n})$ client storage



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where $h_j = \bigoplus_{\ell \in S_j} x_\ell \quad (j \in [T])$

A Two-Server Offline/Online PIR [CK20]

Online phase

Query index $i \in [n]$: find a set S_j that contains i

Offline server

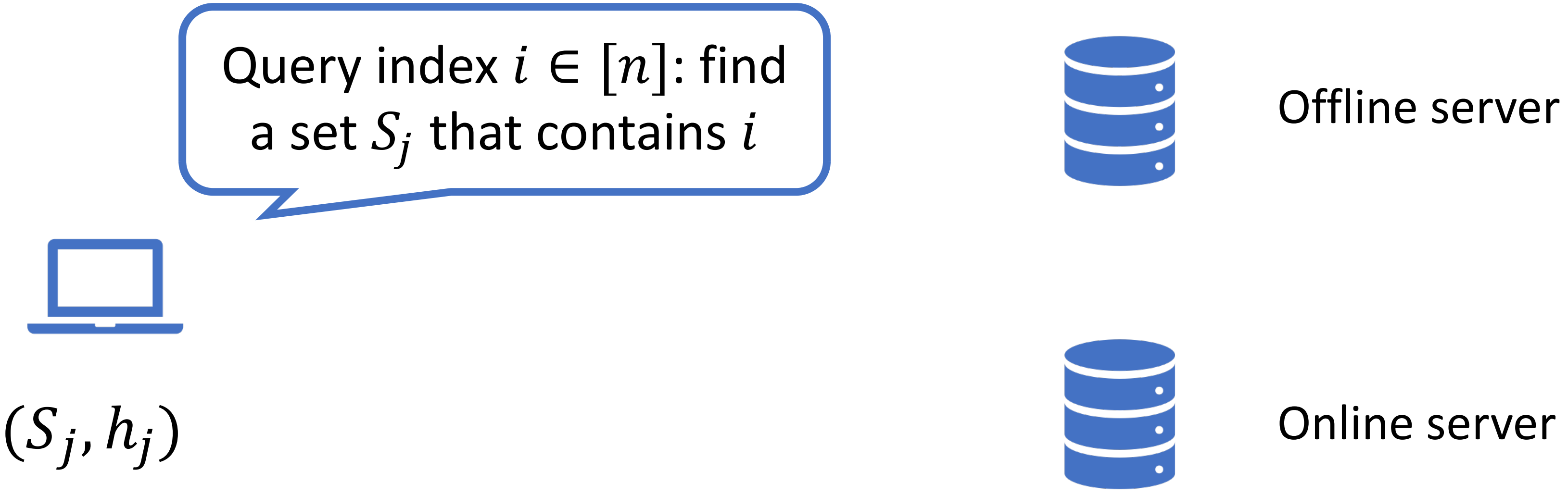
Online server

$(S_1, h_1), (S_2, h_2), \ldots, (S_T, h_T)$
where $h_j = \bigoplus_{\ell \in S_j} x_\ell \quad (j \in [T])$

Membership check: can be done efficiently via one PRP call

A Two-Server Offline/Online PIR [CK20]

Online phase



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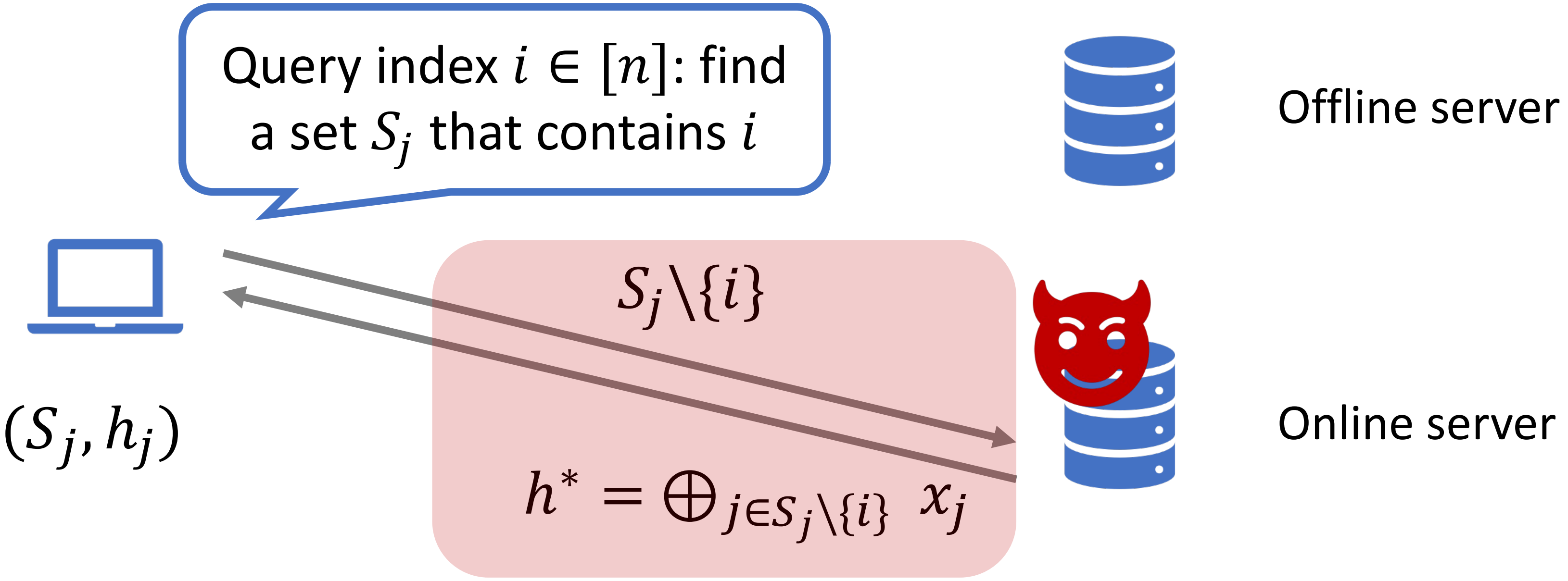
Online phase



A Two-Server Offline/Online PIR [CK20]

Online phase

Offline server

Online server

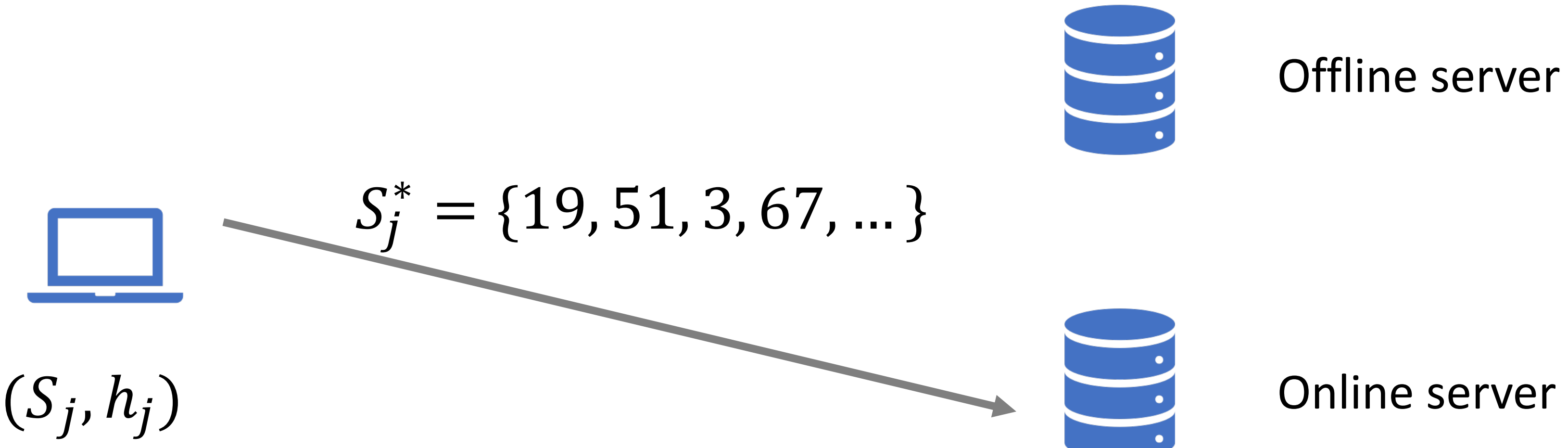
(S_j, h_j)

With *exact* probability $1 - \frac{s-1}{n}$, remove i from S_j ;
With the remaining probability, remove another random element from S_j.

$= S_j^*$

A Two-Server Offline/Online PIR [CK20]

Online phase



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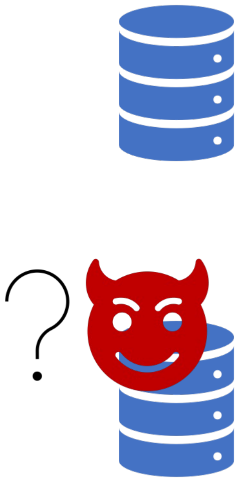
A Two-Server Offline/Online PIR [CK20]

Online phase

(S_j, h_j)

A random subset of $[n]$ with size $s - 1$

S_j^* generated from the query on i



Offline server

Online server

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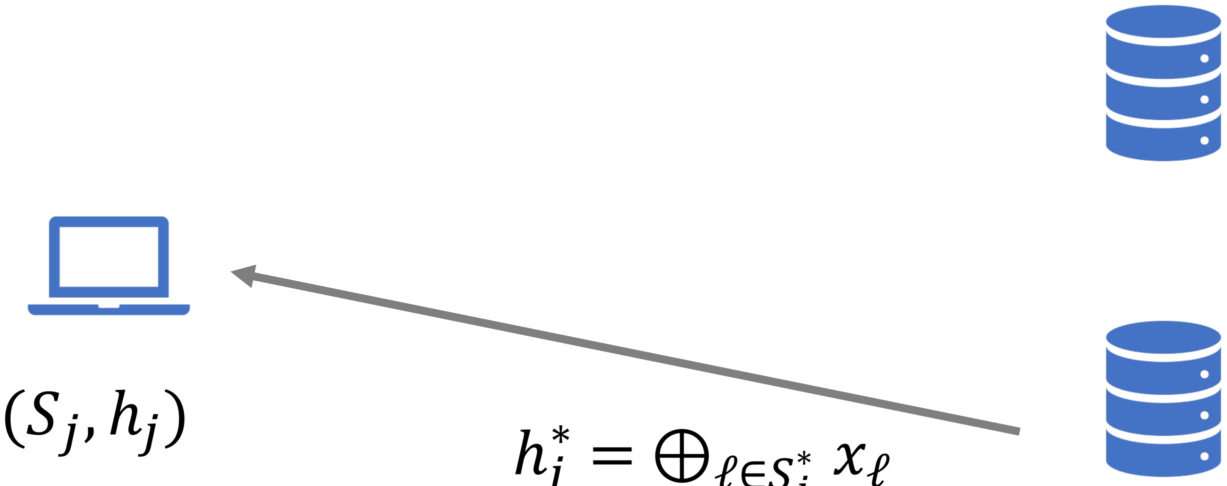
A Two-Server Offline/Online PIR [CK20]

Online phase

$O(\sqrt{n})$ server computation



Offline server

Online server

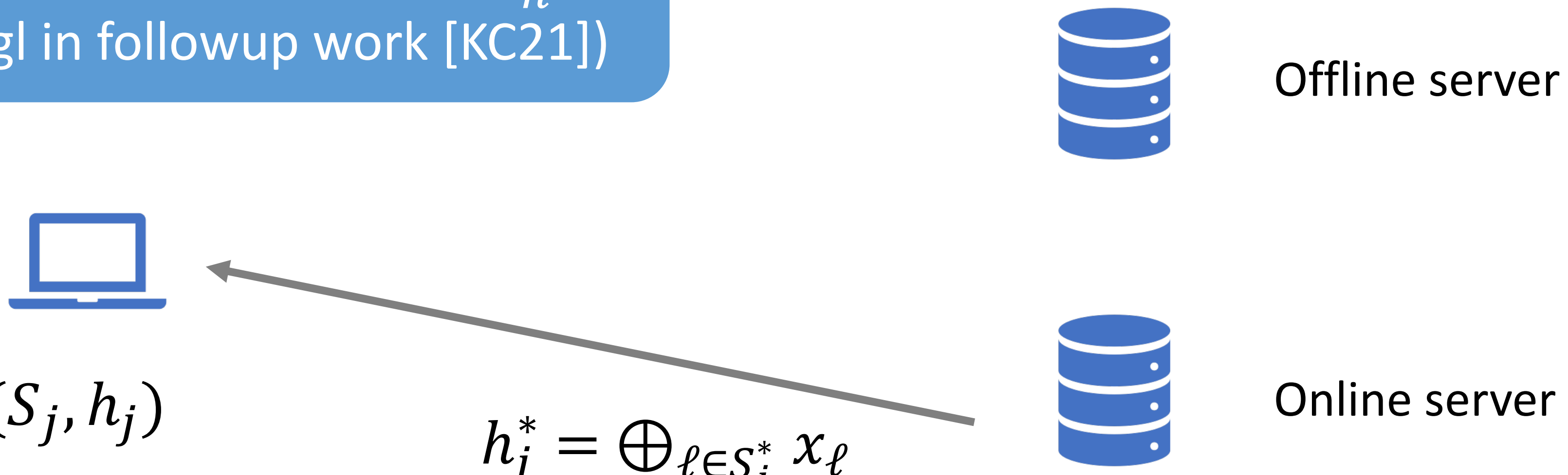
(S_j, h_j)

$h_j^* = \bigoplus_{\ell \in S_j^*} x_\ell$

Compute $x_i = h_j^* \oplus h_j$.

A Two-Server Offline/Online PIR [CK20]

Correctness: with probability $1 - \frac{s-1}{n}$
(can be made negl in followup work [KC21])



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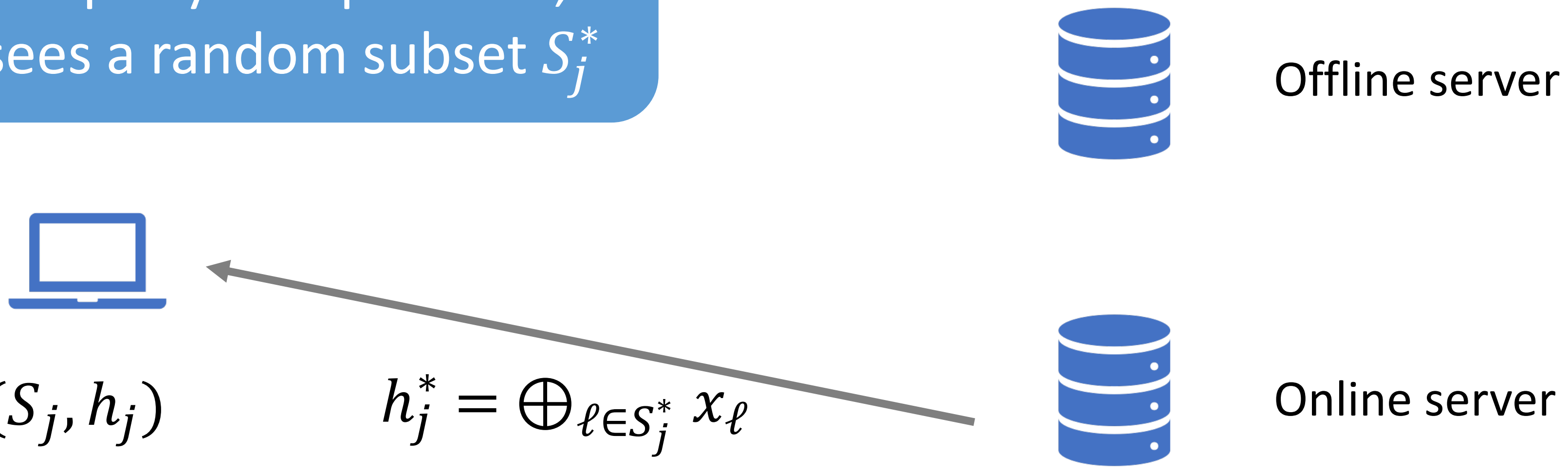
Compute $x_i = h_j^* \oplus h_j$.

A Two-Server Offline/Online PIR [CK20]

Security:
Offline phase is query-independent;
Online server sees a random subset S_j^*



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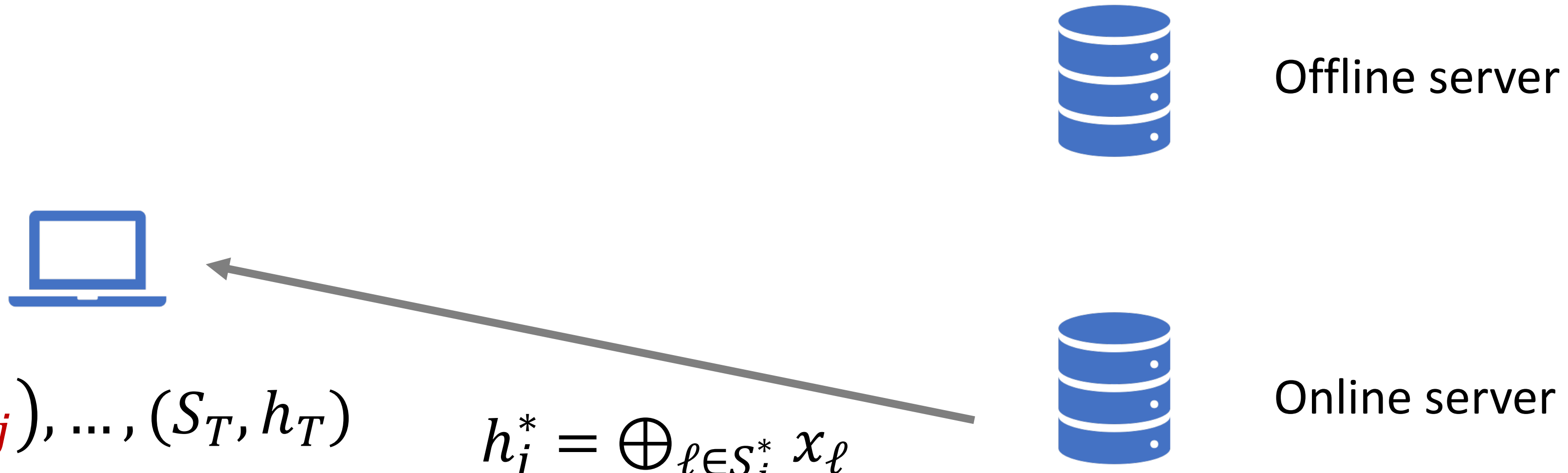


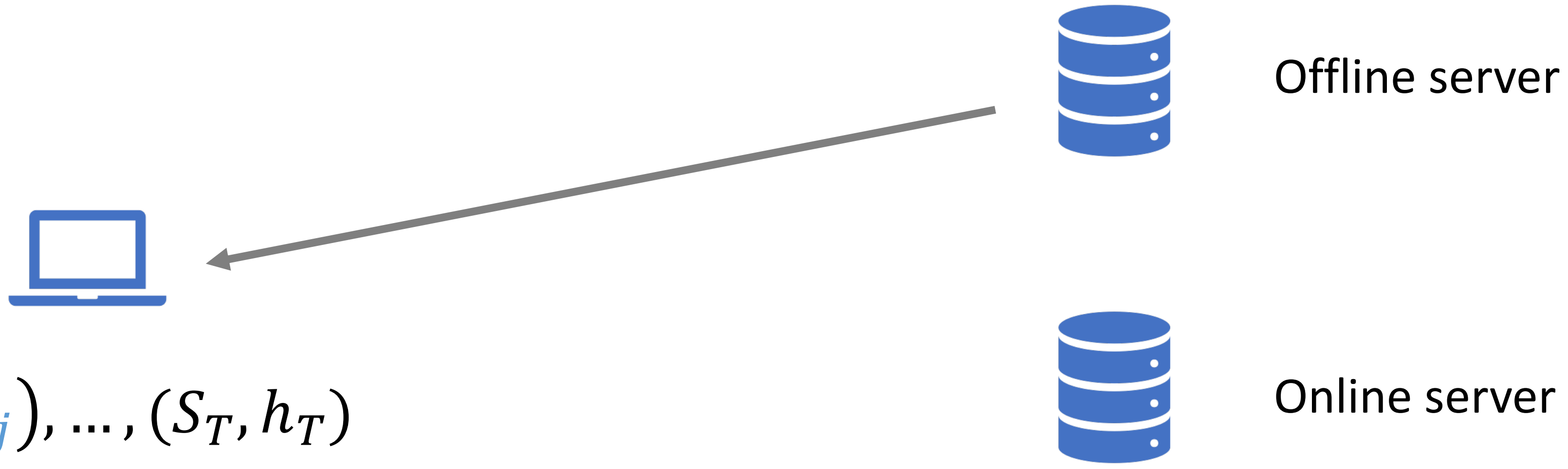
A set cannot be reused

$(S_1, h_1), \dots, (S_j, h_j), \dots, (S_T, h_T)$

Compute $x_i = h_j^* \oplus h_j$.

$h_j^* = \bigoplus_{\ell \in S_j^*} x_\ell$

A Two-Server Offline/Online PIR [CK20]



A fresh subset from offline server

$(S_1, h_1), \ldots, (S_j, h_j), \ldots, (S_T, h_T)$

Compute $x_i = h_j^* \oplus h_j$.

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- Our solution
  - Background of [CK20], [SACM21]
  - Mutable preprocessing based on [CK20]
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Types of Database Changes

- Additions: we will show next
- Deletions: more involved
- In-place edits: easy to see

Update Hints for Additions

Start with a toy version: add a single item

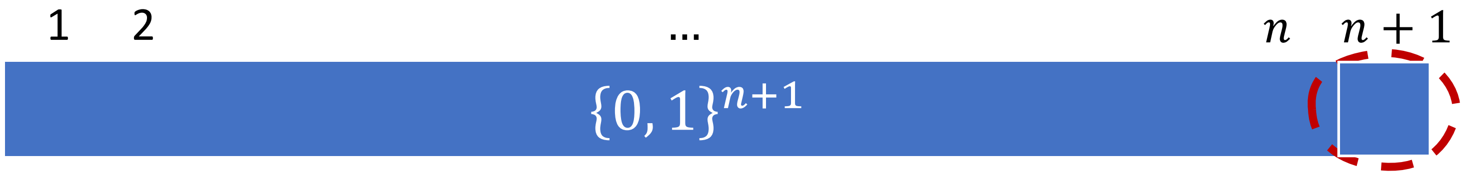
1 2 … n

$\{0,1\}^n$

$$S_1, S_2, \ldots, S_T \xleftarrow{\$} [n]$$
$$h_1, h_2, \ldots, h_T$$

Update Hints for Additions

Start with a toy version: add a single item



$$S_1, S_2, \dots, S_T \xleftarrow{\$} [n]$$
$$h_1, h_2, \dots, h_T$$

$$S_1', S_2', \dots, S_T' \xleftarrow{\$} [n+1]$$
$$h_1', h_2', \dots, h_T'$$

Update Hints for Additions

Start with a toy version: add a single item

1 2 … n $n+1$

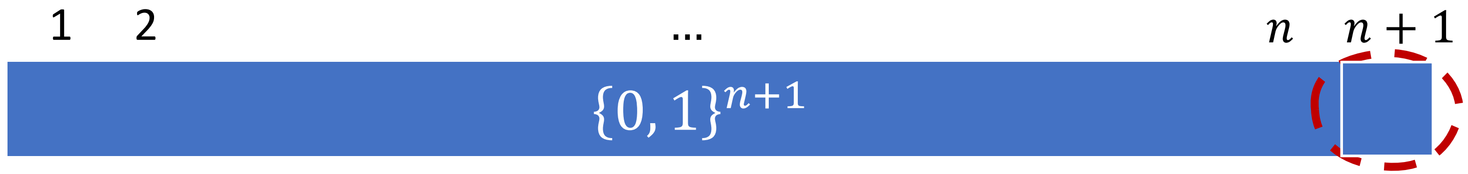
$\{0,1\}^{n+1}$

$$S_1, S_2, \dots, S_T \xleftarrow{\$} [n] \qquad S'_1, S'_2, \dots, S'_T \xleftarrow{\$} [n+1]$$
$$h_1, h_2, \dots, h_T \qquad h'_1, h'_2, \dots, h'_T$$

With small server computation

Update Hints for Additions

Start with a toy version: add a single item



$$S_1, S_2, \dots, S_T \xleftarrow{\$} [n]$$
$$h_1, h_2, \dots, h_T$$

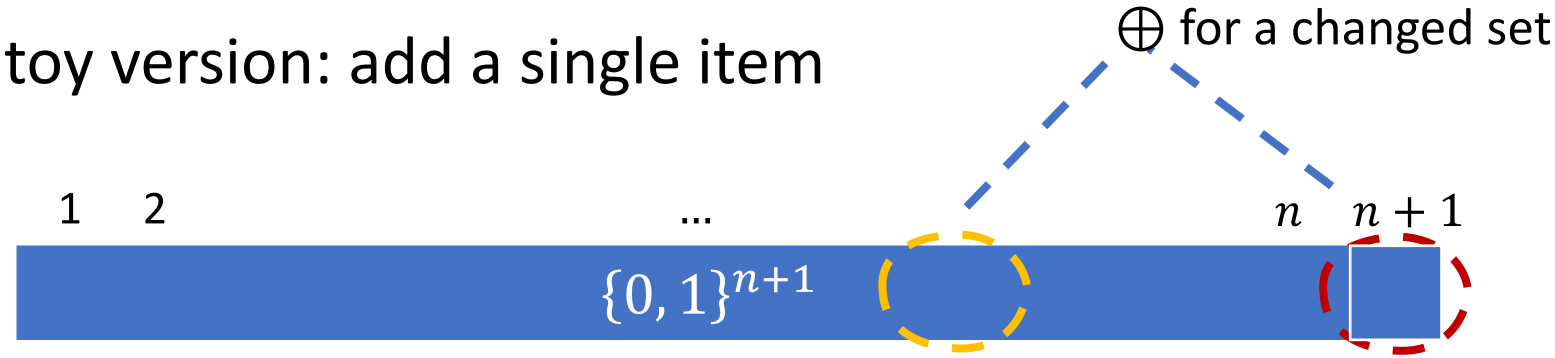
$$S'_1, S'_2, \dots, S'_T \xleftarrow{\$} [n+1]$$
$$h'_1, h'_2, \dots, h'_T$$

Client updates each S_j for $j \in [T]$:

- With probability $p = s/(n+1)$, use $n+1$ to replace a random element in S_j;
- With probability $1-p$, do nothing.

Update Hints for Additions

Start with a toy version: add a single item



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$$h'_1, h'_2, \ldots, h'_T$$

Analysis: roughly s/n portion of sets will change by one element; note that s is very small compared to n

Update Hints for Additions

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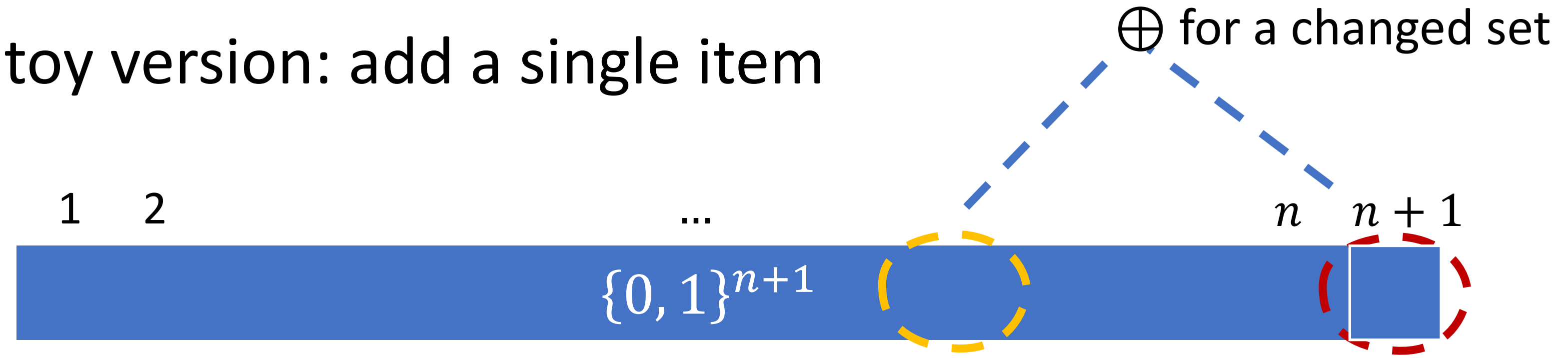

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$$h'_1, h'_2, \dots, h'_T$$

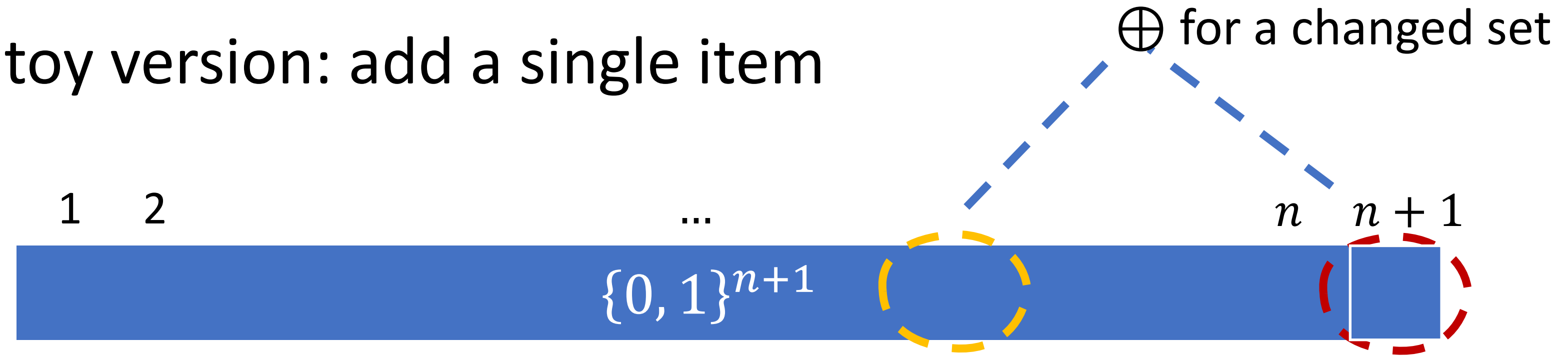
Analysis: roughly s/n portion of sets will change by one element; note that s is very small compared to n

Cost: in expectation λ XOR operations; whereas redoing preprocessing takes $O(\lambda n)$ XOR operations

Assuming $\mathrm{s} = \sqrt{n}$ and $T = \lambda\sqrt{n}$

Update Hints for Additions

Start with a toy version: add a single item



$$S_1, S_2, \dots, S_T \stackrel{\$}{\leftarrow} [n]$$
$$h_1, h_2, \dots, h_T$$

$$S'_1, S'_2, \dots, S'_T \stackrel{\$}{\leftarrow} [n+1]$$
$$h'_1, h'_2, \dots, h'_T$$

Analysis: roughly s/n portion of sets will change by one element; note that s is very small compared to n

Cost: in expectation λ XOR operations; whereas redoing preprocessing takes $O(\lambda n)$ XOR operations

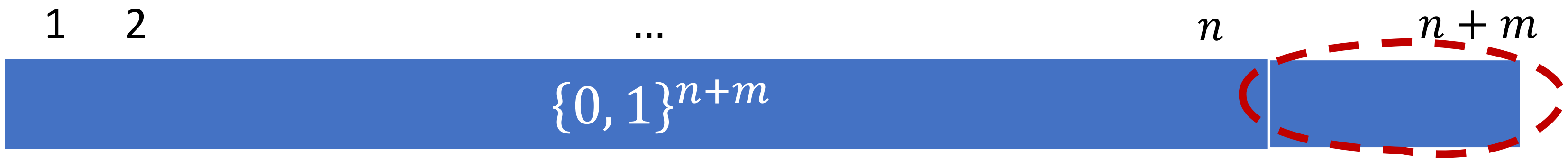
Communication?

Update Hints for Additions

- A toy version of adding a single item
- Can be extended for adding a batch of items
- Can be further extended to adding multiple batches of items

Update Hints for Additions

Adding a batch of items: recursively apply the toy approach



In expectation $O(\lambda m)$ server work

Equivalent to sampling from hypergeometric distribution
For each set S_j:

- Let $w \leftarrow HG(total = n+m,\ featured = m,\ \#samples = s)$
- Randomly choose w elements in $S_j \subset \{1, \ldots, n\}$ to be replaced with w random elements in $\{n+1, \ldots, n+m\}$

Update Hints for Additions

Reducing communication: making the key representation compatible

$$k_1, \dots, k_T \quad \rightarrow \quad S_1, S_2, \dots, S_T$$

Remember [CK20] builds pseudorandom sets from a PRP: $\mathcal{K} \times [n] \rightarrow [n]$:
A random subset in $[n]$ with size s is represented by a key $k \in \mathcal{K}$, and the set is

$$\{\mathrm{PRP}(k, 1), \mathrm{PRP}(k, 2), \dots, \mathrm{PRP}(k, s)\}$$

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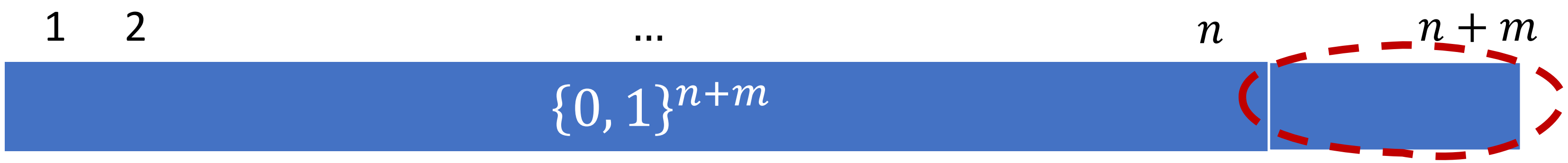
$$Set(k, n, s) \coloneqq \{\mathrm{PRP}(k, 1), \mathrm{PRP}(k, 2), \dots, \mathrm{PRP}(k, s)\}$$

Update Hints for Additions

Reducing communication: making key representation compatible



Server: $2w$ PRP calls per set
Total #PRP calls in expectation $\Theta(m)$

Client: no need to store set elements (indices) explicitly

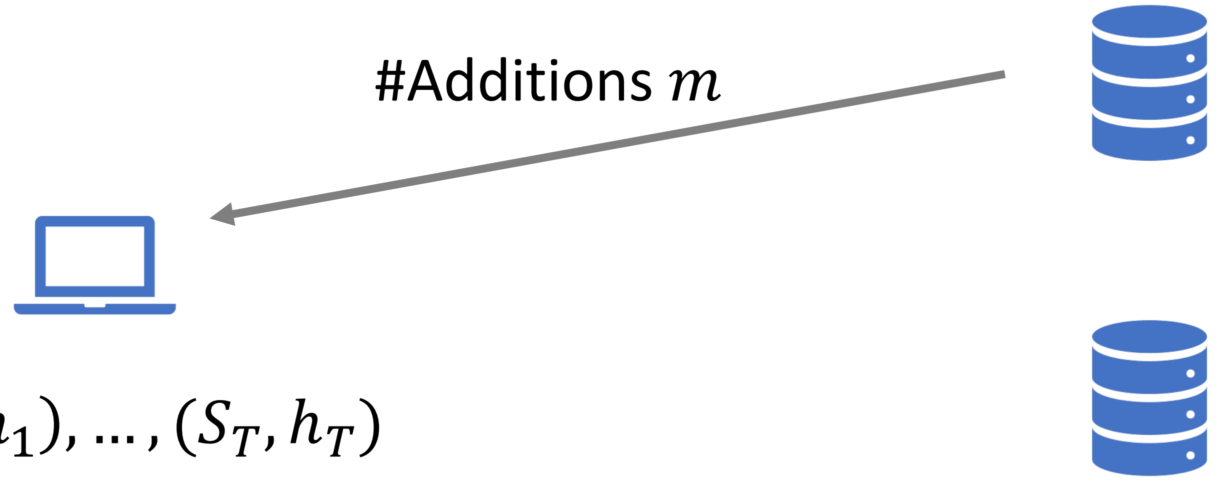
Equivalent to sampling from hypergeometric distribution
For each set S_j:

- Let $w \leftarrow HG(total = n+m,\ featured = m,\ \#samples = s)$
- Let $U^1 = Set(k, n, s-w)$ "kick out w random old elements"
- Let $U^2 = Set(k, m, w) + n$ "add w random new elements"
- Set $S_j \leftarrow U^1 \cup U^2$ "represented by "$k, s-w, w$""

Update Hints for Additions

Preprocessing for updates
“Notify”

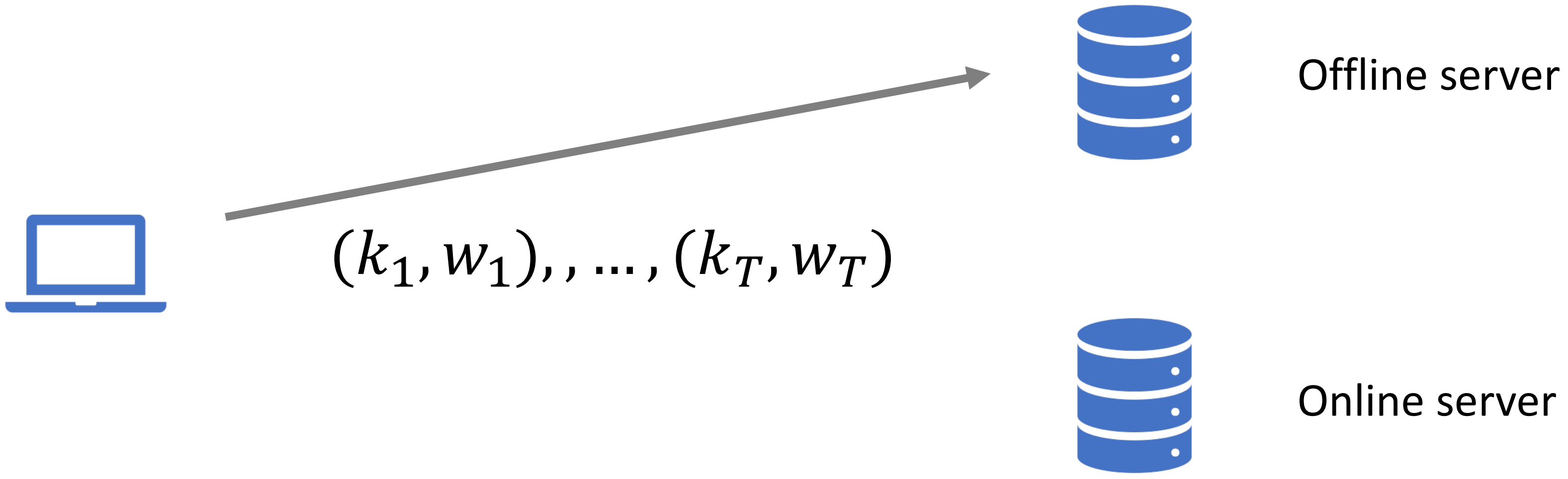
Additions happen at both servers



Existing hints $(S_1, h_1), \ldots, (S_T, h_T)$

Update Hints for Additions

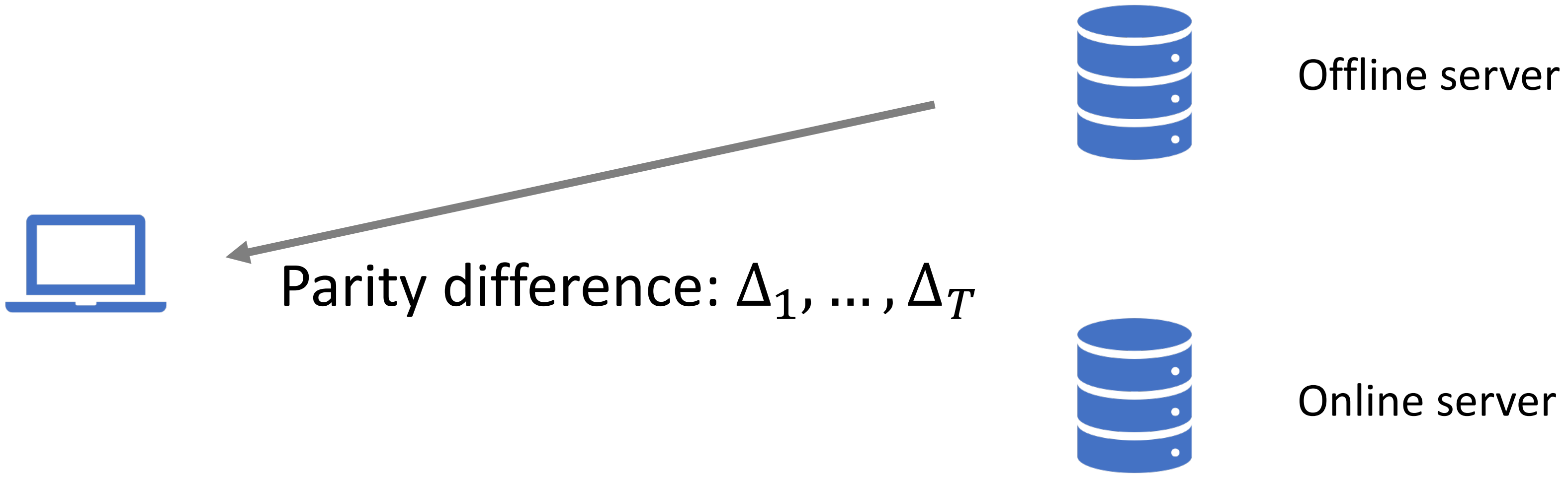
Preprocessing for updates
“Hint request”



Update Hints for Additions

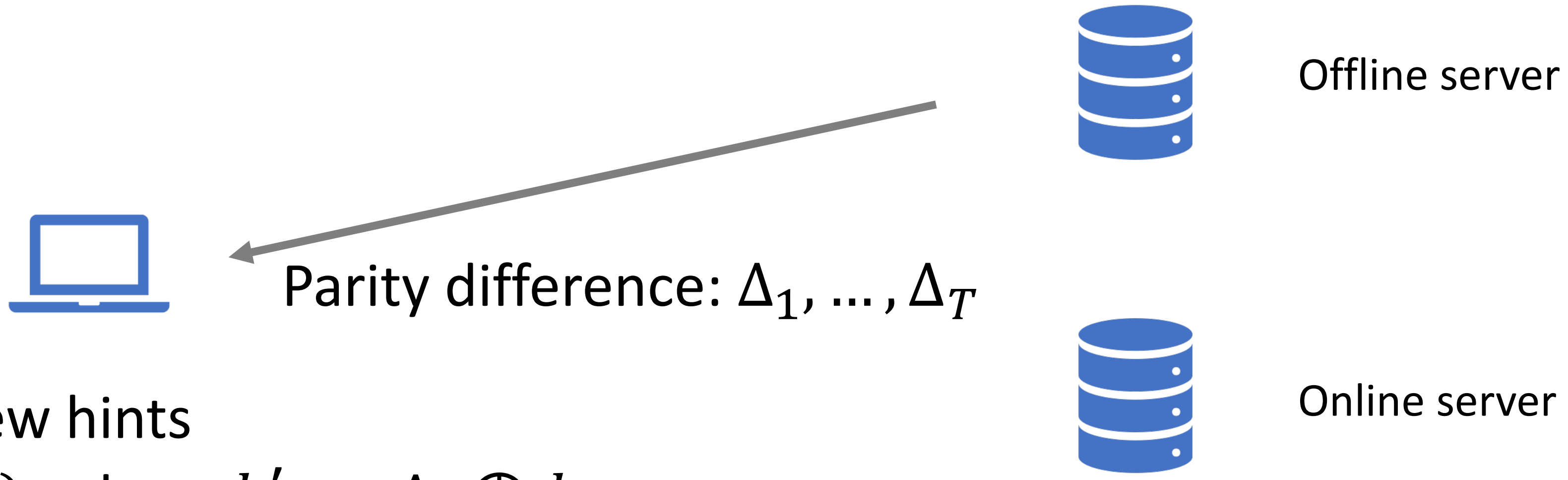
Preprocessing for updates
“Hint response”

$O(\lambda m)$ server computation



Update Hints for Additions

Preprocessing for updates
”Hint update”



Client computes new hints
$(S'_1, h'_1), \dots, (S'_T, h'_T)$ where $h'_j := \Delta_j \oplus h_j$.

Update Hints for Additions

Online phase
(minor change)

Updated database $\{0,1\}^{n'}$, where $n' = n + m$.

Offline server

Online server

Stored hints: $(S_1', h_1'), \ldots, (S_T', h_T')$

Query index $i \in [n]$: find a set S_j' that contains i

Update Hints for Additions

Online phase
(minor change)

Updated database $\{0,1\}^{n'}$, where $n' = n + m$.

Offline server

Online server

$S_j^* = \{2, 15, 9, \dots\}$

(S_j', h_j')

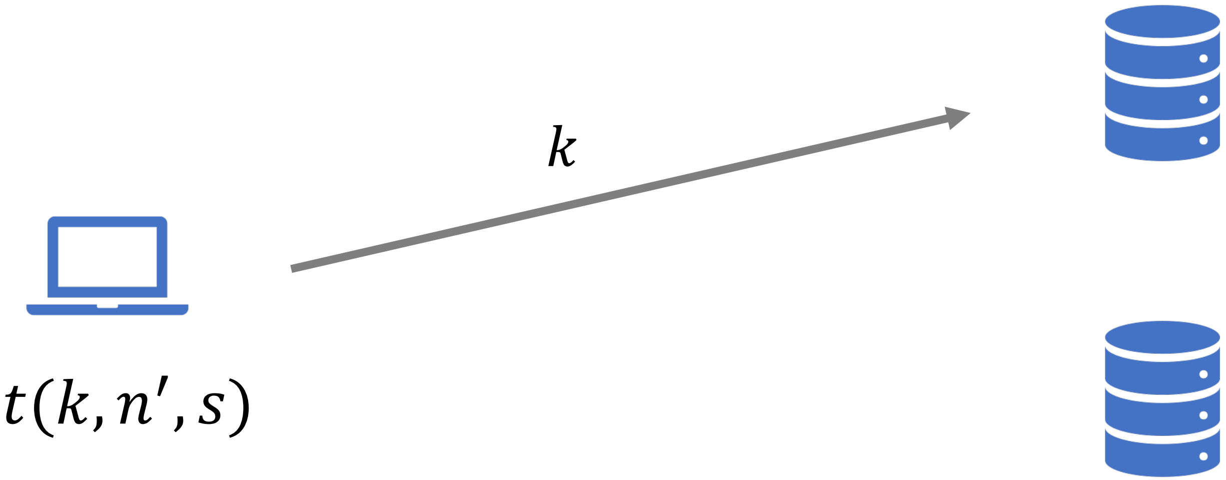
With probability $1 - \frac{s-1}{n'}$, remove i from S_j';
With the remaining probability remove another random element from S_j'.

$= S_j^*$

Update Hints for Additions

Online phase
(refresh)

Updated database $\{0,1\}^{n'}$, where $n' = n + m$.



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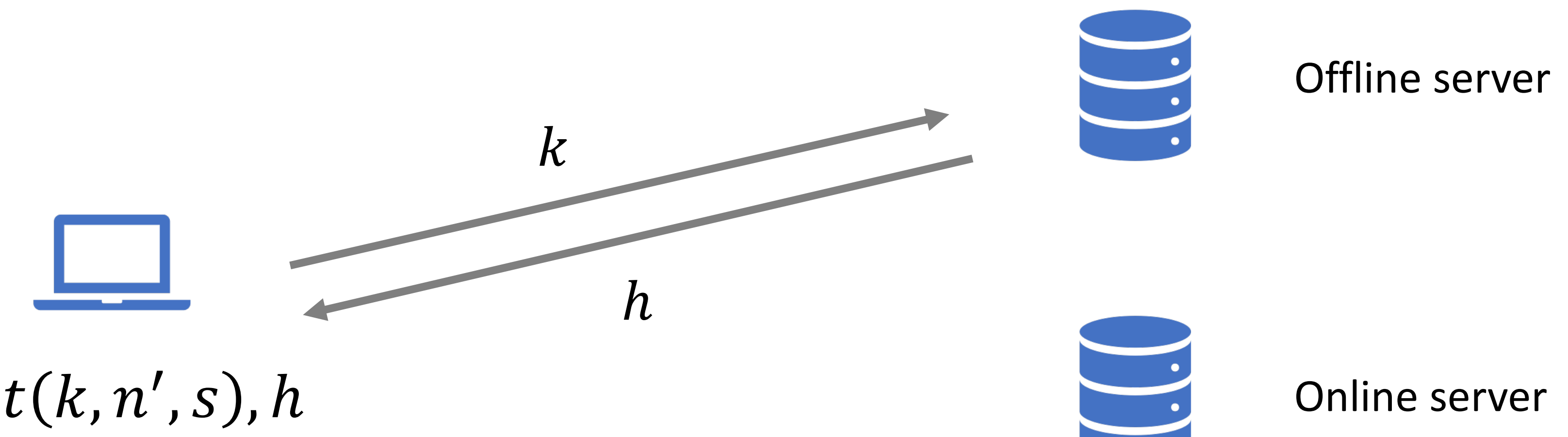

More Technical Details

- Supporting multiple batches of additions
- Supporting in-place edits
- Deletions: cannot actually delete when hints are stored at the client (if client is malicious)

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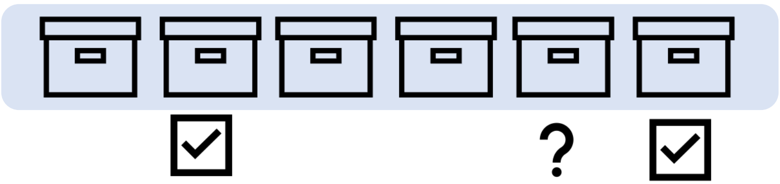
PIR without hints



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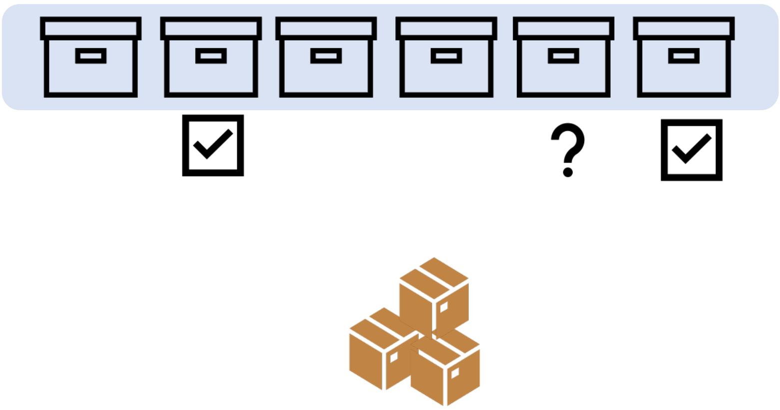

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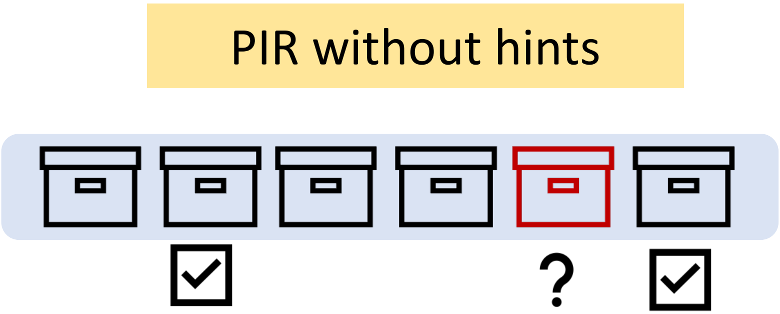

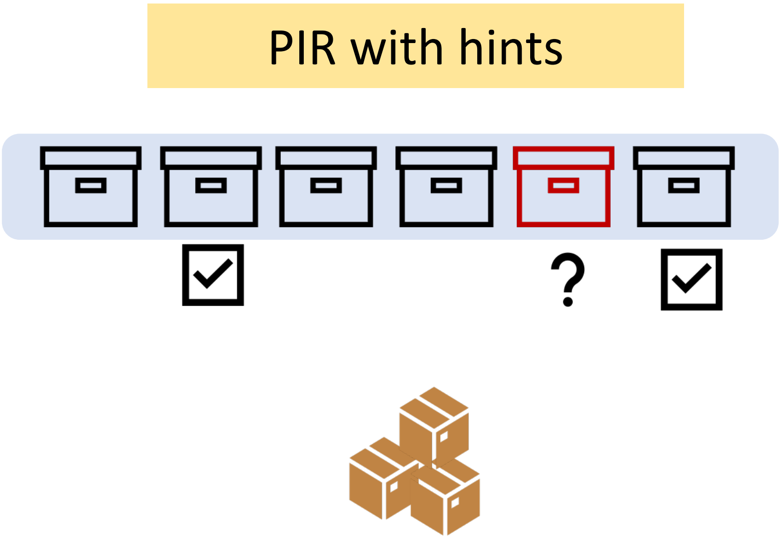

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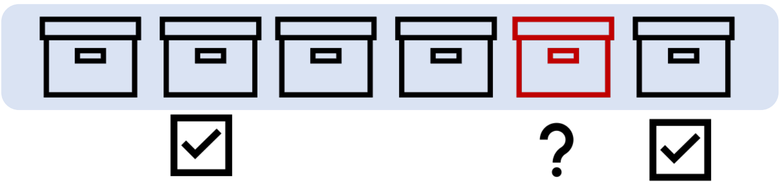

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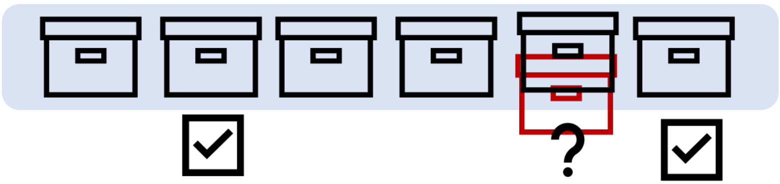


High-level Ideas in [SACM21]

- Instead of sampling a set with fixed size, sample each element into a set with some probability p
- The set size in expectation is np

Mutable Preprocessing from [SACM21]

- Instead of sampling a set with fixed size, sample each element into a set with some probability p
- The set size in expectation is np
- When a new item is added (hence a new index), for each set, sample the new index into the set with probability p

Update sets in a way compatible with the key representation

Independent work

- [KC21, Checklist]
  - Dynamic data structure, black-box construction
  - Amortize the cost over multiple added items
- Ours: make the hints mutable
  - Utilize features of specific protocols
- Depending on concrete parameters (frequency of updates, item size, etc.), provides different benefits

Evaluation: Microbenchmarks

How does our construction save **server cost**?

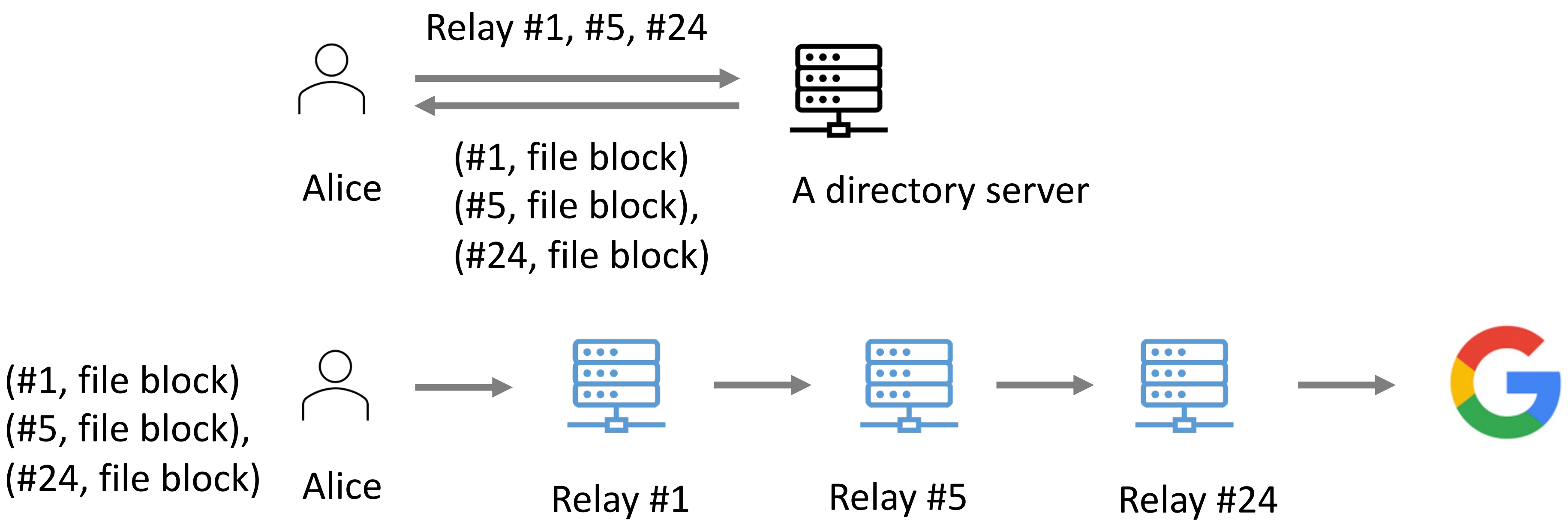
Results for adding 1% data:

Database* size	2^{16}	2^{18}	2^{20}
Initial preprocessing (sec)	3.64	14.52	58.67
Update hints (sec)	0.07	0.25	1.03

*Each data item 32 bytes. Run on a machine with 2 GHz processor and 64 GB RAM, single thread

Evaluation: PIR-Tor Application

- Retrieving relay description files from Tor directory servers
- Why PIR? [PIR-Tor, Sec11]



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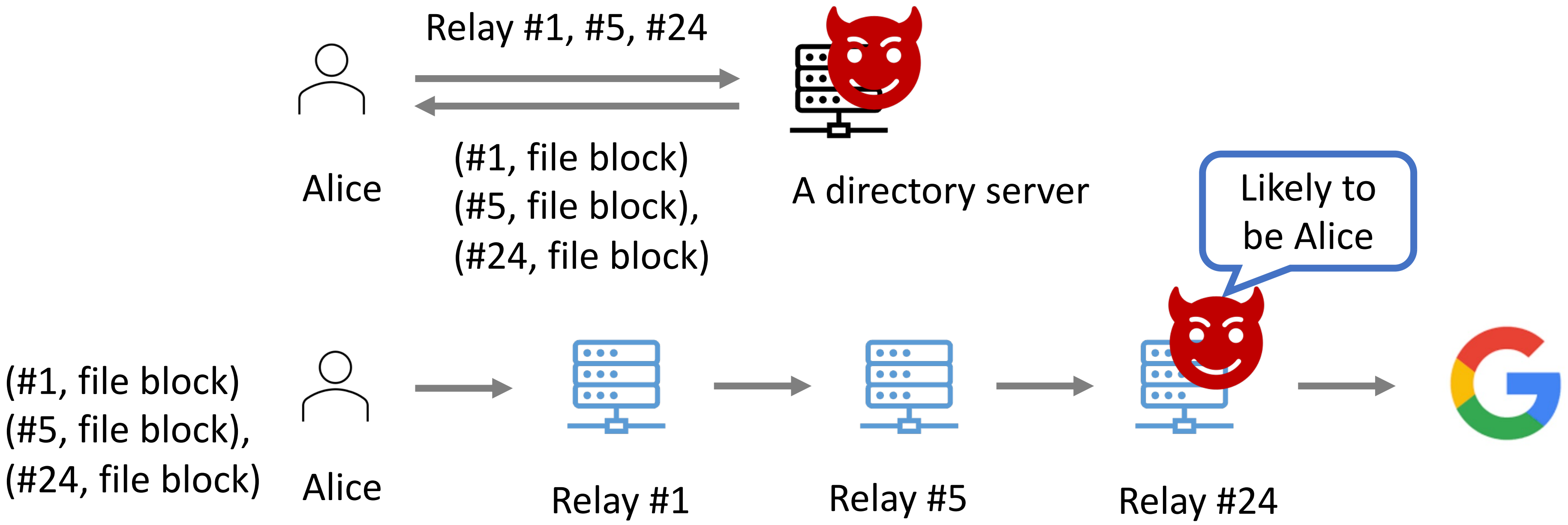

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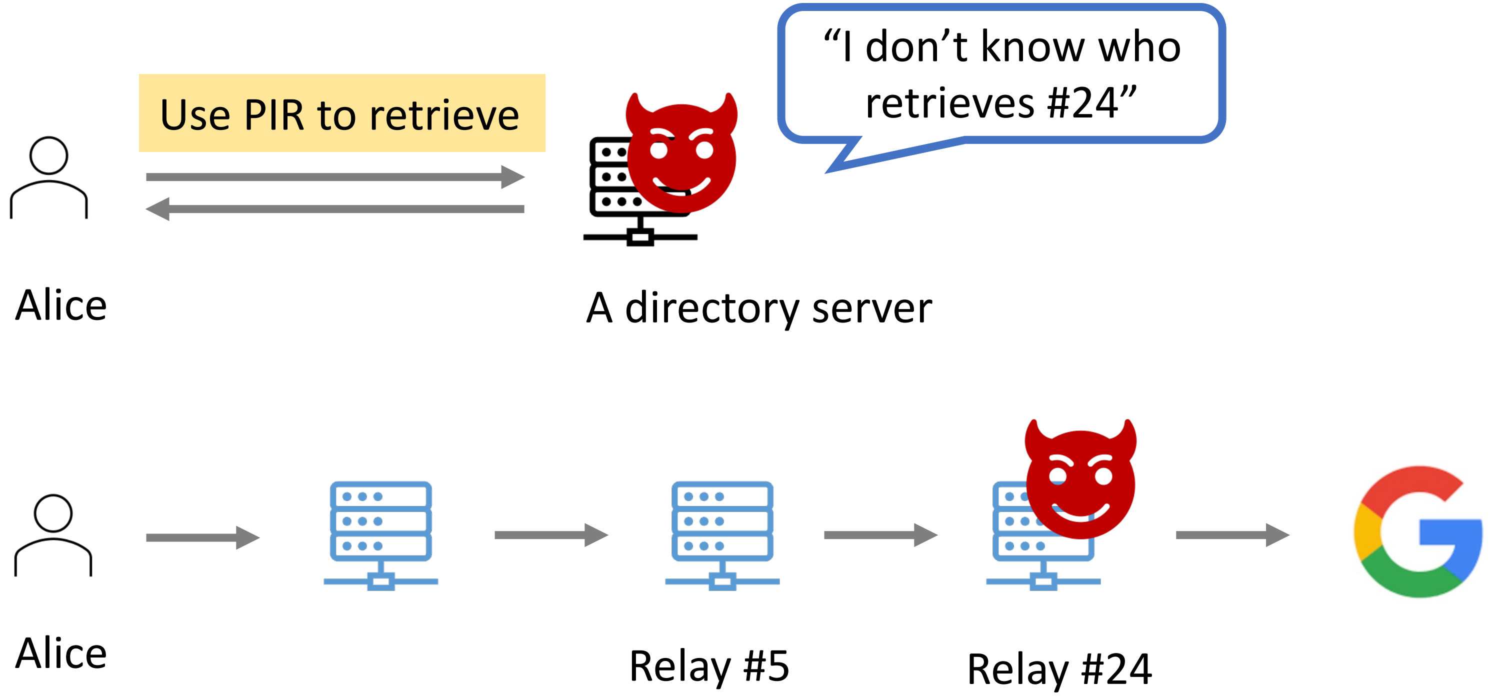

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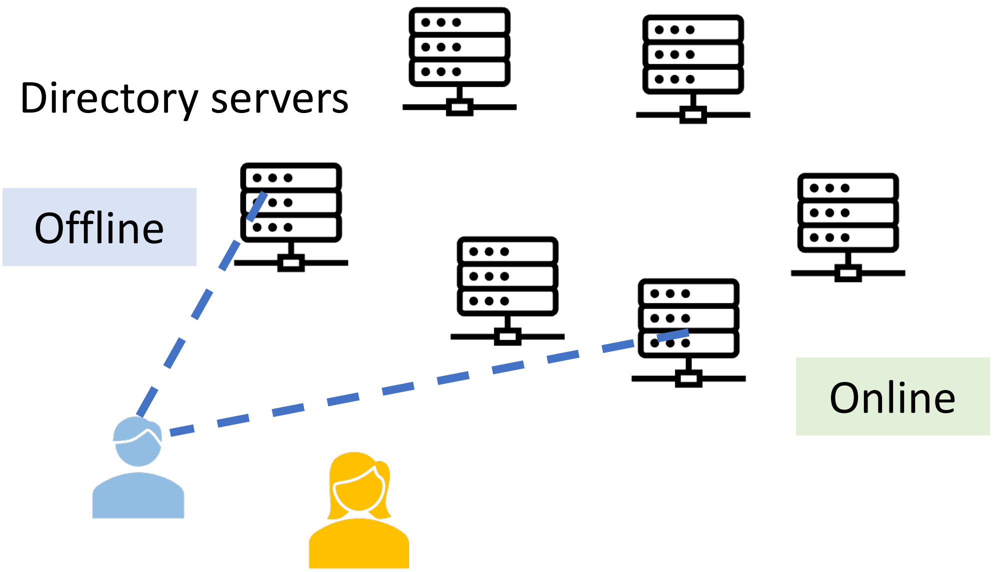
- A server could act as the offline server for one client; and the online server for another client
- Updates are propagated to all the servers



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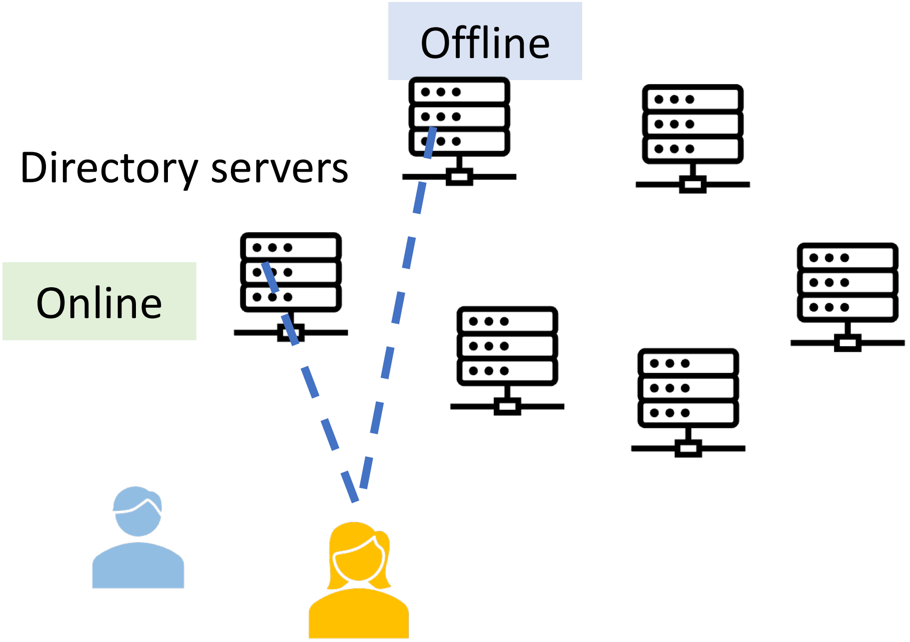

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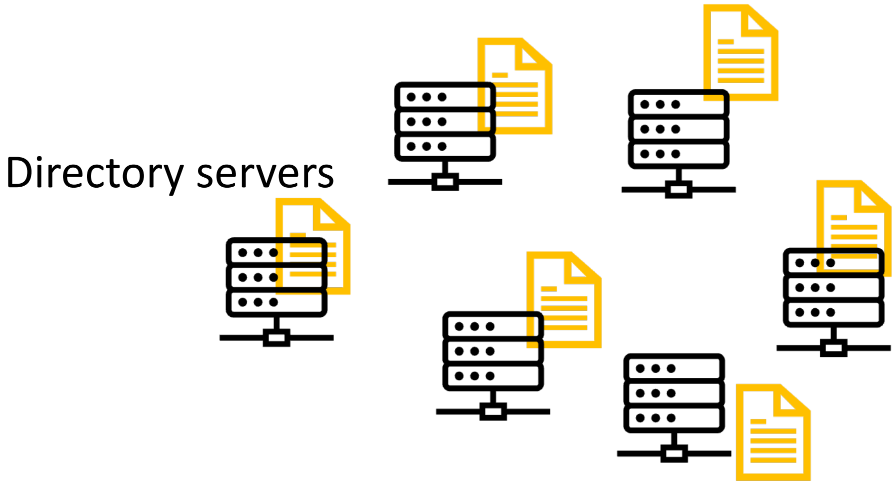

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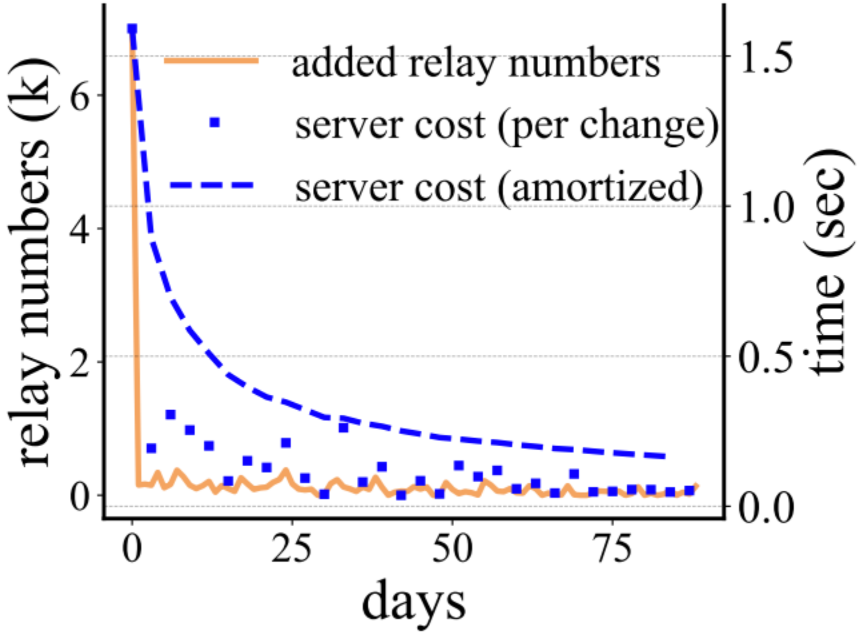
Directory servers



Thank you!